

Model Predictive Controller for Path Tracking and Obstacle Avoidance Manoeuvre on Autonomous Vehicle

Zulkarnain Ali Leman^{1,2}, Mohd Hatta Mohammad Ariff¹, Hairi Zamzuri¹, Mohd Azizi Abdul Rahman¹, Saiful Amri Mazlan¹

Abstract— Some challenging control design problems include non-linear vehicle dynamics, fast sampling time and limited computing resources on automated hardware. MPC has the ability to systematically consider nonlinearity, future predictions and operating constraints of the control system framework. One problem for autonomous vehicles operating on toll roads must be able to do a satisfactory tracking path when avoiding obstacles so that accidents do not occur. This paper will discuss designing tracking path controllers using a predictive controller (MPC) model based on scenario avoidance obstacle on the highway with several variations in speed. The trajectory has been predetermined and the controller must be able to autonomously avoid static obstacles on the road and can track the desired trajectory by controlling the front steering angle of the vehicle. This approach discusses solving a single nonlinear MPC problem for following trajectories and avoiding static obstacle. The vehicle model was developed based on 3 DOF non-linear vehicle model. This controller model was developed based on X, Y global position and yaw rate to get input in the form of steering to the vehicle dynamic system. For path tracking strategy, comparisons with the Stanley controller are done to analyse MPC reliability as non-linear controller in low and middle speed scenario. Simulation results show that the MPC controller has the advantage of a tracking path that is good at mid and high speeds.

I. INTRODUCTION

Path tracking controller research consists of several main topics. The topics is the selection of kinematic and dynamic vehicle models, the design of the control strategy to guide the vehicle and the evaluate the developed controller using certain performance criteria. Most studies [1,2,3,4] use geometric or linear kinematic models to represent the behaviour of vehicle models. In this model ignores the dynamic effects for instance torque, frictional force and tire slip. Some studies [3,4] have also included some dynamic effects of the vehicle and the model uses nonlinear tire force. The effect of contact between tires and roads can be attributed to sources of forces acting on tires that work both horizontally and laterally. To facilitate simulation of vehicle behaviour in handling studies, it often ignores internal and external disturbance factors in the lateral and longitudinal direction. Overall, main challenge in path tracking control is the controller's ability to navigate various types of road curvature, in contrast to different speeds and road conditions. The integrated controls must be developed aims to combine control inputs in the form of steering, throttle and suspension for adjusted lateral and longitudinal control when navigating various road conditions. Sharp, et al. [1] has

discussed mathematical models and optimal controller. Controllers are designed based on the optimal linear discrete time control theory. Error samples are represented by predictive paths, position errors and lateral errors, then change the data that has been saved to the reference steering angle. This method is then applied to autonomous vehicles and motorbikes with a scenario approaching the cornering limit [2]. However, this method does not provide good performance as expected even though it has considered the dynamics of the vehicle and the optimal preview, to compensate for the absence of look forward distance.

There is one type of optimal controller methods that is widely used that is applied to systems with linear or non-linear constraints is the predictive control (MPC) model. MPC uses a mathematical dynamic process model of the system to predict future values and optimize control process performance [3,4]. The model predictive controller (MPC) usually uses a linear or nonlinear plant model to predict the control inputs needed for the plant. This method also performs optimization to get the optimal value for input to the plant. Development and use of MPC for track tracking control can be found in previous researchers' publications. In Falcone et al. [3] discussed about the MPC is used for predictive purposes the wheel steering input for path tracking and obstacle avoidance. Nonlinear and Linear Time Varying (LTV) MPC controller tests using the dSPACE rapid prototyping module. However, the application of this method requires high computational resources because it resolves optimization problems in real time. Bayar, et al. [4, 5] apply MPC to autonomous vehicles for plantation environments, while Tomatsu, et al. [6] applied MPC for tracking of the path on excavators in excavation operations operating at slow speeds. More applications MPC controller for slow speed can be seen in previous studies [7,8,9].

Beal [10] applies predictive control models using custom C-Code tested on autonomous vehicles that can solved optimization. Also, in several studies that discussed solving MPC optimization problems using metaheuristic algorithms. The recent studies [11, 12] showed that using the metaheuristic optimization method to implement the real-time optimization process. Merabti, et al. [11] discusses three types of metaheuristic optimization algorithms to complete optimization of nonlinear MPC for control of tracking the mobile robot path. Borrelli [3] developed MPC combined with path planning based on bicycle vehicle model. Yakub [13] developed MPC based on Borelli concept combined with feed forward controller. Path tracking controller based on bicycle models show some disadvantages including providing less accurate predictions when applied to real vehicles. This paper aims to develop a path tracking controller with input in the form of

¹Vehicle System Engineering, Universiti Teknologi Malaysia, Kuala Lumpur, Malaysia

²Department of Mechanical Engineering, Universitas Sriwijaya, Indonesia

*Corresponding author, e-mail: mohdhatta.kl@utm.my

steering vehicles to 3 DOF non-linear vehicle model. The developed controller was tested by simulating using MATLAB to study vehicle characteristics when running on avoiding obstacles manoeuvre. Then the MPC controller will be compared to the Stanley controller, which is a benchmark of the geometric controller.

II. VEHICLE MODELLING

A. Non-linear Vehicle Dynamic Model

Mathematical modelling of vehicle dynamics is obtained under Newton's second law. The dynamic model of this vehicle defines the forces and moments acting on the vehicle. The dynamics model of the vehicle is also an important factor in designing and analysing the controller that will be used to control the vehicle's yaw stability. In general, dynamic vehicle models are divided into two, namely linear and non-linear dynamic vehicle models as depicted in Figure 1. The following subsections will discuss the nonlinear vehicle model for simulation for controller design purpose.

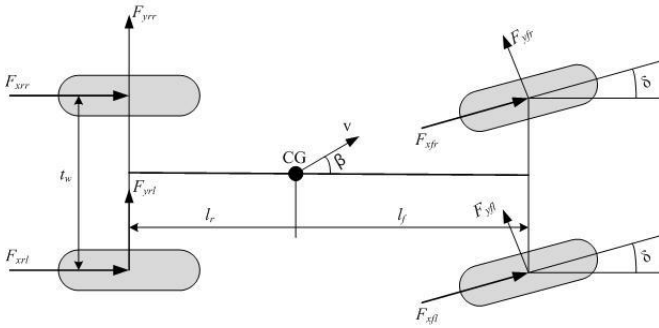


Figure 1 The non-linear vehicle dynamic model

The vehicle nonlinear dynamic models can be used to present actual vehicles. This model is also used to evaluate the controllers that have been designed. In recent years, researches in [14–18] have utilized nonlinear vehicle model for vehicle handling and stability improvement studies. Figure 3 shows the typical nonlinear vehicle model in cornering manoeuvre. The input of this vehicle model is the front wheel steering angle δ_f while the side slip of the vehicle β and the yaw rate $\dot{\psi}$ are the output variables to be controlled. Vehicle parameters consisting of d are vehicle width track, l_f and l_r are the front and rear axle distances to the center of gravity (CG). The longitudinal velocity of the vehicle on the centre of gravity is v . Other vehicle parameter is vehicle mass m , moment inertia I_z and front/rear tire cornering stiffness C_f/C_r .

Longitudinal force F_x depend directly proportional to the tire slip ratio while the lateral force F_y of the tire is proportional to the sideslip angle. Equation 1 defined the motion of the dynamic full vehicle handling model [19,20]. It considers means with mass m_b , inertia moment of vehicle mass on z-axis, I_{CG} , acceleration a_x from motion along x, acceleration a_y from horizontal motion along y and yaw angle, ψ on z-axis. Tires forces on the vehicle are indicated with subscripts l, r, f where $l = \text{left}, r = \text{right/rear}, f = \text{front}$.

$$\begin{aligned} \sum F_x &= m_b a_x \\ \sum F_y &= m_b a_y \\ \sum M_z &= I_z \ddot{\psi} \end{aligned} \quad (1)$$

$$\begin{aligned} F_{xrr} + F_{xrl} + F_{yfl} \cos \delta + F_{yfl} \sin \delta + F_{yfr} \cos \delta + F_{yfr} \sin \delta &= m_b a_x \\ F_{yrr} + F_{yrl} - F_{yfl} \sin \delta - F_{yfr} \sin \delta + F_{yfl} \cos \delta + F_{yfr} \cos \delta &= m_b a_y \\ \left(\sum M_{zif} + [-F_{yrr} - F_{yrl}] l_r + [F_{yfl} \sin \delta + F_{yfl} \cos \delta + F_{yfr} \sin \delta + F_{yfr} \cos \delta] l_f \right) \\ + [F_{yfr} \cos \delta - F_{yfl} \cos \delta + F_{yfl} \sin \delta - F_{yfr} \sin \delta - F_{xrl} + F_{xrr}] \frac{l}{2} &= I_{z,CG} \ddot{\psi} \end{aligned}$$

The equations above show that the forces on the wheel and the traction force is the external forces that affects the dynamics of the vehicle. The forces of the tire come from direct contact between the surface of the tire and the road. Many studies have been done with linearization of lateral and longitudinal forces on wheels. The linearization has been applied to the dynamic model of a full vehicle [21, 22, 23] and bicycle model [18,19, 24,25,26] previously. The lateral and longitudinal forces F_x and F_y are shown respectively in equations (2,3,4). Some studies that consider the behaviour of non-linear tire dynamics show better vehicle response simulation results at high speed vehicles and large steering angles. The motion equations for the vehicle in a global inertial frame or on X-Y axis may be given as

$$\begin{aligned} \dot{X} &= v_x \cos \psi - a_y \sin \psi \\ \dot{Y} &= v_x \sin \psi + a_y \cos \psi \end{aligned} \quad (2)$$

B. Tire Forces

In this paper use non-linear tyre properties based on work of Nagai [27]. In the dynamic nature of the wheel, the effect of load transfer is an important factor to be considered. Load transfer is a characteristic that is affected by longitudinal and lateral acceleration during running and disturbances. The static forces of the wheel are calculated by the following equation. Vertical dynamic forces of the wheel due to the influence of longitudinal and lateral accelerations in each wheel are calculated based on the equation.

$$F_{z0f} = \frac{mgL_r}{2L} \quad F_{z0r} = \frac{mgL_f}{2L} \quad (3)$$

$$\begin{aligned} \Delta F_{zfr} &= -\frac{ma_x h}{2(l_f + l_r)} + \frac{ma_y h l_r}{w(l_f + l_r)} \\ \Delta F_{zfl} &= -\frac{ma_x h}{2(l_f + l_r)} + \frac{ma_y h l_f}{w(l_f + l_r)} \\ \Delta F_{zrr} &= -\frac{ma_x h}{2(l_f + l_r)} + \frac{ma_y h l_f}{w(l_f + l_r)} \\ \Delta F_{zrl} &= -\frac{ma_x h}{2(l_f + l_r)} - \frac{ma_y h l_f}{w(l_f + l_r)} \end{aligned} \quad (4)$$

Where h denotes the height of centre of gravity. The dynamic model of the vehicle 3 degrees of freedom can be seen in the figure 1 and equation 1. In general, vehicles operate by braking and traction during driving on the highway. Based on the concept of friction circle, the addition and reduction of longitudinal and lateral forces will cause a reduction in the

tire's cornering force. Tires with nonlinear characteristics and are influenced by load and braking / traction forces, the cornering style can be calculated using the equations.

$$F_{yi} = K_{xi} \left[\frac{2}{\pi} (F_{z0i} + \Delta F_{zi}) \right] \tan^{-1} \frac{\pi}{2\pi (F_{z0i} + \Delta F_{zi})} C_i \beta_i$$

$$K_{xi} = \sqrt{1 - \left[\frac{F_{xi}}{\mu (F_{z0i} + \Delta F_{zi})} \right]^2}$$
(5)

Where i represent the index indicating front and rear tires, C_i the cornering stiffness and β_i the tires side slip angle. While the longitudinal forces, F_x , of the wheel are calculated based on the concept of friction force circles.

$$\sqrt{F_y^2 + F_x^2} \leq \mu F_z$$

$$F_x = \sqrt{(\mu F_z)^2 - F_y^2}$$
(6)

C. Trajectory Generation

MPC applied to steering controllers has been used in vehicles with different initial longitudinal speeds in a double lane change scenario. This test represents an obstacle avoidance emergency manoeuvre with a certain initial longitudinal vehicle speed. The control input is the front steering angle with the aim to follow the trajectory as close as possible to the desired path by minimizing the lateral deviation of the vehicle's trajectory to the desired path. The desired path is described in term of yaw angle ψ_{ref} and lateral position as function of longitudinal position X [3, 13].

$$\psi_{ref} = \tan^{-1} \left(d_{y1} \left(\frac{1}{\cosh z_1} \right)^2 \left(\frac{1.2}{d_{x1}} \right) - d_{y2} \left(\frac{1}{\cosh z_2} \right)^2 \left(\frac{1.2}{d_{x2}} \right) \right)$$

$$z_1 = \frac{2.4}{25} (x - 27.19) - 1.2 ; d_{y1} = 4.05 ; d_{y2} = 5.7$$

$$z_2 = \frac{2.4}{21.95} (x - 56.46) - 1.2$$
(7)

In extreme situations, when large lateral tracking errors due to spinning or skidding, trajectory vehicles from the simulation results can cause aggressive vehicle maneuvers. The scenario of obstacle avoidance and path tracking can be seen in figure 2. The trajectory is calculated based on equation 7 as the concept given in the paper [3]. The MPC controllers presented in this section, have been tested through simulation on dry surface. The controller has been tested on passenger car with parameters that can be seen in table 1. The controller was run on straight road with an initial velocity 10 meter per second, 15 meter per second and 20 meter per second. The purpose of this scenario is to understand the structure of lane changes and trajectory following performance for avoidance of single static obstacle.

To evaluate controller performance with variable vehicle speed conditions that are applied to various test cases, it usually uses tracking errors. The total cross track error is calculated using equation root mean square error (RMSE).

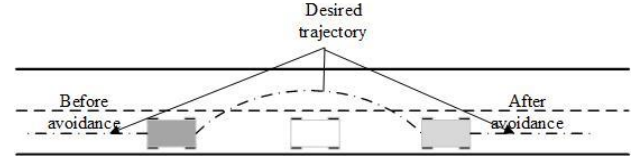


Figure 2 Double lane scenario for simulation test.

TABLE 1. VEHICLE PARAMETERS USES IN SIMULATIONS

Parameter	Value
m	2032 kg
l_f	1.26 m
l_r	1.90 m
I_z	6286 kg/m ²
C_f	40.200
C_r	62.800

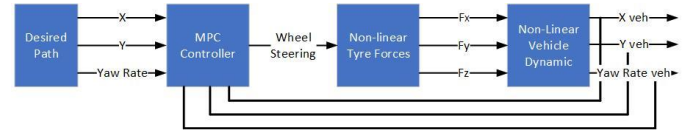


Figure 3 Model predictive control for non-linear vehicle dynamic model

To test the performance of the MPC controller, double-lane change manoeuvres performed at different speeds have been simulated. The aim is to follow the double lane changes as close as possible while avoiding obstacles. The lateral position Y_{ref} and the angle yaw ψ_{ref} as a function of the longitudinal position X , as described in Figure 3 are the desired path parameters in the MPC controller used in this study.

III. METHODOLOGY

A. Model Predictive control

Designing MPC controller computes the front wheel steering angle at the four wheels, such that is followed as close as possible at a given longitudinal speed based on the vehicle models present in section 2.1-2.2. The following cost function:

$$J(\xi(t), \Delta u) = \sum_{i=1}^{H_P} \|X_{t+1,i} - X_{ref,t+1,i}\|_Q^2 + \sum_{i=0}^{H_C-1} (\|\Delta u_{t+1,i}\|_R^2 + \|u_{t+1,i}\|_S^2) \quad (8)$$

The reference signal, $X_{ref} = [\psi_{ref}, X_{ref}, Y_{ref}]$ represents the desired output. The Q , R and S are weighting matrices of appropriate dimensions.

B. Stanley controller

A standard Stanley controller was presented by Snider, et al. [24] which the controller considers two properties such as the heading error ϕ and the lateral error, e as shown in Eq. (9). Measurement of heading and lateral errors is made through the center of the front steering wheel shaft to the closest point on the trajectory. Configurations and parameters of the Stanley controller can be seen in Figure[24].

$$\delta(t) = \phi + \tan^{-1}\left(\frac{k_e(t)}{v(t)}\right) + k_{yaw}(\dot{\psi} - \dot{\psi}_{ref}) \quad (9)$$

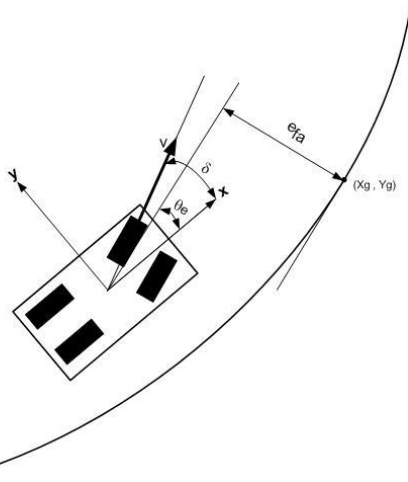


Figure 4: Configurations and parameters for Stanley controller

The stability of this control law evaluated at $e=0$, $v>0$ and $0 < \delta < \frac{\pi}{2_{max}}$ using Lyapunov criteria. In addition to strengthening the stability of the control law was added steady state yaw, ϕ_{ss} , yaw rate error $\dot{\psi} - \dot{\psi}_{tra j}$.

IV. RESULTS AND ANALYSIS

This section shows and discusses controller performance simulation results in the form of a control signal, measuring control efforts and tracking errors. The control horizon and prediction horizons were set to 2 and 10 steps. The gain Q value are 3x3 matrix diagonal 25. The gain R , S , k and k_{yaw} value are 25, 25, 10 and 25 respectively.

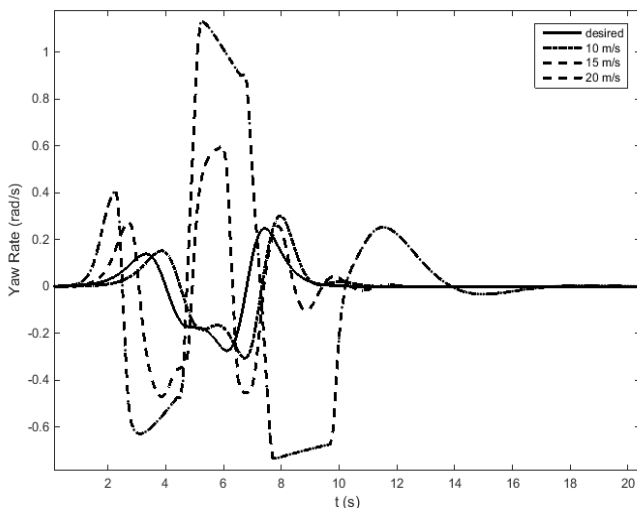


Figure 5 Simulation results: At speed of 10 m/s, 15 m/s, and 20 m/s the MPC controller can track desired yaw rate.

The double lane change used in paper to test handling manoeuvre. The double track change test is an obstacle

avoidance test that describes changes in the vehicle lane to a predetermined trajectory. Figure 5 shows the simulation results of the vehicle yaw rate obtained using control law using MPC. For vehicle stability in manoeuvring, it is important to control the dynamic motion of the vehicle. For this reason, yaw rate and vehicle side slips are important to be controlled. When the vehicle is travelling at medium and high speeds, the reference trajectory becomes more difficult to follow. In addition, to perform obstacle avoidance manoeuvres combined with path tracking, the controller will force the vehicle to deviate from the reference. This will cause the system to become unstable especially for one-level controllers operating at high speed vehicles. However, this did not occur as seen in ref [3,13] when the side slip reference was made at zero and the front steering control input was combined with the yaw moment.

To see a comparison of the performance of the MPC controller and Stanley can refer to table 2. Root mean square (RMSE) lateral position and yaw rate error for MPC controller and Stanley controller summarised in table 2. Table 2 reported the maximum deviation of each proposed controller with constant longitudinal vehicle speed. Table 2 shows that MPC controllers with 3 DOF non-linear vehicle models are more able to follow the desired yaw rate and position compared to Stanley controllers at low and medium vehicle speeds.

TABLE II. ROOT MEAN SQUARE YAW RATE AND LATERAL POSITION

Vehicle speed	MPC controller		Stanley Controller	
	RMSE (Yaw Rate)	RMSE (Lateral Position)	RMSE (Yaw Rate)	RMSE (Lateral Position)
10 m/s	0.0636	0.0891	1.2910	0.3155
15 m/s	0.1353	0.1098	1.2910	0.2881
20 m/s	0.1353	0.1932	1.2910	1.4504

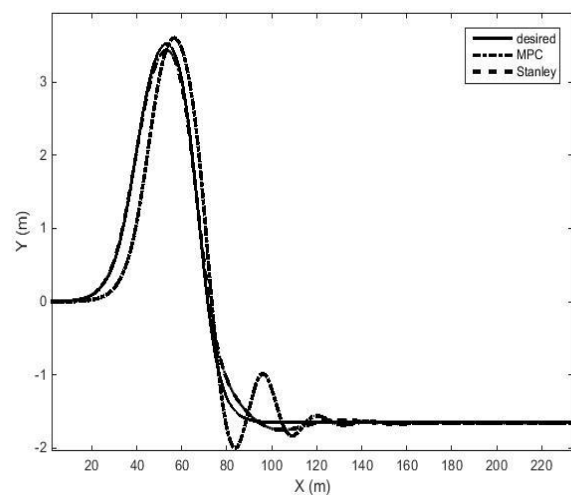


Figure 6 Simulation results: At speed of 15 m/s, the MPC controller can stabilize the vehicle in the double lane manoeuvre

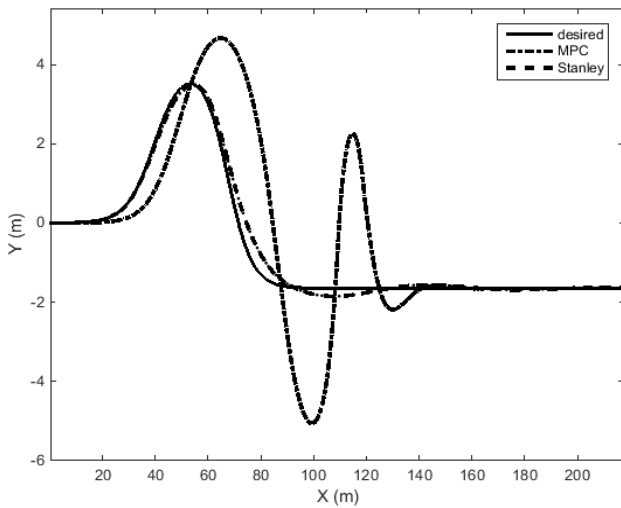


Figure 7 Simulation results: At speed of 20 m/s, the MPC controller can stabilize the vehicle in the double lane manoeuvre

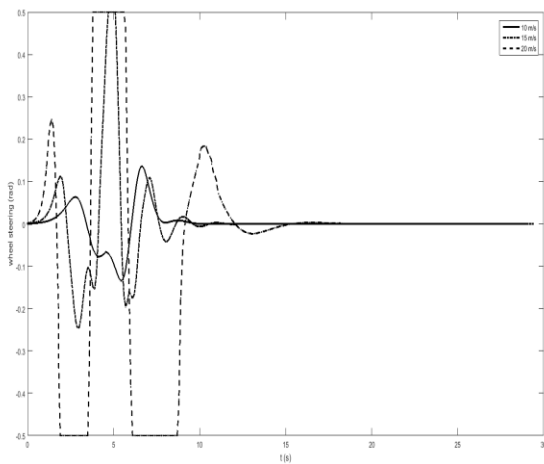


Figure 8. Wheel steering input for avoidance obstacle manoeuvre

Figures 6 and 7 showed a comparison of the trajectory tracking simulation results with MPC and Stanley controllers at speeds of 15 meter per second and 20 meter per second. At speeds 15 meter per second and 20 meter per second, vehicles can still perform avoidance obstacles maneuvers compared to Stanley's controllers with road surface coefficient of 1. However, at the speed of 20 meter per second the vehicle with MPC controller experiences a lateral deviation from the trajectory at the time after avoiding the existing obstacles. This is due to an increase in value from the side slip angle of the vehicle. Figure 8 shows the wheel steering as input control values of the MPC controller. Input control consists wheel steering with constraints ± 0.5 rad. It can be seen that to do double lane change manoeuvres, yaw moment also requires further consideration. Yaw moment is needed to prevent the vehicle from being thrown from the desired trajectory due to the lateral acceleration that appears. In critical conditions or manoeuvres, improper steering and braking control input will cause the vehicle to become unstable and have an accident.

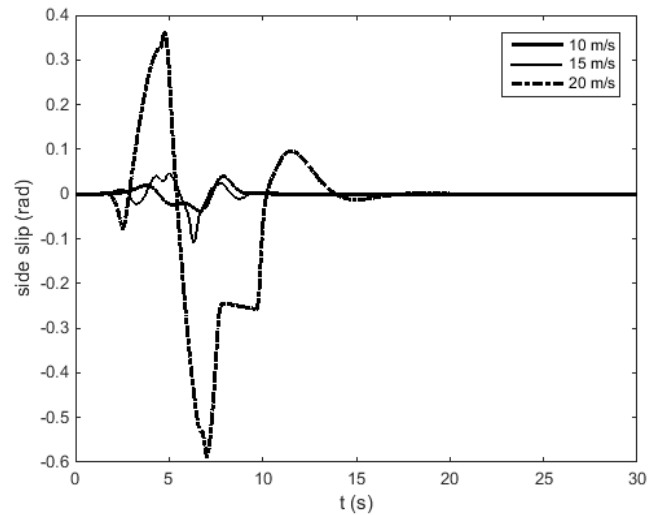


Figure 9 showed that side slip references are zero, this means that the vehicle when doing a turning manoeuvre doesn't slip. It can be seen that the vehicle side slip when performing a avoidance obstacle manoeuvre is not close to zero. This happens to MPC controllers that are designed not to consider vehicle side slips. The value of the side slip angle of the vehicle increases with the increase in vehicle speed. This causes the MPC controller to not be able to provide the right steering input to perform the obstacle avoidance manoeuvre.

CONCLUSION

In this paper have been proposed a path tracking controller for autonomous vehicle at various speed on avoidance obstacle. It is aimed to simulate driving handling and to predict the stability of vehicle at the same time. For this purpose, the path tracking controller proposed in this study was designed to follow the path using MPC controller. Path tracking controller had designed based on 3 DOF non-linear vehicle model for avoidance obstacle scenario.

The performance of the controller was verified through the MATLAB. It was showed that the small lateral position error at 10 meter per second and 15 meter per second vehicle speed. The controller can follow the value of the yaw rate well at a speed of 10 meter per second, while at speeds of 15 meter per second and 20 meter per second gives a larger error.

The method of designing the speed command using only the curvature of the path does not take into consideration the dynamic response characteristic of the vehicle, so that the vehicle may be overturned, or the vehicle may be damaged by a large impact during the actual driving. Therefore, future studies will be conducted to calculate the optimum travel speed considering the dynamic stability of the vehicle in the field using the path-tracking controller proposed in this study.

ACKNOWLEDGMENT

The work presented in this study is supported by Ministry of Higher Education Malaysia through UTM Transdisciplinary Grant, VOT: 05G44.

REFERENCES

- [1] R. Sharp, D. Casanova, and P. Symonds, "A mathematical model for driver steering control, with design, tuning and performance results," *Vehicle System Dynamics*, vol. 33, pp. 289-326, 2000.
- [2] [2] R. Sharp, "Rider control of a motorcycle near to its cornering limits," *Vehicle System Dynamics*, vol. 50, pp. 1193-1208, 2012.
- [3] P. Falcone, F. Borrelli, J. Asgari, H. E. Tseng, and D. Hrovat, "Predictive Active Steering Control for Autonomous Vehicle Systems," *Control Systems Technology, IEEE Transactions on*, vol. 15, pp. 566-580, 2007.
- [4] G. Bayar, M. Bergerman, A. B. Koku, and E. i. Konukseven, "Localization and control of an autonomous orchard vehicle," *Computers and Electronics in Agriculture*, vol. 115, pp. 118-128, 7// 2015.
- [5] G. Bayar, M. Bergerman, and A. B. Koku, "Improving the trajectory tracking performance of autonomous orchard vehicles using wheel slip compensation," *Biosystems Engineering*, vol. "In Press", 2016
- [6] T. Tomatsu, K. Nonaka, K. Sekiguchi, and K. Suzuki, "Model predictive trajectory tracking control for hydraulic excavator on digging operation," in *Control Applications (CCA), 2015 IEEE Conference on*, 2015, pp. 1136-1141.
- [7] A. S. Yamashita, P. M. Alexandre, A. C. Zanin, and D. Odloak, "Reference trajectory tuning of model predictive control," *Control Engineering Practice*, vol. 50, pp. 1-11, 5// 2016.
- [8] I. Prodan, S. Oлару, F. A. C. C. Fontes, F. Lobo Pereira, J. Borges de Sousa, C. Stoica Maniu, et al., "Predictive Control for Path-Following. From Trajectory Generation to the Parametrization of the Discrete Tracking Sequences," in *Developments in Model-Based Optimization and Control: Distributed Control and Industrial Applications*, ed Cham: Springer International Publishing, 2015, pp. 161-181.
- [9] G. V. Raffo, G. K. Gomes, J. E. Normey-Rico, C. R. Kelber, and L. B. Becker, "A Predictive Controller for Autonomous Vehicle Path Tracking," *IEEE Transactions on Intelligent Transportation Systems*, vol. 10, pp. 92-102, 2009.
- [10] C. E. Beal, "Applications of MPC to Vehicle Dynamics for Active Safety and Stability," PhD, Department of Mechanical Engineering, Stanford University, 2011.
- [11] H. Merabti, K. Belarbi, and B. Bouchemal, "Nonlinear predictive control of a mobile robot: a solution using metaheuristics," *Journal of the Chinese Institute of Engineers*, vol. 39, pp. 282-290, 2016/04/02 2016.
- [12] T. Xue, R. Li, M. Tokgo, J. Ri, and G. Han, "Trajectory planning for autonomous mobile robot using a hybrid improved QPSO algorithm," *Soft Computing*, pp. 1-17, 2015.
- [13] F. Yakub and Y. Mori, "Comparative Study of Autonomous Path-Following Vehicle Control Via Model Predictive Control and Linear Quadratic Control," *Journal of Automobile Engineering*, vol. 229(12), pp. 1695-1713, 2015.
- [14] R. Wallace, A. Stentz, C. E. Thorpe, H. Maravec, W. Whittaker, and T. Kanade, "First Results in Robot Road-Following," in *IJCAI*, 1985, pp. 1089-1095.
- [15] T. Kanade, C. Thorpe, and W. Whittaker, "Autonomous land vehicle project at CMU," in *Proceedings of the 1986 ACM fourteenth annual conference on Computer science*, 1986, pp. 71-80.
- [16] T. J. Gordon & M. Lidberg, "Automated driving and autonomous functions on road vehicles" *International Journal of Vehicles Mechanic and Mobility*, vol. 53, pp 958-994, 2015.
- [17] SAE-J3016, "Taxonomy and definitions for terms related to on-road motor vehicle automated driving systems," O.-R. A. V. S. Committee, Ed., ed, 2014.
- [18] O. Amidi, "Integrated Mobile Robot Control," Robotics Institute, Carnegie Mellon University CMU-RI-TR-90-17, 1990.
- [19] G. M. Hoffmann, C. J. Tomlin, D. Montemero, and S. Thrun, "Autonomous Automobile Trajectory Tracking for Off-Road Driving: Controller Design, Experimental Validation and Racing," in *American Control Conference*, 2007. ACC '07, 2007, pp. 2296-2301.
- [20] J. Wit, C. D. Crane, and D. Armstrong, "Autonomous ground vehicle path tracking," *Journal of Robotic Systems*, vol. 21, pp. 439-449, 2004.
- [21] J. Wang, J. Steiber, and B. Surampudi, "Autonomous ground vehicle control system for high-speed and safe operation," *International Journal of Vehicle Autonomous Systems*, vol. 7, pp. 18-35, 2009.
- [22] E. J. Rosseter, "A potential field framework for active vehicle lanekeeping assistance," stanford university, 2003.
- [23] C. Hu, H. Jing, R. Wang, F. Yan, and M. Chadli, "Robust output-feedback control for path following of autonomous ground vehicles," *Mechanical Systems and Signal Processing*, vol. 70-71, pp. 414-427, 3// 2016.
- [24] J. M. Snider, "Automatic Steering Methods for Autonomous Automobile Path Tracking," Robotics Institute, Carnegie Mellon University CMU-RI-TR-90-17, 2009.
- [25] S. Hima, B. Lusseti, B. Vanholme, S. Glaser, and S. Mammar, "Trajectory tracking for highly automated passenger vehicles," in *International Federation of Automatic Control (IFAC) World Congress,(Milano)*, 2011, pp. 12958-12963.
- [26] P. Zhao, J. Chen, Y. Song, X. Tao, T. Xu, and T. Mei, "Design of a control system for an autonomous vehicle based on Adaptive-PID," *International Journal of Advanced Robotic Systems*, vol. 9, 2012.
- [27] M. Nagai, M. Shino and F. Gao, "Study on Integrated Control of Active Front Steer Angle and Direct Yaw Moment," *JSAE Review*, vol. 233, pp. 309-315, 2012..