# Developing an Impartial Game by Mathematical Approach 

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#### Abstract

In this study, we developed an impartial game namely Nim by using a mathematical approach. The game consists of multiple ways of movement for player winning the game. A total of five players participating in the game testing. The experiment shows that the proposed impartial game for learning process is easy to use.


## 1. Introduction

In this era of technology, video games are a form of entertainment for people to enjoy themselves. Video games are created for the purpose of entertainment and the development human thinking. Mathematical games or puzzle games are a type of game genre that helps in developing human mind hand help them increase awareness and focus. Nim game is impartial mathematical game consisting of two players partake in it and tokens or any type of medium placed in heaps and players makes their move to make sure that they are players to make the last move to end the game, for, this version of the game is called the circular nim game [1]. Another version of nim game with different rules set to it like the End-Nim, which, instead of using heaps, it uses row and stack of boxes which each player can pick up one or more until the last box where pickup and won by that player [2]. Mis`ere-Nim game also has a popular rule of where, if the last player picks up a token is the loser instead. This is the rule set up with this version the game and there is no draw in this version due to a condition is added, and it is called "the ending condition" [3]. In this study, we develop a nim game that is based on combinatorial impartial mathematical approach.

## 2. Related Works

Video games nowadays not only seen as entertainment device but also a method of education. Though teacher nowadays rarely used games anymore like they used to from back game-based learning method was hype, there are still teachers that still implements them but only after the real "work" is done [4]. Game nowadays is a constructive method that allows learners to engage in learning materials that is authentic and safe environment [5].

## 3. Methodology

Nim-game is a combinatorial impartial mathematical game as it have a certain rule build into it that's decide whether the last player wins or loses. Given a set of players $\mathrm{T}=\{1, \ldots, t\}, t \geq 2$, partition $T=A \mathrm{U}$ $B$ into two parties, $k$ natural numbers $0 \leq \mathrm{n}_{1} \leq \mathrm{n}_{2} \leq \ldots \leq \mathrm{n}_{\mathrm{k}}, k \geq 2$, and a size of heap $m \epsilon \neq 0$. The players move in succession $t, t-1, \ldots, 1$. A single move consists in subtracting either $n_{1}$ or $\ldots$ or $n_{k}$ from the size of heap. The player rendering the size of heap non-positive makes his party winning. The winning condition $\mathrm{m} \leq 0$ instead of $\mathrm{m}=0$ serves to keep the playing rules simple [6].

### 3.1. Development

The main goal is to compete within two players, and there is a player making last move to end the game. Note that the intrusions of values equal to $2 k$ or larger within the blocks corresponding to positions with all the heaps of size smaller than $2 k$. Also, within each row or column the sequence appears to be ultimately known as arithmetico-periodic [7,8]. For the strings of the first type, with $n$ even, the second player wins by copying the opponent's move but on the other side of the string so these positions are in $P$. When the $n$ is odd, these positions are in $N$ since the first player can win by deleting one of the stacks and then following the strategy.

### 3.2. Game Map

The game uses a min-max approach and requires certain rules. The winner is determined if the nodes are forced to win in MAX state which the node $=1$ or the nodes $=0$. Min-max procedure uses a recursive or backtracking method for decision making. It enables the players with all the possible best move to win the game. The min-max procedure in Figure 1 delivers a strategy that involves the opponent to get the minimum advantage while the player gets the maximum advantage.


### 3.3. Game Rules

The game requires two players which can be both human (Figure 2) or in this case, or one human player and one computer player (Figures 3, 4 and 5). There is a finite se of positions available in the game. The rules will specify which game positions for each player can move to. Then the players will start their individual alternate moves. Accordingly the game ends when a player cannot make further move.


Figure 2. Game Play between players


Figure 3. Game play move by computer


Figure 4. Game move made by human player


Figure 5. Game results won by computer

## 4. Testing

We distribute a Google survey form to players who play this game as shown in Figure 6. Feedback is then observed from them to improve our game. Then, responses about this games are recorded and analyzed as shown in Figure 7. 70\% of respondents are interested playing this game. All respondents are satisfied with its graphical user interface (GUI), and they agree to recommend this game to other persons. Furthermore, $50 \%$ of respondents also suggest that this game is very interesting.


Figure 6. Survey of Nim Game


Figure 7. Result of survey

## 5. Conclusion

In this study, we developed an impartial game using mathematical approach namely Nim game. Overall based on the feedback from the players, we can conclude that that this game has operated well.

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