The Effectiveness of Integrating Geometer's Sketchpad Software in Phase-Based Geometric Learning

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***Abstract*—The aim of this study was to study the effect of learning activities based on the van Hiele's phases of learning geometry in the environment of GSP software on students' geometric thinking. A quasi-experimental design was employed in which participants were assigned to either the treatment or the control groups. Ten students, five from each group, were randomly chosen for interviews in order to identify the early levels of their geometric thinking. The items used in these interviews were the items contained in the van Hiele's Geometry Test (vHGT). Students in both groups were found to have achieved a good van Hiele first level of thinking in the pre-interview. In the post-interview, students were asked for their views whether a square was a parallelogram, and whether a square was a rhombus. However, for both of these items, almost all the students in the control group provided inaccurate answers. Meanwhile, three students in the treatment group agreed and provided clear justifications for those items. This finding indicates that activities in geometry topics based on the van Hiele's phases of learning geometry and aided with GSP software can be applied to the students in order to enhance their geometric thinking levels.**

Keywords—van Hiele's model, geometric thinking, Geometer's Sketchpad (GSP)

I. Introduction

Geometry is among the topics that contribute a high percentage in the secondary school syllabus. According to [1], geometry is an important topic in Mathematics and also one of the basic skills to be mastered by students [2]. According to [3], students are required to learn geometry starting from primary schools. Hence, the emphasis on geometry topics from the elementary school level is important because 40% of the mathematic topics in secondary schools are related to geometry. However, students' practice in school geometry curriculum in these days focuses only to recognize and name geometrical forms and learn to write symbols for simple geometric concept [4]–[7] as well as giving focus that require students to memorize definitions and characteristics of shapes [8][9]. [10][11] stated that more than half of seventh grade students believed that learning geometry is based on rules and assumed that they learn geometry just by memorizing. Moreover, for geometry topics contained in secondary school textbooks, we often find the emphasis on theorems and proofs. Such approach in textbooks assume the students are in the fourth level of geometric thinking namely deduction, while the study showed that more than 70% of students who entered high school were only at the first or second level of geometric thinking [12]. This view is supported by [13] and [14] who asserted that the main reason of the failure of the approach in the textbook on the topics of geometry in improving students' geometric thinking is the content in the textbook, which is delivered at a higher geometric thinking than the existing level of students' geometric thinking. Delivery of the curriculum also affects teaching practice. Mathematics is seen as something that is readily available and the teacher's task is to transfer what is available in the textbook with the most effective methods to students [15]. The task of a student is to absorb the mathematical content delivered by teachers as best as possible [16]. The conflict between students, geometry content of textbooks and teachers can only be solved when students, textbooks and teachers are on the same geometric thinking level [17].

As a result, the current teaching and learning practices of geometry are unable to help improve students' geometric thinking [18][19]. In fact, according to [5], van Hiele believed that by using the traditional approach, geometric thinking of secondary school students is not at a higher level than the level of thinking that students should have. There are a few studies in Malaysia which show that the level of geometric thinking of secondary school students in Malaysia is still at a low level [20]–[22]. One of the reasons why students are having difficulty to achieve a higher level of geometric thinking is teachers' teaching involved a higher level of geometric thinking than the level of students' thinking [13][14]. This

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happens because the teachers are on the fourth level, but most of the secondary school students are at level one and two. According to [23], learning without understanding is like learning by memorization, and memorizing the algorithm of routine questions is counted as not achieving any level of geometry thinking. Therefore, this study aims to study the implementation of effective geometric learning strategies in improving the students' geometric thinking skills. This study also aims to test whether or not geometric learning activities based on the phases suggested in the van Hiele model in the GSP software environment are able to improve the student's geometric thinking.

II. Geometry Learning Phases

Van Hiele model was introduced by Dina van Hiele-Geldof and Pierre van Hiele [36]. It consists of five levels of geometric thinking. Geometric thinking for primary school students is conducted from the first level to the third level, while the fourth and fifth levels are performed on high school students [24]. In the first level, which is visualisation, students recognise shapes based on their holistic appearances. In the second level which is analysis, students can analyze the form based on the components and the relationship between the components. While at the third level which is non-formal deductions, students can make connection between the characteristics of the forms and the rules by giving informal arguments. Students can also prove the theorem deductively and make connections between the theoretical networks at this level. The fifth level of van Hiele's model is rigor. At this level, students can set the theorems in different postulate systems. In addition, students can also analyze and compare the systems. The van Hiele levels of geometric thinking are created through the students' level of maturity in the study of the geometry subject from the introduction of simple to more difficult shapes. According to van Hiele, students will be assisted by appropriate instructions to pass through the five stages of the van Hiele level of geometric thinking. Students should not move beyond any level of thinking if they have not passed the previous level of thinking. The van Hiele model has introduced the van Hiele geometry learning phases to help students improve their geometric thinking. The learning phases consist of five phases namely information, guided orientation, explicitation, free orientation, and integration. Students have to go through the five phases of learning to reach every level of van Hiele's geometric thinking. Explanations of each phase are provided in Table 1.

TABLE I. The Five Learning Phases

Phase 1	Information	interaction between teachers and students through discussion is emphasised
Phase 2	Guided Orientation	students explore through planned activities
Phase 3	Explicitation	students can explain and express their views about the observed structure
Phase 4	Free Orientation	students can solve more complex tasks
Phase 5	Integration	students summarise what they have learned in order to build a new overall picture

III. Geometer's Sketchpad (GSP)

It is important to tackle the problem and reconfigure the strategies to rebuild students' interest in achieving geometric thinking. [25] recommended that the use of software technology is advantageous to help children think geometrically and to generate abstract reasoning skills. Besides, through technological tools, students can learn collaboratively with self-exploration [26]. This can be achieved through the use of the dynamic software, Geometer's Sketchpad (GSP). Extensive research has reported that GSP is really useful to increase the geometric thinking levels among students [1][5][6]. [27] in their studies observed the changes in discourse through the implementation of the dynamics geometry software, GSP within a period of four weeks in the United States. After the period of observation, they highlighted that the teaching methods of teachers changed from static discourses to dynamic interactions. They also mentioned in their studies that the shapes' manipulation in GSP alters the students' imagination and gives them a clearer image of shapes' transformation. [28] found that the implementation of GSP showed promising implications to form-three students' level of geometric thinking. [29] in their exploratory case study also showed a positive feedback on the GSP use. Based on the van Hiele's model of geometrical thinking, the research used phased-based instructions supported by GSP in order to investigate the students' geometrical thinking on the shapes of equilateral triangle and regular polygons. They highlighted that, through the interventions, the students' geometrical thinking on those polygons was significantly increased and enhanced.

IV. Objective

Therefore, this study aimed to investigate the effect of learning activities, which are based on the van Hiele's phases of learning geometry and on the environment of Geometer's Sketchpad (GSP) software, on students' geometric thinking using a qualitative approach.

V. Methodology

A. Research Design

A quasi-experimental pre-test/post-test design was employed in which participants were assigned to either the treatment or the control groups. Table 2 shows the design of the study.

TABLE II. Research Design

Group	Pre-test	Treatment	Post-test
Control (X_1)	O	X_1	O_1
Treatment (X_2)	O	X_2	O_1

A total of 94 students from a school in Negeri Sembilan, Malaysia were involved in the study and they were divided into two groups: (1) 47 students in the control group (X_1); and (2) 47 students in the treatment group (X_2). Table 3 shows demographics of the samples. It can be seen that 55 students were male students and 39 were female students. All the students involved in the study obtained a grade of A in mathematics in the UPSR Examination.

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TABLE III. SAMPLE DEMOGRAPHICS

		Group			
		Control		***Treatment***	
		n	***%***	***n***	***%***
Gender	Male	26	27.66	29	30.85
	Female	21	22.34	18	19.15
Mathematics Grade in UPSR	A	47	50	47	50
	B	-	-	-	-
Race	Malay	45	47.87	47	50
	Indian	2	2.13	-	0

For the treatment group, as shown in Fig. 1 students went through the same phase with different type of activities twice to move from level one to level two and then moved from level two to level three. This measure was taken because the researchers took into account the views of previous researchers who argued that lower secondary students are usually able to achieve up to three levels of van Hiele's geometry thinking, which is informal deduction [30]. The activities were conducted with the assistance of Geometer's Sketchpad (GSP). The topic involved was Transformation (Form Two), which includes the subtopics of Translation, Reflection, Rotation, and Quadrilaterals. For the control group, students learnt the same topic using a conventional approach. The teaching and learning process lasted for six weeks.

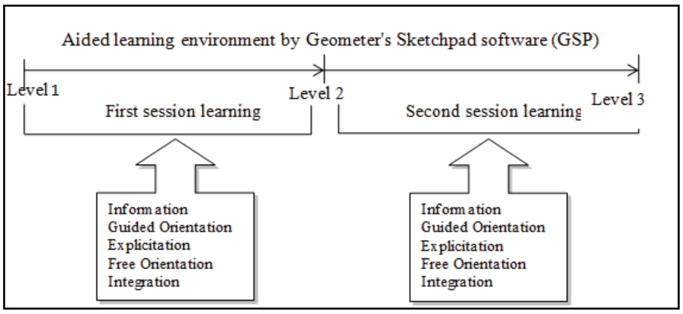

Fig. 1. Van Hiele's phases of learning geometry in the study.

B. Data Collection

Ten students, five from each group (A, B, C, D & E from the control group, and F, G, H, I & J from the treatment group), were randomly chosen for interviews in order to identify the early and final levels of their geometric thinking. The items used in these interviews were the items contained in the van Hiele's geometry test (VHGT)[31]. In this study, the Malay version of VGHT from the study by [20] was used. Each student was asked ten items in which item one, two, six, and seven represented the level-one item, which is visualisation; item three, four, eight, and nine represented the level-two item, which is analysis; and item five and ten were the level-three item, which is informal deduction. Table 4 summarises the interview questions, showing the level of items tested and their description.

TABLE IV. SUMMARY OF INTERVIEW QUESTION

Item	**Level tested**	**Description**
1,2,6 & 7	Level 1- visualisation	Student will be asked to draw a few quadrilaterals
3,4,8 & 9	Level 2- analysis	Student will be asked to explain the characteristics of certain quadrilaterals
5 & 10	Level 3 – informal deduction	Student will be asked to explain the relationship between quadrilaterals

The interviews between the researcher and the students were then transcribed for analysis.

VI. FINDINGS

C. Early Thinking Level of Control Group Students

For the first item, students were asked to draw a rectangle. Student A succeeded in drawing a picture of this rectangle by identifying the sample drawing of a rectangle based on its overall form. The same went for item two, which asked the students to draw a parallelogram. Student A made it to draw the picture of a parallelogram (Fig. 2). Two students did not recognise a rectangle. They got confused between a square and a rectangle. However, all the five students had no problem identifying a parallelogram.

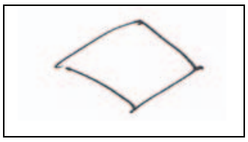

Fig. 2. Sample of a student's answer.

Item three asked the students to explain the characteristics of a rectangle. Student A and B only answered that it had two pairs of sides with equal lengths. For these answers, the researcher found that the students only provided explanation based on the overall shape and not focused on other important characteristics.

"left and right measures the same and top and bottom measures the same as well (referring to the sides)"

"this rectangle… is not like a square… a square has all sides measured the same but a rectangle has only two… two with exact measurement… two which measures the same"

The same applied to item four, which asked the students to explain the characteristics of a parallelogram whereby student E answered the following, showing that the student was only able to explain the characteristics of a parallelogram based on its shape (Fig. 3):

"for a parallelogram, the opposite sides are the same (referring to one pair of sides) and this one is also the same (referring to the other pair of sides) and their corners are not right angles."

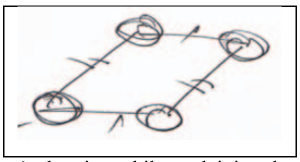

Fig. 3. Sample of a student's drawing while explaining the characteristics of a parallelogram.

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"This and this are the same (referring to a pair of opposite sides). This and this are the same (referring to another pair of opposite sides)"

Item five asked the students whether a rectangle is a parallelogram. All of the students disagreed because both shapes were different. They were only able to differentiate the shapes based on the outside appearance and not from the characteristics of these shapes.

"I don't think so because this parallelogram does not have the same shape as a rectangle."

Item six asked the students to point out which shape is a rectangle based on four shapes given, and item seven asked the students to choose which shape is a rhombus from the three shapes given. Almost all of the students did not have problems recognising a rectangle and a rhombus as most of them were able to correctly point out both shapes from the choices given. However, one student got confused between a rectangle and a square. For item eight and nine, most of the students were only able to associate with the characteristics of a rectangle and rhombus based on their physical characteristics and not focused on other specific characteristics.

"a rhombus… is like a diamond but… it is like a square but diamond shaped"

"it is not 90^o …" "maybe 90… 90 (referring to the inner angles)

For the last item, the students were asked whether a rectangle is a rhombus. All of the students disagreed because according to them, both have different shapes.

After analysing, it can be summarised that students A, B, D, and E had achieved a good level of visualisation, a below-average level of analysis, and none of them achieved the third level of thinking, which is informal deduction. Meanwhile, student C achieved a weak level of thinking for the first and second level and did not reach the third level of the van Hiele geometric thinking.

D. Early Thinking Level of Treatment Group Students

Students in the treatment group were labelled as F, G, H, I and J. For the first item which is a level one item, students were asked to draw a picture of a rectangle. Most of them were able to draw a picture of a rectangle well. However, one student was confused between a square and a rectangle. For item two which is still a level one item, students were asked to draw a picture of a parallelogram. Three out of five students were able to draw a picture of a parallelogram correctly. Two students did not know anything about a parallelogram at all. For the third item which is a level two item, students were asked to explain the characteristics of a rectangle. Only two students gave an answer. Among the answers given,

"each corner is 90^o"

"the top and bottom lines are parallel. This line (referring to the right side) and this (referring to the left side) are parallel"

However, the answers only provided a picture of the overall appearance of the shape. Two students only explained the answer based on the overall appearance. One student explained the characteristics of a square following a wrong answer to the first item due to him being confused between a rectangle and a square.

For the fourth item, which is a level two item, the students were asked to explain the characteristics of a parallelogram. Most of the students answered correctly; however, the answers given were not sufficient as they explained very little. The following is an example,

"characteristics of a parallelogram… these sides (referring to a pair of sides) are the same." "also the angles inside here make 360^o." "this is the same as this (referring to a pair of angles on opposite sides). This is the same as this (referring to another pair of angles on opposite sides)."

The fifth item is a level three item on whether a rectangle is a parallelogram. Most of the students disagreed because both rectangle and parallelogram have different shapes. Nearly all of the students had no problems recognising the shape of a square and a rhombus from other choices of shapes given. For item 8, which is an analysis level item, the students were asked to explain the characteristics of a square. They gave the correct answer, but their answers were insufficient. However, the answers given gave an indication on their level of analysis thinking (Fig. 4).

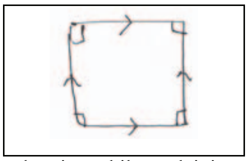

Fig. 4. Sample of a student's drawing while explaining the characteristics of a square.

"a square has each angle at 90^o."

"each of its corners and its lines (referring to sides) are the same size"

The ninth item is also an analysis level item, which asked students to explain the characteristics of a rhombus. Most of the students only related the characteristics of a rhombus to its overall appearance. The last item is a third level item, which is informal deduction whereby the students were asked whether a square is a rhombus. Nearly all of the students disagreed that a square is a rhombus because both have different shapes.

After analysis, five students namely student F, G, H and I were found to have achieved a good van Hiele first level of thinking. Meanwhile, student J achieved an average first level of thinking. Student F, G, H, and J achieved a weak second level of thinking, which involved analysis, and did not achieve the third level of thinking, which is informal deduction. On the other hand, student I achieved an average second and third level of thinking.

E. Final Thinking Level of Control Group Students

Item one, two, six, and seven represented level one of the van Hiele geometric thinking items, which is visualisation. The students were able to draw a picture of a rectangle for

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item one and a parallelogram for item two. Meanwhile, for item six and seven, all of the students correctly chose a square and a rhombus from the choice of shapes given. Item three, four, eight, and nine represented the second level of the van Hiele geometric thinking item, which is analysis. Item three asked the students to explain the characteristics of a rectangle. Three students made it to give a rather good explanation. Among the answers given were as follows,

"it has different angles, this one is 90º… this one is 90º …it has two equal sides… if this is 9 cm, this one is 9 cm too… same goes for these (referring to another pair of sides)" "it has two sides which measure the same. Also, each corner has the same angle and it has two symmetrical axes."

Two of the students were only able to explain the characteristics of a parallelogram in item four based on the overall appearance. An example, *"it has two sides measuring the same. That is all."* And *"a parallelogram has… same as this rectangle… these are the same length, this two is the same length, and these other two are the same length but not the same"*. For item 8 which asked the students to explain the characteristics of a square, most of the students were unable to provide all the characteristics of this shape. Among the answers given include the drawing shown in Fig. 5.

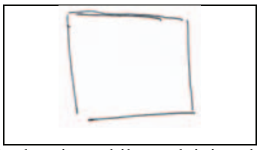

Fig. 5. Sample of a student's drawing while explaining the characteristics of a square.

"the characteristics of a square are… it all measures the same" "this (referring to the sides) is the same is this and this and this" "it has a 90º angle too. They are all the same… 90º, 90º, 90º, 90º … symmetry lines, there are 1, 2, 3, 4… 4 symmetry lines"

The characteristics given were not complete and did not focus on other more important characteristics.

Item nine, which is also a level two item namely analysis, asked the students on the characteristics of a rhombus. The students gave various answers. There were students who only gave the characteristics of a rhombus based on its overall appearance. There were also students who gave characteristics of a rhombus whereby in their list, there were several incorrect characteristics given. There were students who described the characteristics of a rhombus well; however, their answers were incomplete. Nevertheless, there was one student who explained the characteristics of a rhombus well. Item five and item ten represented third level geometric thinking item, which is informal deduction. Item five asked the students for their views on whether a square is a parallelogram, and item ten asked the students on whether a square is a rhombus. However, for both of these items, almost all the student provided inaccurate answers.

As a conclusion, all the students A, B, C, D, and E achieved a high level of thinking for the first level. Almost all the students A, B, C, and D achieved a moderate second level of thinking, which is analysis. Only one student, student E achieved a high second level of thinking. However, none of the students in the control group achieved the third thinking level, which is informal deduction.

F. Final Thinking Level of Treatment Group Students

This section discusses the final thinking level of students in the treatment group. For item one and two, which are first level item, all the students had no problem in drawing a picture of a rectangle and a parallelogram. The same applied for item six and seven, which are level one item whereby all of the students had no problem choosing the correct pictures namely a square and a rhombus. Item two and three represented level two item. Item two asked the students to explain the characteristics of a rectangle. Almost all of the students succeeded in providing as many characteristics of a rectangle as possible. Among the answers include the following:

"a rectangle, all sides are the same length. The length of a rectangle is the same as the length of its side. The diagonal lengths are also the same. The measurements of the lengths are the same too. Its inside angles are all the same, which is 90º"

For the fourth item, the students were asked to explain the characteristics of a parallelogram. Only one student managed to give as many characteristics of a parallelogram as possible. Two students were able to state the characteristics of a parallelogram, but it was incomplete and did not focus on the more important characteristics. The answers given were as follows:

"a parallelogram, its opposite sides are the same length" "the opposite angles are the same. The diagonal lines are not the same length."

"a parallelogram, the opposite sides are the same length, and the opposite angles are equal"

These answers were correct and clearly reflected a certain level of thinking. However, it was incomplete and the justifications were not enough. Item eight and nine are also a level two item, which is analysis. Item eight asked the students to explain the characteristics of a square. The students were asked about the characteristics of a rhombus in item nine. Among the answers given for item eight was the drawing shown in Fig. 6.

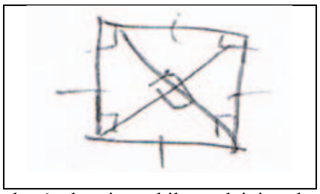

Fig. 6. Sample of a student's drawing while explaining the characteristics of a square.

"a square, the inner angles are 90º. The diagonal lines are the same length" "The diagonal lines also have a 90º angle" "all the sides are the same length" "the diagonal lines split into equal halves."

However, for item nine, there were a few incorrect characteristics stated by the student. For example:

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"a rhombus… its sides are all the same… the diagonal lines are the same. Also, the sides are parallel"

Item five and ten represent third level item, which is informal deduction. In item five, the students were asked whether a square is a parallelogram. Meanwhile, item ten asked the students for their opinion on whether a square is a rhombus. Three students agreed and gave clear justifications for item five. One of the justifications given was as follows:

"agree, because there are characteristics of a square that are also found on a parallelogram" "on the diagonal lines dividing into two, the opposite lines are the same length, the opposite angles are the same"

In this answer, the student was able to classify the shape of a square in logical manner. The same applied for the following answer given when asked to state whether a square is a rhombus:

"agree, there are characteristics of a square that are also found on a rhombus" "its sides are the same length, it has two diagonal lines, when the diagonal lines are standing straight, it will form a 90º angle"

After analysing the transcript of the interview, three students namely students F, I, and J succeeded in achieving all three levels of thinking, which are visualisation, analysis, and informal deduction. Student G achieved the first level that is complete, an average second level, and did not reach the third level of thinking. Meanwhile, student H succeeded in achieving the first level that is complete and an average second and third level.

VII. Discussion and Conclusion

Based on the findings it is found that learning activities based on the phases of van Hiele geometry, are able to improve the geometric thinking level of students. The integration of the GSP software into these activities is also seen as helping to improve the students' level geometric thinking. According to [32], GSP is the best software in geometric learning that can help improve the students' level of geometric thinking, at least at the third level. The integration of dynamic geometry software in these geometric learning phases is consistent with previous studies conducted in Malaysia. Among them is the study by [33] that identified the students' level of van Hiele's thinking in Cube and Cuboid topics after being exposed to geometric learning phases using GSP. In this study, the students' level of geometric thinking increased at least at one level. Before the sample from each control and treatment group was selected for interview in this study, a descriptive analysis was first conducted. The findings show that no students in both groups showed a decline from the initial level to the final stage of their geometric thinking. However, most of the students in the control group did not show any change in the initial and final levels of geometric thinking. At the same time, most of the students in the treatment group showed an increase in the level of geometric thinking both from level 1 to level 2 and from level 1 to level 3. This finding also shows that the progress from one level of thinking to another level of thinking is more dependent on the learning rather than on the students themselves [34]. In addition, [34] also notes that transferring students from one level of thinking to a higher level of thinking is the process initiated by the students themselves. Although it is not a natural process, it is influenced by the teaching and learning process [35]. This study used a qualitative approach through interviews in identifying students' geometric thinking. This study is more accurate to determine the level of geometric thinking as compared to using a written test that contains multiple-choice questions. However, the interview protocol used was taken from an established instrument [31] that has a high level of validity. The findings of this study suggest that geometry contents and learning activities organized based on the learning phases proposed by the van Hiele model are able to increase the level of students' geometric thinking. In addition, in line with the objective of the geometer's sketchpad software to develop students' geometric thinking, they can be used as a medium in the implementation of the geometric learning phases. However, this study only focused on the first three levels of geometric thinking in the van Hiele model. Therefore, future studies should test the effectiveness of the phases of geometric learning at the higher level of students' geometric thinking. Besides, other types of geometry software can also be used, such as GeoGebra which is free and open-source software. In addition to the topic of quadrilaterals, other geometry topics, such as Triangles, Circle, Transformation, Isometry, can also be applied in the van Hiele's phases of geometric learning.

Acknowledgments

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References

[1] A. H. Abdullah and E. Zakaria, "The activities based on van Hiele's phase-based learning: experts' and preservice teachers' views," Journal of Mathematics and Statistics, vol. 8, no. 3, pp. 385-395, 2012.

[2] National Council of Teachers of Mathematics (NCTM), Principles and Standards for School Mathematics, Reston: VA, 2000.

[3] M. S. Abu, M. B. Ali and T. T. Hock, "Assisting primary school children to progress through their van Hiele's levels of geometry thinking using Google SketchUp," Procedia-Social and Behavioral Sciences, vol. 64, pp. 75-84, 2012.

[4] E. Yetkin, "Student difficulties in learning elementary mathematics," ERIC Clearinghouse for Science, Mathematics, and Environmental Education, DIGEST, 2003.

[5] I. Noraini, Pedagogy in Mathematics Education, 2nd ed. Kuala Lumpur: Utusan Publication Sdn. Bhd, 2005.

[6] S. Olkun, N.B. Sinoplu, and D. Deryakulu, "Geometric explorations with dynamic geometry applications based on van Hiele levels", International Journal for Mathematics Teaching and Learning, vol. 1, no. 2, pp. 1–12, 2009.

[7] E. Halat, "Reform-Based Curriculum & Acquisition of the Levels", Eurasia Journal of Mathematics, Science & Technology Education, vol. 3, no. 1, pp. 41-49, 2007.

[8] M.T. Battista, "Learning geometry in a dynamic computer environment", Teaching Children Mathematics, vol. 8, pp. 333–339, 2002.

[9] I. Noraini, "The impact of using Geometer's Sketchpad in Malaysian students' achievement and van Hiele geometric thinking", Journal of Mathematics Education, vol 2, no. 2, pp. 94–107, 2009.

[10] L. Ames, The effect of incorporating Geometer's Sketchpad in a high school geometry course to improve conceptual understanding, inductive

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reasoning and motivation. Unpublished Master's Thesis. William Paterson University of New Jersey, 2011.

[11] V. L. Kouba, "Results of the fourth NAEP assessment of mathematics: Measurement, Geometry, Data Interpretation, Attitudes, and Other Topics", Arithmetic Teacher, vol. *35*, no. 9, pp. 10-16, 1988.

[12] D. J. Brahier, Teaching Secondary and Middle School Mathematics, 2nd ed. Boston, MA: Pearson Education, Inc, 2005.

[13] M. De Villiers, Rethinking proof with the Geometer's Sketchpad, Berkeley, CA: Key Curriculum Press, 2003.

[14] K. Khairiree, "Exploring geometry with the Geometer's Sketchpad," in Integrating Technology in The Mathematical Sciences, A.H. Yahya, B. Adam, M.I. Ahmad Izani, H.L. Koh and H.C. Low,Eds. Pulau Pinang: Universiti Sains Malaysia Press, 2004, pp. 145-153.

[15] A. Z. Noor Azlan, The Development and Challenges of Education in The 21st Century, Ceramah Profesor Perdana, Universiti Teknologi Malaysia, 2005.

[16] N. P. Nik Azis, Agenda Tindakan: Penghayatan Matematik KBSR dan KBSM, 1st ed. Kuala Lumpur: Dewan Bahasa dan Pustaka (DBP), 1992.

[17] R. R. Parsons, Teacher Beliefs and Content Knowledge: Influences on Lesson. Crafting of Pre-Service Teachers During Geometry Instruction, Unpublished PhD's Thesis. Washington State University, 1993.

[18] A. Craft, Creativity Across The Primary Curriculum: Framing and Developing Practice, 1st ed. London: Routledge, 2000.

[19] A. H. Abdullah and M. Mohamed, "The use of interactive software (IGS) to develop geometric thinking", Jurnal Teknologi, vol. 49, pp. 93-107, 2008.

[20] B. L. Tay, A van Hiele-Based Instruction and Its Impact on The Geometry Achievement on Form One Students, Unpublished Master's Project. Universiti Malaya, 2003.

[21] L. T. Hong, van Hiele Levels and Achievement in Writing Geometry Proofs among Form 6 Students, Unpublished Master's Project, Universiti Malaya, 2005.

[22] R. Jamaludin, Penilaian Tahap Pemikiran Geometri van Hiele Pelajar Tingkatan Dua, Unpublished Master's Project, Universiti Malaya, 2006

[23] J. A. van de Walle, Elementary and Middle School Mathematics: Teaching Developmentally. Boston: Pearson Education, 2001

[24] W. F. Burger and J. M. Shaughnessy, "Characterizing the van Hiele levels of development in geometry", Journal for research in mathematics education, pp. 31-48, 1986.

[25] A. H. Abdullah, J. Surif, N. H. Ibrahim, M. Ali and M. H. Hamzah, "The development of MyGSP: an online resource for teaching mathematics based on Geometer's Sketchpad (GSP)", Asian Social Science, vol. 10, no. 22, pp. 227–240, 2014.

[26] P. Shadaan and K. E. Leong, "Effectiveness of using GeoGebra on students understanding in learning circles," Malaysian Online Journal of Educational Technology, vol. 1, no. 4, pp. 1-11, 2013.

[27] N. Sinclair and V. Yurita, "To be or to become: how dynamic geometry changes discourse," Research in Mathematics Education, vol. 10,no. 2, pp. 135-150, 2008.

[28] N. Idris, "The effects of Geometer's Sketchpad on the performance in geometry of Malaysian students' achievement and van Hiele geometric thinking," Malaysian J. Math. Sci., vol. 1, no. 2, pp. 169-180, 2007.

[29] C. M. Chew and C. S. Lim, "Enhancing primary pupils' geometric thinking through phase-based instruction using the Geometer's Sketchpad," Asia Pacific Journal of Educators and Education, vol. 28, pp. 33-51, 2013.

[30] C. I. Saifulnizan, Development of Learning Modules using Interactive Geometry Software, Unpublished Master Thesis. Universiti Teknologi Malaysia, 2007.

[31] Z. Usiskin, Van Hiele levels of achievement in secondary school geometry, Final report of the Cognitive Development and Achievement in Secondary School Geometry Project. Chicago, University of Chicago, 1982.

[32] W. Finzer, and D. Bennett, "From drawing to construction with the Geometer's Sketchpad," Mathematics Teacher, vol. 88, no. 5, pp. 428-431, 1995.

[33] C. M. Chew, Assessing Pre-Service Secondary Mathematics Teachers' Geometric Thinking, Proceedings of the 5th Asian Mathematical Conference, Malaysia, 2009.

[34] P. M. van Hiele, Structure and Insight. A Theory of Mathematics Education, Orlando, FL: Academic Press, 1986.

[35] M. Mason, The van Hiele Levels Of Geometric Understanding, Professional Handbook For Teachers, geometry: Explorations and Applications, Boston: McDougal Inc, 1998.

[36] P. van Hiele, The Child's Thought and Geometry, Brooklyn, NY: City University of New York, 1959.

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