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Validation of foam filled tube model and optimization result using Weibull distribution

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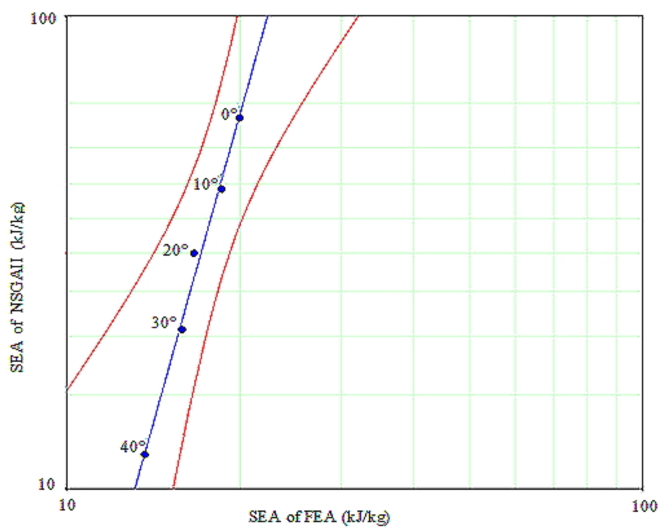
Abstract

Passenger safety and low fuel consumption rate are the most important factors that need to be considered when designing modern transportations. This study validates the crash behavior and optimum values of foam-filled structures under the dynamic oblique impact using the Weibull distribution. The optimization method aimed to absorb maximum energy with minimum peak crushing force. Furthermore, the metamodel and optimization techniques such as RBF and NSGA-II were used to ensure accurate validation of the Weibull distribution method. The result showed that the finite element model is comparable to the experimental data in the reference, while the metamodel method, which is directly verified, affects optimization results. The Weibull distribution method shows that the optimum value and the simulation have good accuracy or R^2> 0.85.

Keywords

crashworthiness, validation, optimization, Weibull distribution

Graphical Abstract



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1 INTRODUCTION

Thin-walled structure in the transportation industry has been widely used as an energy absorber for crashworthiness. It is recommended to protect passengers from severe injuries and death (Sun et al. 2010). The main features of this structure are affordable, lightweight, energy absorbers (Li et al. 2014). Certain research analysts have thoroughly investigated thin-walled circular tubes under axial loading. Baroutaji et al. (2015), researched the behavior, effects of impact velocity, and characteristics of tubular energy absorbers. In their study, mathematical equations were derived from numerical methods and validated through experiments. The results from this study are intended to predict the conductivity of crushed circular tube during lateral loading. In order to achieve this design, foams have demonstrated an excellent ability to absorb energy as it undergoes large deformation at constant loading (Zheng et al., 2014). Foam-filled thin-wall structure improves crushing stability and collapse mode as well as its crashworthiness capability. According to research carried out on the energy absorption of a thin-walled circular tube, it was discovered that different foam types nonuniform deformation and densities (Kavi, Tasoy & Guden 2006). Zhang et al. (2012) studied the foam filled thin-walled column structure design. The use of foam as a filler boosts the structure's crashworthiness capacity without altering its weight, thereby improving its effectiveness on the vehicle body. The double structure profile consists of two tubes with uniform cross-sections and concentrically placed with or without a foam filler (Djamaluddin et al., 2018). Experimental studies and numerical methods of this topology are carried out in a three-point bending test (Guo & Yu 2011) and conditions under static quasi-binding loading (Li, Yu & Guo 2012). In addition, Li et al. (2013) researched the bending behavior of a thin, hollow walled tube by applying the three-point dynamics experiment.

The thin-walled structure's energy absorption capacity is heavily influenced by design parameters such as size, shape, and mechanical properties of the materials. Therefore, it is necessary to determine their optimum values. The metamodel method algorithm represents input and output relationships based on finite element analysis (Hou et al. 2008) and (Djamaluddin et al., 2015). This technique is an effective tool commonly used to construct an approximation function for crashworthiness design and aids engineers in achieving optimum parameters of foam-filled thin-walled structure (Fang et al., 2014). According to Bi et al. (2010), optimized foam-filled columns with respect to tubular geometry and different foam densities was carried out to achieve the maximum SEA and Mean Crushing Force (MCF), (Djamaluddin et al., 2016) (Djamaluddin et al., 2019). The optimization results show that SEA increases in both single-cell and three-cell tubes due to the thickness of the tubular walls and foam densities. In accordance with previous parametric studies, Song et al. (2013) carried out a research on crashworthiness optimization of foam-filled tapered tubes using metamodel techniques and variables such as tubular wall thickness, tapered angle and foam density.

Furthermore, research analysts tend to focus on the Multi-Objective Optimization (MOO) of a thin-walled tube filled with foam. Hou et al. (2009) reviewed a rectangular tube-shaped design in accordance with its energy absorption and peak crushing strength using the MOO method. Zhang et al. (2012) researched the crashworthiness design of double-sided rectangular tubes. The MOO method shows that the foam-filled double structure has a more effective crashworthiness capacity than the single design. Yang and Qi (2013) also optimized the crashworthiness of empty and foam-filled square tubes under oblique loading. They discovered that foam-filled tubes have a more effective crashworthiness capacity under the pure axial loading, than the empty tubes under oblique loading. However, different approaches existent in mathematical programming such as genetic algorithm (GA), Non-dominated Sorting Genetic Algorithm version II (NSGA II), and Particle Swarm Optimization (PSO) have been used to resolve crashworthiness optimization problems for different structures.

Validation of finite element models compared with experiments from references using the Weibull distribution. Furthermore, the Weibull distribution is used because it can work with small samples and provide to plot graphs easily from data (Barabadi 2013). The parameters used are the peak crushing force and specific absorption energy with different loading angles for validation of optimization results.

2 METHODS AND MATERIALS

Figure 1 shows the geometric configuration of the model. The profile of the double structure consists of two tubes which have uniform cross-sections and are concentrically placed (Goel 2015). The purpose of this study is to investigate the energy absorption capability of double tubes as opposed to single foam tubes.

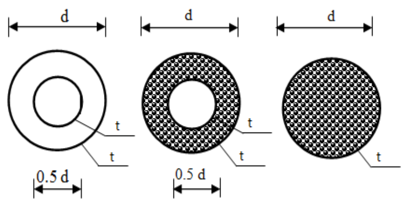


Figure 1: double tube configurations: (a) empty double tubes (ED), (b) double foam filled tubes (FD), (c) single foam filled tube (SF)

This study reported that the double circular composite tubes configuration are subjected to dynamic loading angles of 0°, 10°, 20°, 30°, and 40°. In the structural simulation of finite element models, each tube's foundation is completely assembled to rigid walls, which serve as boundary constraints. This support is constrained until the tube is immobilized, and a rigid wall fence destroys its top. Figure 2 shows the geometric schematic of the tubes with length (l) 200 mm (Yang & Qi 2013). The structure has uniform dimensions with the experimental specimens of MTS810 machines (Li, Yu & Guo 2012). This schematic has been used by other research analysts to simulate elliptical tubes under oblique loading (Gao et al., 2016). In this section, the effects of energy absorption and deformation mode are determined by the structure under dynamic oblique loading. Tube modeling for energy-absorbers such as front-side rails of the passenger vehicles is completely clamped (Kilicaslan 2015). The upper end of the tubing in the rigid wall impact has a constant velocity (v_o) of 10 m / s (Hu et al., 2017) and an oblique dot load angle of 0°, 10°, 20°, 30° and 40° (Gao et al. 2016).

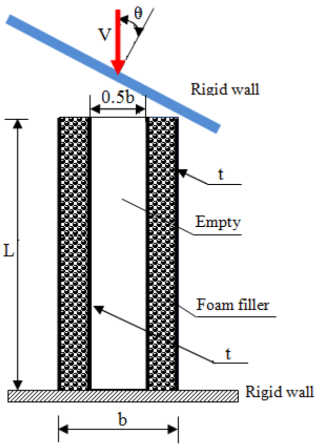


Figure 2: Schematic of structures

The analysis of finite element models is carried out using ABAQUS. This section involves modeling the three types of tubular structures, namely empty tubes, foam-filled double, and single tubes, as shown in Figure 1. The geometry of the double structure has a wall thickness (t) of 1.5 mm (Guo & Yu 2011), and the diameters of the outer and inner tubes are 38 mm and 24 mm, respectively (Li, Yu & Guo 2012). Subsequently, its foam has a density of 5.34×10^{-7} kg / mm^3, in accordance with the experimental data reported by Alulight International Germany (Ahmad, Thambiratnam & Tan 2010). The input data is selected because it is used to develop a finite element model of the foam. The geometric parameters of the tubes under the oblique dynamics loading are consistent with the specimen from ASTM E208 (Fang et al. 2015). The ABAQUS software was used to develop the foam-filled tubular wall model and to predict the response of thin-walled tubes against a rigid wall, as shown in Figure 2. The entire model in this study consists of thin-walled tubes, foam, and the wall's upper and lower surfaces. Furthermore, the tube wall and foam are modeled with 3D Deformable Shell Extrusion and the 3D Deformable Solid Extrusion, respectively. The upper (impactor) and lower (base tube) surfaces of the rigid walls are modeled using the 3D Deformable Shell Planar plane. The thin-walled structure is modeled using a four-node shell element and five-point integration along with its thickness (Tarlochan et al., 2013). It is also modeled using linear elastoplastic materials in accordance with the criteria of Von Mises (Meguid, Attia & Monfort, 2004). The Belytschko-Lin-Tsay four-node shell element and five-point integration along its thickness to develop the model.

The detailed parameters of the foam material used in the simulation are shown in table 1 and 2. Figure 2 represents the three-dimensional modeling developed based on the conditions of oblique loading reported by other research analysts (Gao et al., 2016). The boundary conditions of the foam and base of the tube are translational shifts and rotational degrees of freedom (Gao et al., 2016). The model was analyzed and refined to ensure the simulation is perfect. The size of elemental mesh was 2 mm for both shell and foam models. They were selected based on the results from convergence studies to ensure accuracy during the deformation process (Tarlochan et al., 2013). The contact algorithm is used to simulate the relationship between all components. It aids in the deformation of the structure during contact with rigid and foam walls; however, this algorithm's duration is short. According to Tarlochan et al. (2013), the interaction between the tube wall, with or without foam filler, is modeled as an infinite slide-contact algorithm. The frictional coefficient between the tubes and foam walls under static and dynamic quasi-loading is 0.3 and 0.2, respectively. On the contrary, a frictionless relationship exists between the tubes and rigid walls (Ahmad, Thambiratnam & Tan 2010).

Table 1 Mechanical properties of aluminum alloy 6063 T6

Parameters	Values
Density (ρ)	2.7 × 10^{-6} kg/mm^3
Young Modulus (E)	68200 MPa
Poisson Ratio (v)	0.3
Yield Stress (σ_y)	184.4 MPa
Maximum Stress (σ_u)	215.5 MPa

Table 2. The foam material parameters for the simulation

Parameters	Values
Foam Density (ρ_f)	5.34 × 10^{-7} kg/mm^3
Young Modulus (E)	625 MPa
Poisson Ratio (v)	0.1
Yield Stress (σ_y)	1.8 MPa

The optimization methodology is proposed based on three levels, as shown in Figure 3. In the first stage, the nonlinear finite element analysis is carried out on a double circular tube model under dynamic loading conditions. The initial aspect of the optimization procedure was developed based on the D-Optimal design experiment (Zhang et al., 2014) and the RBF metamodel technique (Gao et al., 2016). The input data is determined by considering the design variable of the DOE sampling point and the constraint value. Metamodel techniques are applied in order to avoid complexity because it leads to multi-objective optimization (Yin et al., 2014a). RBF is used to determine the objective function of this study because it has been proved effective in resolving optimization problems (Fang et al., 2014). In addition, the D-Optimal design is applied to produce sample points in the domain RBF function. However, its developed model needs to be verified because the Metamodel technique has a direct impact on the optimization outcome. The RBF model needs to be accurate and validated by selecting a sample point (Yang & Qi, 2013). One of the parameters for determining the reliability of the model and finite element of SEA and PCF reactions is the relative error value (Li et al. 2015). Objective optimization methods applied to NSGA II and Pareto front. Subsequently, the D-Optimal experimental design was selected because it produced accurate metamodel outcomes (Zhang et al., 2014). The RBF technique was chosen because it showed a better precision than the other metamodels (Yin et al., 2014b). It has been used for the optimization of previous crashworthiness structures (Fang et al. 2015). NSGA II is also used in this research because it has an efficient and effective algorithm and a fitness function for optimization problems (Zheng et al., 2014). RBF has been used for several purposes, such as objective function approximation (Zhang, Zhang & Wang 2016).

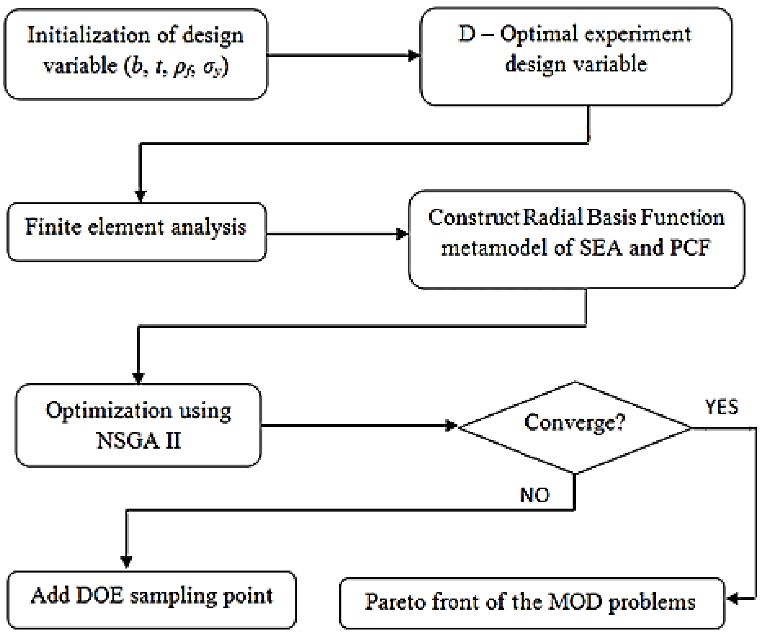


Figure 3: *Flowchart of optimization for tubes*

An optimization method is needed in place of the results from the simulation to determine the optimum value of crashworthiness, which is verified using the probability analysis to ensure its accuracy. In addition, results from the analysis of finite element and the experimental data from previous references (Li, Yu & Guo 2012) were verified using RMSE (1) and R^2 (2).

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2}{n}} \quad (1)$$

$$R^2 = \frac{\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)}{\sum_{i=1}^{n}\left(y_i - y_i^{'}\right)} \quad (2)$$

Based on the probability and reliability analysis, the Weibull distributions (3) aim to examine the validity of finite element and NSGA II optimization methods. Meanwhile, statistical methods such as *RE* and R^2 are used for the validation of metamodel techniques, which is investigated in terms of optimization to determine the accurate decision value capacity as carried out in previous researches (Barabadi 2013).

$$RE = \left|\frac{y_i - \hat{y}_i}{y_i}\right| \times 100\% \quad (3)$$

where, *RMSE* is average absolute error percentage, R^2 is coefficient of correlation, RE is relative error, *n* is the number of samples in the set, y_i is the real value, $\hat{y}_i$ is the forecast value and $y_i^{'}$ is average of the real values.

The Weibull distribution is the preferred method for the verification of the thin-walled structure. It is aimed at validating the RBF metamodel and NSGA II optimization methods. The validation of the probability analysis carried out using the Weibull distribution is in line with research carried out by Sakin and Ay (2008).

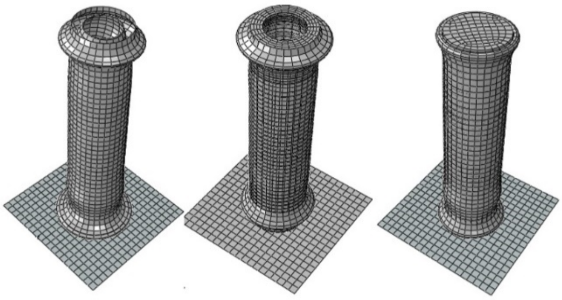
$$f(x) = \frac{\beta_c}{\eta}\left(\frac{x}{\eta}\right)^{\beta_c - 1} e^{\left(\frac{x}{\eta}\right)^{\beta_c}} \quad (4)$$

where η is the scale parameter, β_c is the shape parameter and x is the design variable.

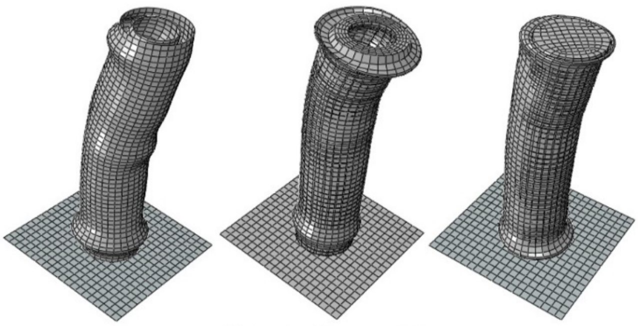
3 RESULT AND DISCUSSION

3.1 Deformation modes

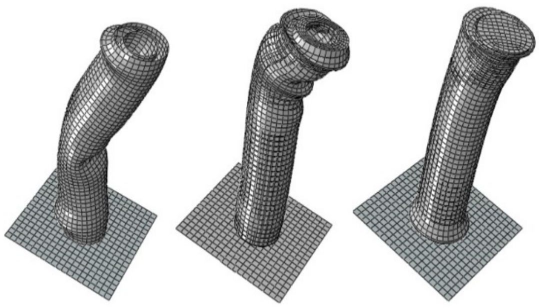
Figure 4 shows the numerical reaction or deformation modes for empty double, foam-filled double and single tubes under oblique loading with an impact angle of 0°, 10°, 20°, 30°, and 40°. All numerical models of the tubular structure in the progressive fall mode are stacked by rigid walls with loading angles of 0° and 10°, while 30° and 40° are categorized in the global bending modes. The loading angle of 20° is a transitional area between the progressive fall and the global bending collapse modes. The examined deformation modes show that the tube configuration has similar geometry and material as those under oblique static quasi-loading. The deformation mode for the 0° loading angle is a progressive axial collapse mode. Assuming it is 10°, the tubes form folds progressively, and at the final stage, they bend slightly towards the center, as shown in Figures 4a and b. At 30° and 40°, the bend is more significant at the tube portion where global mode occurs, as shown in Figures 4c and d. This modifying mode reduces the structure's energy absorption capacity.



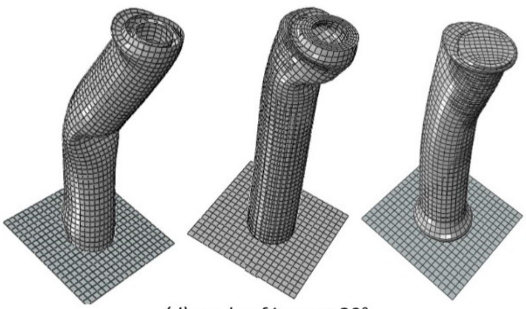
(a) angle of impact 0°



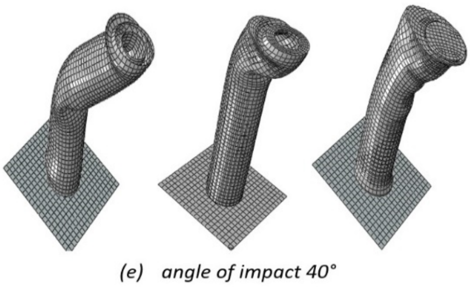
(b) angle of impact 10°



(c) angle of impact 20°



(d) angle of impact 30°

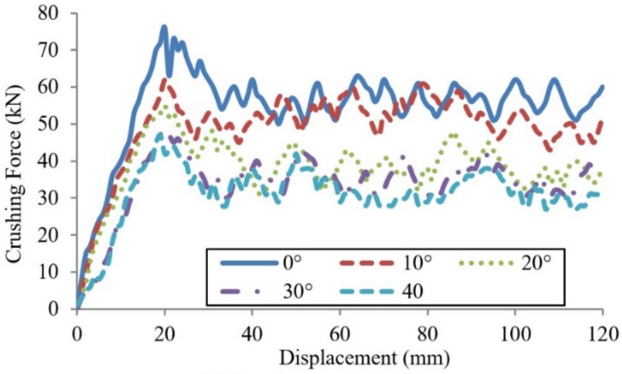


(e) angle of impact 40°

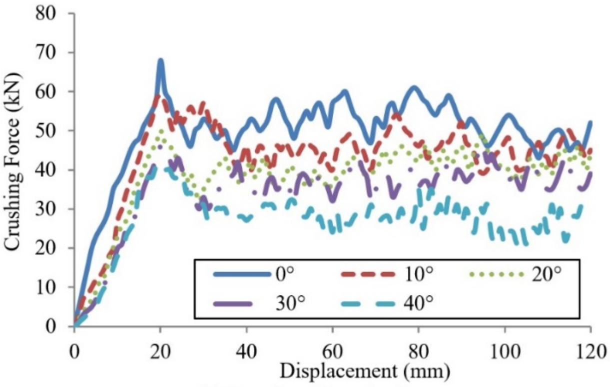
Figure 4: The deformation modes for empty double tubes (ED), double foam tubes (FD) and single foam filled tubes (FS) under oblique impact

3.2 Angle Effects

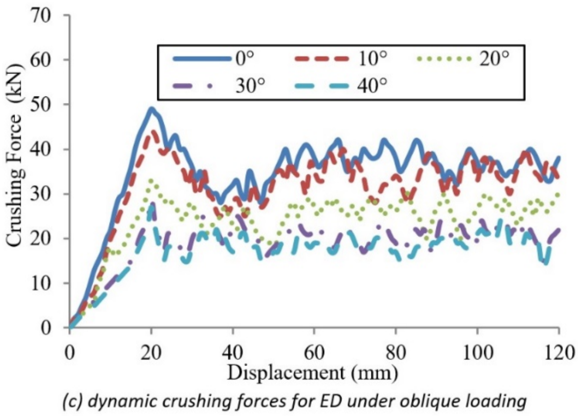
The graph of crushing force plotted against displacement for the three types of tubes, namely empty double tubes (ED), double-filled foam (FD) tubes, and foam-filled tungsten tubes (FS) under different angular loading is shown in figure 5. It is evident that the load angle has an impact on the structure. Generally, FD's dynamic crushing force has the ability to absorb energy compared to ED because because of the contact effect between foam and the tube wall (Li, Yu & Guo 2012). Figure 5 shows the maximum crushing force on each tube under pure axial loading or 0°. The peak value on the F40° is 26.45 kN. Generally, the graphic form of dynamic crushing force and the displacement of the different tubes have s D tube is 76.53 kN. However, the value of the minimum crushing force on the ED tube under the loading angle of imilar axial and oblique loading conditions. Therefore, angular loading parameters play an insignificant role in the folding mechanism of the tubular structure, as reported by Mirzaei et al. (2011). According to Fig. 5, tubes under axial loading are excellent energy absorbers than those under oblique loading (Tarlochan et al., 2013). On the contrary, when subjected to small loading angles better crashworthiness performance than large ones. They also tend to absorb a lot of energy; however, assuming the loading angle is greater than the dynamic crushing force, the energy absorption efficiency decreases (Li et al., 2014).



(a) FD under oblique loading



(b) FS under oblique loading

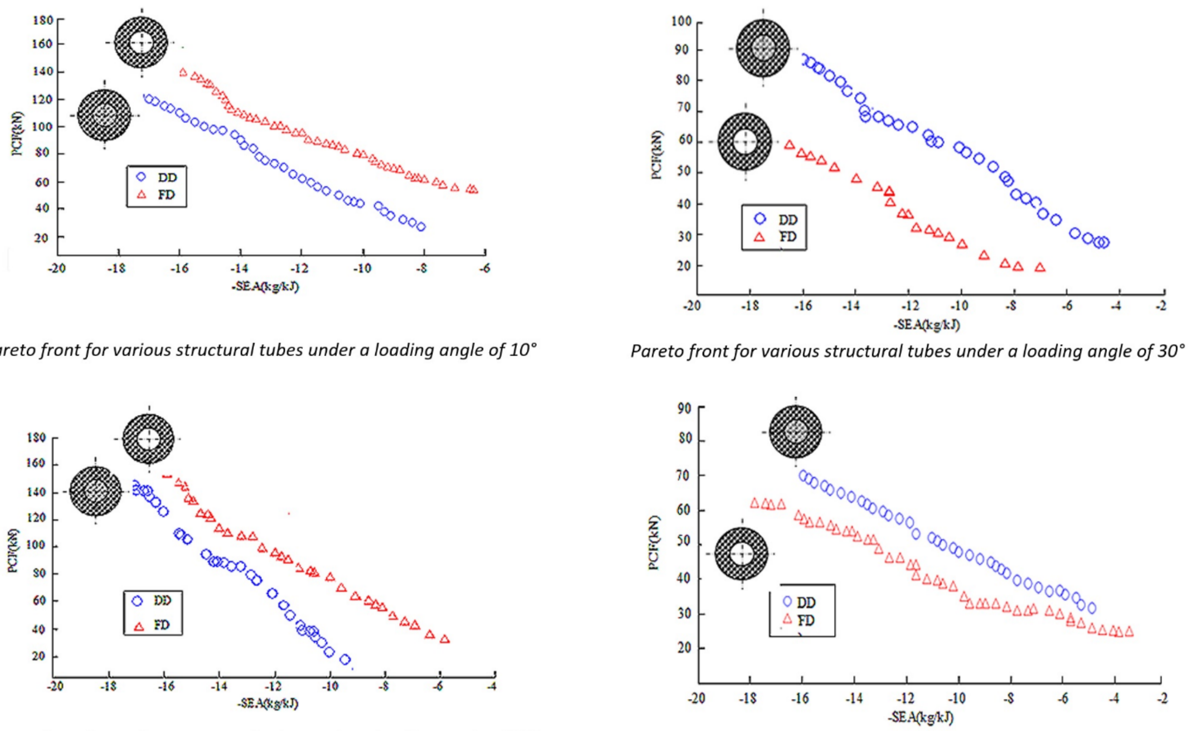


(c) dynamic crushing forces for ED under oblique loading

Figure 5: *ffect of dynamic loading angles for structures under oblique loading*

3.3 Comparison of circular tubes foam-filled under oblique dynamic impact

In this section, the double tubes containing full foam (DD) and half-foam (FD) are optimized under four load angles of 10°, 20°, 30°, and 40°. The SEA and PCF are designated as the objective function for each loading angle and the criteria for critical accident suitability. The tubes have similar dimensions, boundaries, and loading conditions, as shown in Figure 6. Its optimization is based on design variables such as the thickness and density of the tubular wall and foam, respectively. The tube under oblique blast shows that FD has a better survival capacity than DD due to deformation mode. Generally, double foam structures such as FD and DD are good energy absorbers because of the interactive effect of foam fillers and internal and external tubes (Zhang et al., 2012). The optimum value is obtained when a double-filled circular tube is under oblique dynamics. The simulation's speed is 10 m/s (Gao et al. 2016) under a loading angle of 10° and 40°. In addition, the results from figures 6 shows the maximum optimum value of the FD and DD tubes. The optimum value of DD tubes has an effective crashworthiness performance at loading angles 10° and 20°. Conversely, the FD tubes' optimum value has an effective crashworthiness performance at loading angles 30° and 40° (Yang & Qi 2013). The maximum optimum value of SEA on DD tubes is 172.49 kJ / kg at the loading angle of 10 °, while the minimum value is 9.82 kJ / kg at the loading angle of 40°. The maximum optimum value of PCF on DD tubes is 83.43 kn at a loading angle of 10°, while the minimum value is 3.98 kn at a loading angle of 40° (Zhang et al., 2014). The maximum and minimum difference for SEA is 2.97% and 1.23%, respectively.



Pareto front for various structural tubes under a loading angle of 10°

Pareto front for various structural tubes under a loading angle of 30°

Pareto front for various structural tubes under a loading angle of 20°

Pareto front for various structural tubes under a loading angle of 40°

Figure 6: Pareto front for various structural tubes under different loading angle

This study's results were carried out to determine the accuracy of the prediction technique and the applied crashworthiness optimization method. The selection of prediction methods is based on specific criteria stated in previous researches (Li, Yu & Guo 2012). It is essential to determine those that offer an accurate approximation of data. The validation of the Finite element model and optimization values are required to ensure the accuracy of the study carried out. Weibull distribution was used to compare the analysis of finite element and optimum value. The two parameters are dependent on the methods used for validation in order to distribute the data set effectively. Barabadi (2013), reported that it is important based on the characteristics of various alternative distributions in data validation.

3.4 Validation of Finite Element Model

According to a research conducted by Li, Yu and Guo (2012), finite element models are compared with experimental data to ensure that the analysis results are accurate for the optimization design of the crashworthiness structure, as shown in figure 7. The dimensions of the model are similar to the experimental data (Li, Yu & Guo 2012), consisting of twin foam-filled tubes of lengths 90 mm, with outer and inner diameters of 38 mm and 24 mm, respectively.

The thickness of the inner and outer walls of the tubes was 2.0 mm and 1.2 mm respectively under the quasi-static static loading (0°, 5°, 10°, 15°, and 20°).

In accordance with Li, Yu & Guo (2012), correlation analysis was carried out using statistical methods to calculate and determine the accuracy of R^2 values as well as the relationship between specific energy absorption and energy absorption as shown in Figures 7. This experimental data serves as the basis for the validation of a foam-filled double-tube numerical model subjected to oblique loading. According to Figures 7, the correlation coefficient or R^2 values for SEA and EA are 0.9296 and 0.9512, respectively.

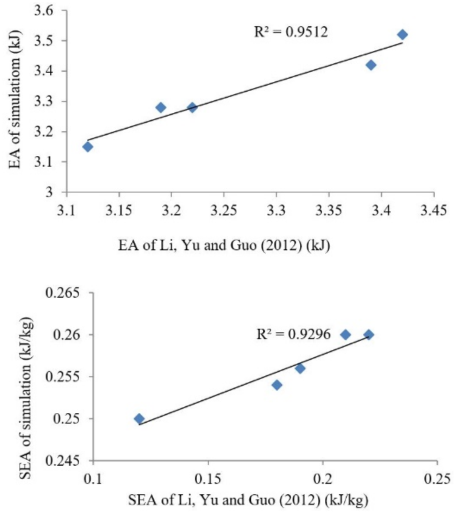


Figure 7: Validation of Simulation and experiment result by Li, Yu and Guo (2012)

Weibull distributions are used to determine the probability of foam-filled double tubes under progressive crushing conditions or loading angles, ranging from 0° to 20°, as shown in figures 8. The plots in the graph provide information on the results from the probability of simulation and experiments carried out by Li, Yu, and Guo (2012). Additionally, the Weibull distribution offers accurate data for optimum value analysis. Probability values were estimated based on a 95% confidence level, which was selected because it represents the real population parameters with repeated sample data (Barabadi 2013).

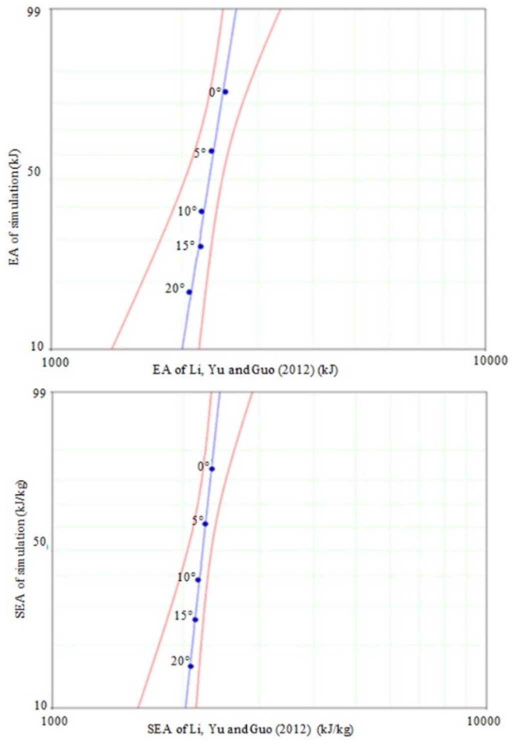
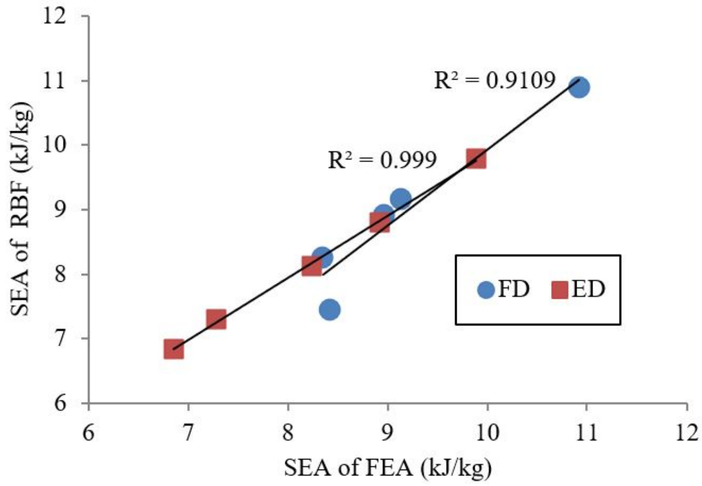


Figure 8: EA and SEA analysis between simulation and experiment result using weibull distribution

3.5 Validation of Metamodel RBF

Correlation analysis was carried out using statistical methods to determine the accuracy of R^2 values and correlation coefficient of FEA and RBF outcome for SEA and PCF, as shown in Figures 9. Furthermore, the correlation coefficient or R^2 between FEA and RBF simulation data is greater than 0.9109 for SEA and 0.9971 for PCF. This shows that RBF is highly accurate, as stated in the study by Acar et al. (2011).



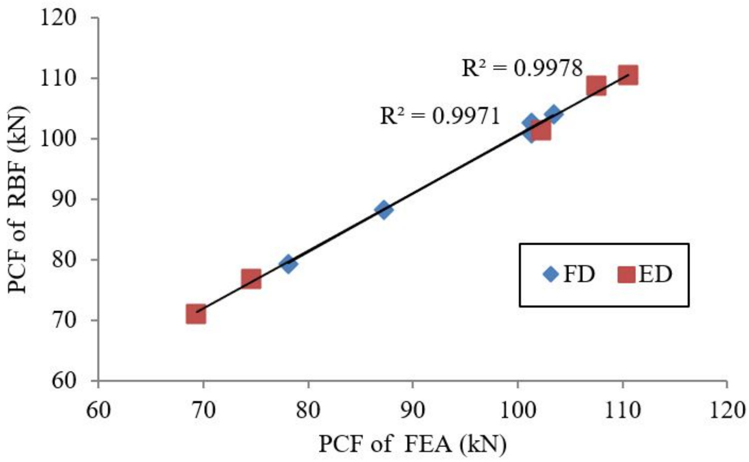


Figure 9: Correlation of SEA and PCF for FEA and RBF

3.6 Validation of Optimization Result

Weibull distribution is used for verification analysis and optimization of values obtained from the following optimizers, namely NSGA II and Pareto front. This is based on the difference in angular loadings of the crashworthiness parameters, namely, SEA and PCF. Numerical simulation data from the variables are foam density and thickness of tube wall with the following given design parameters, namely, 2.2 × 10-7 kg / mm^3 ≤ ρf ≤ 7.1 × 10-7 kg / mm^3 and 1.5 mm ≤ ti ≤ 2.5 mm as well as 200 mm length. Weibull distribution was selected due to the suitability of data distribution in probability analysis as shown in Figure 10. The probability value is estimated using a correlation coefficient of 95%, which shows the data analysis (Barabadi 2013). The SEA and PCF values obtained from FEA and NSGA II were used to determine the relationship between these two parameters under the oblique dynamic load, as shown in Figure 10. The correlation coefficient value is less than 0.95; however, assuming the value is high, it implies the accuracy between SEA and PCF. Figures 10 also shows the use of Weibull distribution to determine the correlation between optimized and simulation values. The plot in both diagrams provides information on the probability of tubing with different loading angles. Figures 10 show that the probability values are estimated at 95% confidence level, which shows the actual population parameters. It is therefore concluded that both FEA and NSGA II values have good accuracy.

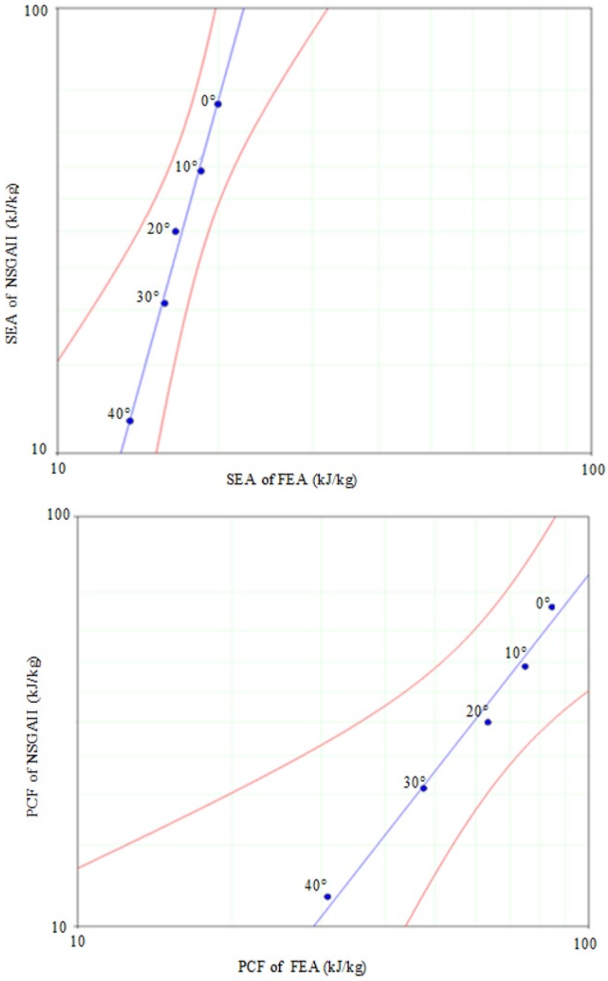


Figure 10: SEA and PCF analysis between NSGAII and FEA result using weibull distribution

4 CONCLUSION

The crashworthiness design of a thin-walled structure made of aluminum foam-filled circular tubes was explored. Active crashworthiness criteria, such as specific energy absorption (SEA) and peak crushing force (PCF), were calculated under axial and oblique impact loads. Optimal problems of various objectives in accordance with the basic functions of the network (RBF) were developed using finite element analysis (FEA). The outcome of the maximum SEA and minimum PCF values under the pure axial loading conditions are 19.98 kJ / kg and 63.54 kN, respectively. Similarly, values below 40 ° obtained a maximum SEA value of 10.16 kJ / kg and a minimum PCF of 3.98 kN. It was discovered that increasing the angle of load on the circular tube produces lesser SEA and PCF values. The NSGA-II and Pareto front were used for multi-objective optimization of SEA and PCF of circular tubes.

The results from simulation and optimal methods are compared, such as the RBF and NSGA II metamodels. The estimation of metamodel was carried out using Relative Error (RE) and correlation coefficient (R2). The results from the analysis show that the RE and R2 used in the estimation of these RBF metamodels are less than 2.28% and greater than 0.9 respectively, this shows that the RBF model for the objective functions of SEA and PCF produces sufficient accuracy for design optimization. Correlations between simulations and experiments need to be carried out to determine the accuracy of the analysis from the data plotted in the graph. The results from PCF and MCF are accurate and have an R2 value of 0.9. Weibull distribution probability analysis showed that the correlation between optimization and simulation has a good accuracy value of R^2> 0.85. Weibull distribution probability method and reliability analysis are the preferred technique for determining the optimum value of a protected structure.

Author Contributions: Investigation, F Djamaluddin, Z Ahmad, F Mat.

Editor: Marcílio Alves.

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