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Binary Linear Programming Model in Solving Bus Crew Problem as Tactical Fixed Task Scheduling

Binary Linear Programming Model in Solving Bus Crew Problem as Tactical Fixed Task Scheduling

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Abstract. In this paper, we consider a crew scheduling problem (CSP) of bus transportation with the objective of minimizing the cost of crew members. We address a different time frame and different type of crew members. The maximum total working limit for each type of crew member has been set. The problem can be formulated as a Tactical Fixed Task Scheduling Problem (TFTSP) where the ready time and due date of the tasks are fixed in advance. A Binary Linear Programming (BLP) model is used to obtain an optimal solution for the problem. We conduct a testing and implement the model using LINGO 17.0 software. Results of the computational testing give optimum values for the problem. Hence, a feasible crew scheduling is obtained.

1. Introduction

Task scheduling is one of the challenging industrial problems. This problem typically arises in manufacturing and service industries. The purpose of the scheduling is to determine the allocation and the sequence of the operations to the target. A desired performance measure will be used to present the efficiency of the assignment. The difficulty of the problem is that the scheduling needs to satisfy a set of requirements as well as a range of environmental constraints. The problem is known to be NP-complete.

Crew Scheduling Problem (CSP) is one of a real-life examples that involves task scheduling in service industries. Industries that relate to this CSP are bus transportation, airline, telecommunication, railway and many others. In this paper, we consider a bus crew problem with the existence of different route, different time frame, different type of crew member and different cost applied.

The bus crew scheduling problem can be stated as follows: We are given a set of n tasks, $N_i = N_1$, N_2, …, N_n that are to be assigned on a set of m drivers, $Q_k = Q_1$, Q_2, …, Q_m. The crew duties are the independent tasks with different route and different start time and end time. The drivers are said to be unrelated, when the processing time are different among the drivers. Figure 1 depicts an example of a task graph for crew scheduling problem. Each node represents a location and each edge represents a bus route with the start and end time.

We refer to this problem as scheduling problem with unrelated parallel processors in minimizing cost and denoted as $R|r_i,e_i|c_{min}$. We develop a Binary Linear Programming (BLP) model to solve the problem. This paper is organized as follows. Section 2 presents the literature review of the recent work in the CSP field. Then, in Section 3, the detail of the CSP is discussed with the mathematical formulation. Section 4 conducts a case study using the model. Lastly, Section 5 summaries the results.
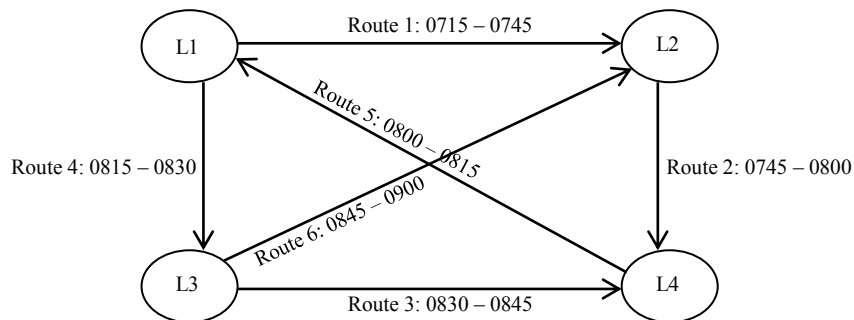
Figure 1: A directed task graph

2. Literature Review

The CSP is an area that still been studying until now. For example, recently, Ciancio et al. [2] proposed integrated approach to solve scheduling problem for local public bus companies. Their objective is to assign drivers to vehicle schedules. Hoffman and Buscher [4] focused crew scheduling in regional rail transit. They modeled an arc flow formulation to minimize crew costs that satisfies operating conditions and legal requirements. The intensive review on the crew scheduling can be found in Deveci and Demirel [6] that focused in airline crew; and Suyabatmaz and Sahin [1] in railway crew.

In this research, CSP is considered to be an NP hard problem (Boyer et al. [8]). The bus CSP has been addressed by Oztop et al. [3]; Boyer et al. [8]; and Chen and Niu [5]. Oztop et al. [3] considered bus CSP with different types of vehicles that require different crew capabilities. They introduced eligibility constraint in the model based on the competencies to use certain vehicle types. In Boyer et al. [8], they proposed Variable Neighorhood Search Algorithm to integrate vehicle scheduling problem and CSP in bus transportation systems. They considered labor regulation constraints in the model. Chen and Niu [5] developed a new heuristic procedure using tabu search algorithm in minimizing the total idle time. They generated the initial solution from the time shift of trips and work intensity constraints. We continue the bus CSP with minimizing the cost and detemining the optimal number of drivers needed.

3. Mathematical Model

In this section,we provide a mathematical model for CSP which attempts to assign the bus crews to the trips without overlap (ie. one bus crew will be assigned to one task only at one time). The model also able to determine the optimum number of bus crew and hence minimize the total cost. Our CSP problem can be defined as TFTSP, where the ready time and the deadline of the task are fixed approximately known. Ready time is the start time for the bus crews start their trip. Deadline is the finish time for the cycle trip where it define as their ready time plus processing time. The processing time of the task assigned to the crew are include the working time and the travel time to reach the starting point of the route. The drivers cannot exceed the maximum total working time limit. In the model, task preemtion is not allowed. There are no interruption once the driver has started until he finish the cycle trip. The sequence of the tasks assigned in a shift cannot be migrated to another driver to avoid overlaps. One trip can be done by one crew only. All crew members are assume available at time 0715 and must covered all tasks. Cancellation or delaying in the task assigned are not permitted. However, the driver can have the idle time during their shift. Each trip can be completed within the time frame given. Any unusual traffic flow is not considered in the model. We declare two types of bus crew for this CSP which are full time and part time. Each type of bus crew have their own working time limit and certain fixed cost is incurred.

From all these descriptions and assumptions, our CSP can specifically be denoted based on the established three-filed notation of Graham et al. [7] as $R|r_i,e_i|c_{min}$, i.e. the task scheduling problem for minimizing the cost on unrelated parallel processors with the model requires ready date and end date to

be known in advance. In this case, the tasks are the bus trips that need to be accomplished and the unrelated parallel processors are referred to the bus crews.

3.1. Notations
The following notations are used for the problem under consideration.
Sets:

N Set of tasks

Q Set of crew members (drivers)

I_i Set of incompatible task for task $i \in N; I_i \subset N$

Parameters:

r_i Ready time of task $i \in N$

e_i Deadline of task $i \in N$

a_{ij} Setup time between tasks $i \ and \ j, i, j \ \in N$

W Working time limit

c_k Fixed cost of driver $k \in Q$

Decision Variable:

$x_{i,k}$ 1 if eligible task i is assigned to driver k, 0 otherwise

y_k 1 if driver k is used, 0 otherwise

3.2. Mathematical Formulation
In this section, a CSP mathematical model is developed and modified from Oztop et al. [3]. Our model did not consider spread time limit as in Oztop et al. [3]. The BLP model for the problem can be written as follows:

Minimize $\sum_{k=1}^{Q} c_k y_k$
subject to:

$$\sum_{k \in Q} x_{i,k} = 1 \qquad\qquad , \forall i \in N \qquad\qquad\qquad (1)$$
$$x_{i,k} + x_{j,k} \leq 1 \qquad\qquad , \forall i, j, I_i \in N, k \in Q \qquad (2)$$
$$\sum_{i \in N}(e_i - r_i)x_{i,k} \leq W y_k \qquad , \forall k \in Q \qquad\qquad\qquad (3)$$
$$x_{i,k}, y_k \in \{1,0\} \qquad\qquad , \forall i \in N, d \in D \qquad\qquad (4)$$

Constraint (1) is to ensure that all tasks are covered. Constraint (2) is to ensure that incompatible task pair are not assigned to the same driver. Note that setup times between tasks are considered in definition of the incompatibility set. Thus, the setup time parameter (a_{ij}) does not appear in the formulation. The spread time parameter (S) does not appear in the model due to the same reason. Constraint (3) is to guarantee that the sum of the processing time of task assigned to a driver does not exceed the total working time limit. Constraint (4) is to ensure the value obtained is binary.

4. Experimental Design
This section carries out a computational testing of the BLP model. First, we design and implement a case study to evaluate the performance of the model. Then, we report the results of BLP.

4.1. A Case Study
This case study is intended to demonstrate the whole process of CSP. The case study consists of 6 routes that come from 13 colleges in UTM (i.e. KDOJ, KTGB, KTR, KTHO, KTDI, KDSE, KRP, KTF, KP, K9, K10, KTC and FBME) and need to complete Lingkaran Ilmu (LI) route as shown in Figure 2. Each route caters a cluster of collages that located next to each other. For example, KDOJ and KTGB are considering as one cluster and have been assigned in route 1 to LI. We consider 6 drivers and 18 trips with different time frame between 0715 until 1700. These 18 trips represent the tasks that assigned to the drivers. The job data for the CSP is shown in Table 1. The start time and end time is displayed for

each route and the time values in the table are set in 24-hours format. The task list T_i of the trip that assigned to the drivers is in ascending order. T_1 will start with $R1$ which starts 0715 to 0735, followed by T_2 with $R2$ which starts at 0735 to 0755 until the final task T_{18} for $R6$ which starts at 1600 to 1700.

Figure 2: Path of Lingkaran Ilmu

Table 1. Instance for CSP case study

Route, Rn	$r_i - e_i$
$R1$: KDOJ & KTGB – LI	0715 – 0735, 1015 – 1045, 1400 – 1420
$R2$: KTR, KTHO & KTDI – LI	0735 – 0755, 1045 – 1115, 1420 – 1440
$R3$: KDSE, KRP & KTF – LI	0755 – 0815, 1115 – 1145, 1440 – 1500
$R4$: KP – LI	0815 – 0835, 1145 – 1215, 1500 – 1520
$R5$: K9, K10, KTC – LI	0835 – 0915, 1215 – 1255, 1520 – 1600
$R6$: FBME – LI	0915 – 1015, 1255 – 1400, 1600 – 1700

4.2. Computational Results

We implemented the zero-one integer problem and Branch and Bound (B&B) that ran with LINGO 17.0 package. The BLP gives the optimum result for the instance problem. We now present our results of the BLP model. There are two types of drivers that we considered in the testing, which are part time drivers and full time drivers. The objective of the experiment is to find the optimal numbers of drivers to be assigned with minimum cost.

4.2.1 Case 1: $c_k = 1500$ for $k = \{1, 2, 3, 4, 5, 6\}$ and $W = 4$

For part time drivers, the maximum working time limit is 4 hours with a cost of RM1500 if the driver is assigned. From LINGO 17.0, the optimal number of driver needed in performing these 18 tasks was only 4 out of 6 crews with cost RM6000 and average working time is 145 minutes. In this case, driver 1, 2, 3 and 5 will be assigned to a different task with different start time and end time without clash. The driver might have a long setup time or idle time between the tasks. For driver 1, the ready time for his first task, T_3, is at 0755 from KDSE, KRP & KTF and end at 0815 at LI. Then, ready time for task 6 from FBME is at 0915. The last task is at 1520 until 1600. For the driver 2, he needs to perform T_5, T_8,

T_{10} and T_{14}. The start time for the T_5 is at 0835 and finishes the final task T_{14} at 1440. For driver 3, he needs to perform tasks T_1, T_{11}, T_{13} and T_{18} with ready time for T_1 is at 0715 and end the trip T_{18} at 1700. For the driver 5, he need to complete T_2, T_4, T_7, $T_{9,}$ T_{12} and T_{16} from 0735 until 1520. A feasible schedule for the whole trip is obtained as in Figure 3 where D1, D2, D3 and D5 are refer to driver 1, driver 2, driver 3 and driver 5 respectively.
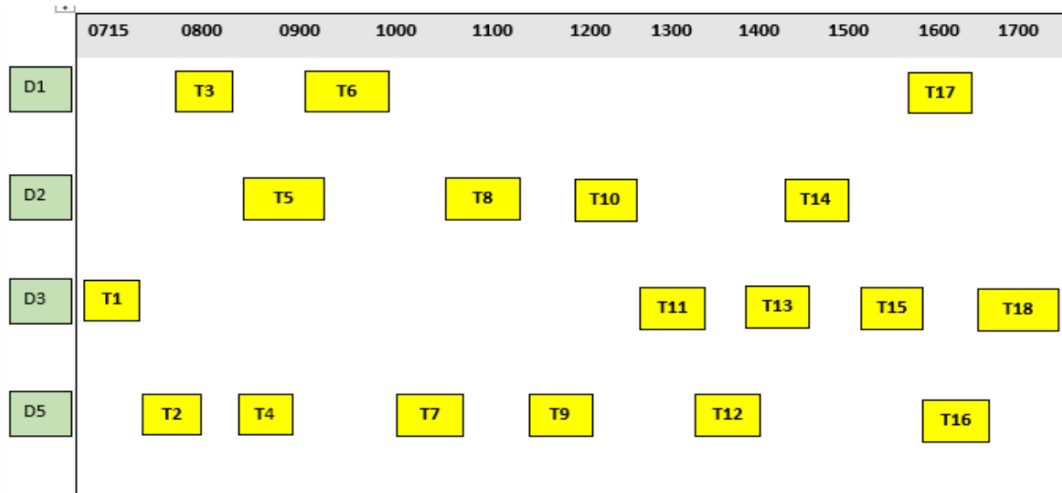

Figure 3: Schedule for part time drivers

4.2.2 Case 2: c_k = 3000 for k = {1, 2, 3, 4, 5, 6} and W = 9

Case 2 is for full time drivers where the maximum working time limit is 9 hours and RM3000 is the cost if the driver is assigned. In this case, there are no incompatible tasks since all the task have different start time and end time. The result obtained from LINGO 17.0, only 2 full time drivers needed to implement all the 18 tasks with ready time 0715 to 1700 per day with cost RM6000. The average work times of both crews are 290 minutes. Driver 1 need to perform task T_1, T_3, T_5, $T_{7,}$ T_9, T_{11}, $T_{13,}$ T_{15} and T_{17}. The earliest start time is at 0715 and ends at 1600. While, driver 2 needs to perform tasks T_2, T_4, T_6, $T_{8,}$ T_{10}, T_{12}, $T_{14,}$ T_{16} and T_{18}. There are setup times between the tasks given. Figure 4 illustrates a schedule for full time drivers.
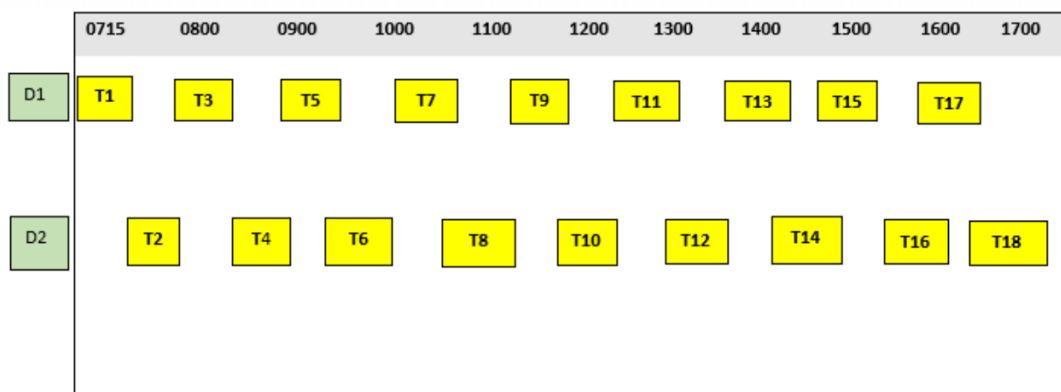

Figure 4:Schedule for full time drivers

5. Summary

In this paper, a case of CSP for bus transportation problem is identified. The objective function of the problem is to find the optimal number of bus crew as well as minimize the total cost. Our strategy in

solving the CSP is formulating a BLP model. A case study is conducted using 6 routes in Universiti Teknologi Malaysia. This study considered 2 cases, which are full time and part time drivers. The results show that only four part time or two full time drivers can be assigned and give a minimum cost.

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