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Emerging patterns and problems of higher-order thinking skills (HOTS) mathematical problem-solving in the Form-three assessment (PT3)

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Higher-order thinking skills (HOTS) mathematical problem-solving was first introduced in the Form-3 assessment (PT3) in 2014. However, to date, there have been no studies of students' ability to solve the mathematical problems in this assessment. Therefore, this study investigated the emerging patterns and problems of HOTS mathematical problem-solving in the PT3. This investigation was a case study and classified under a qualitative research approach. Oral reporting (i.e. thinking aloud protocol) was used to obtain the data. The participants were 10 Form-3 students who were candidates for PT3 in 2015. They were students in a secondary school in a district in Johor Bahru. The results show that students who successfully solved the HOTS mathematical problems produced the same process starting with understanding, followed by phases of planning, implementation, and ending with the final answer. The students who failed to answer the HOTS mathematics questions produced a solution pattern starting with understanding followed by planning and implementation. Based on the patterns, this study also identified the problems that emerged in every step of the HOTS mathematical problems-solving processes and discusses how they could be overcome and improved.

Keywords: Form-three assessment (PT3); higher-order thinking skills (HOTS); mathematical problem-solving; Model of Polya

Introduction

An education system plays an important role in the development of knowledgeable and highly skilled human capital that can fulfil the needs of developing countries like Malaysia and South Africa. Therefore, education holds the responsibility to deliver appropriate knowledge and skills to the society. In this regard, transformation in the Malaysian education system may benefit education in both countries. The Malaysia Education Development Plan (PPPM) 2013–2025 emphasises the concept of higher-order thinking skills (HOTS) in order to produce students who possess high and competitive values that may meet the demands of the twenty-first century (Ministry of Education [MOE], 2012a). Higher-order thinking skills have been considered as the highest level in the human cognitive process. It takes place when a person obtains new information and then saves, organises, associates and uses it with existing knowledge in order to achieve a goal or a solution in complicated situations (MOE, 2012a).

Problem-solving in mathematics is one of the important elements that should be emphasised in the process of teaching and learning mathematics. Polya (1957) states that the problem-solving process involves the use of heuristic applications. This process should not be predictable as the solution would then no longer be heuristic. The National Council of Teachers of Mathematics ([NCTM], 2000) states that the mathematical problem-solving process may help students acquire new mathematical knowledge. Therefore, problem-solving is one of the elements emphasised in the Mathematics Curriculum Framework of Secondary Schools in Malaysia (Curriculum Development Centre, 2002). Similarly, according to Petersen (2016), the Mathematics curriculum currently used in South African classrooms emphasises problem-solving in order to develop critical thinking.

The previous analysis of The Trends in International Mathematics and Science Study (TIMSS) in 2007 showed that Malaysia ranked low compared to other countries in terms of average achievements in Mathematics. Out of 60 participating countries, Malaysia was placed 26th with a score of 474 (500 is the TIMSS mathematics scale average). In TIMSS 2011 Malaysia recorded a decline by scoring only 440. The same situation happened in South Africa. According to Howie (2004), Reddy (2006) and Siyepu (2013), South African Grade 8 learners performed poorly in the TIMSS. Table 1 shows a comparison of the 8th grade Malaysian and South African Mathematics scores in TIMSS from 1999 to 2011 with an international average score (Mullis, Martin, Foy & Hooper, 2016).

Table 1 Comparison of the 8th grade Malaysian and South African Mathematics scores in TIMSS from 1999 to 2011 with an average score of TIMSS and an international average score

Year	Malaysia	South Africa	International average score
1999	519	-	487
2003	508	285	467
2007	474	-	450
2011	440	352	467

According to the Descriptions of the TIMSS International Benchmarks of Mathematics Achievement (Mullis et al., 2016), Malaysian students are only able to use basic mathematical knowledge in simple situations. They are able to solve problems with sentences involving a one-step solution only. Although they can interpret and read graphs or tables, their understanding is limited to simple algebraic relations and basic geometrical concepts. According to Hashim, Razali and Jantan (2003), HOTS questions provided in TIMSS may challenge students' thinking skills and help them strengthen their knowledge. However, Malaysian students are having difficulties in solving these questions (Jayarajah, Saat & Rauf, 2013). Therefore, teachers need to provide an appropriate learning environment in the classroom so that their students can handle HOTS questions more effectively (Brissenden, 1980). According to Schoenfeld (1985) students are innately and potentially capable of solving problems. However, they may not possess the appropriate skills to manage strategies to solve a given problem. Therefore, teachers play an important role in educating their students in problem-solving.

Taking into account the Malaysian students' poor performance in solving mathematics problems in international assessments like TIMSS and The Programme for International Student Assessment (PISA), the implementation framework of HOTS in a classroom was introduced. One of the components requires several changes in assessment. The National Education Assessment System (SPPK) dictates three changes in assessment: (1) providing alternative assessments; (2) reviewing an examination-oriented assessment system; and (3) strengthening the quality of the assessment and evaluation systems. This is implemented through the incorporation of assessment results from schools with those from centralised bodies. The Malaysia Examination Board (LPM) has also improved the existing assessment system, which includes incorporating HOTS questions in the PT3. Form-three assessment is a summative assessment that evaluates the academic performance of students at the lower secondary level. The subjects assessed in PT3 are English, Bahasa Malaysia, History, Mathematics, Islamic Education, Science, Geography, Integrated Living Skills (Agriculture, Home Economics, Technical and Trading Skills, and Entrepreneurship) and five additional languages (Chinese, Arabic, Kadazan Dusun, Tamil and Iban). Form-three assessment assesses the scores of these subjects from Form One to Form Three based on the goals, objectives and content of the subjects.

This study aimed to investigate the emerging patterns of HOTS mathematical problem-solving in the PT3. To achieve this aim, the researchers chose the Model of Polya, which is the most basic model in solving mathematical problems. Through this model, the assessment process of HOTS problems is expected to become easier. In the Integrated Curriculum of Secondary School (KBSM), the problem-solving model used was based on the Model of Polya (MOE, 2003). In the current study, the researchers identified the problems that emerged in every step of the HOTS mathematical problems-solving processes and discussed how to overcome and improve them.

Literature Review

In the preliminary report of the PPPM 2013–2025 (MOE, 2012b), the MOE plans to launch the Standard Based Curriculum for Secondary Schools (KSSM) by 2017. The school curriculum, therefore, needs to be revised in order to incorporate balanced knowledge and skills. This revision includes problem-solving skills that need to be developed comprehensively and integrated across the Mathematics curriculum. More crucially, national examinations and assessments need to be reviewed to shift the focus to HOTS.

Problem-solving in mathematics

Problem-solving in mathematics is a branch of mathematical knowledge. In the learning of mathematics, problem-solving is the most important aspect that should be taught. Goldstein and Levin (1987) define problem-solving as a high-level cognitive process that requires the control of basic and routine skills. Applying existing knowledge and skills in order to identify a different and unusual method is often a problem-solving process required for an individual. Krulik and Rudnick (1989) state that problem-solving is a complex and difficult skill to learn. It consists of a series of tasks and thinking processes that are associated with the formation of a set of heuristics. Students are expected to develop new knowledge and skills through the application of various strategies in a problem-solving process.

Problem-solving can generate two forms of thinking: systematic and logical thinking. Systematic thinking focuses on the parts of a whole using a methodical, step-by-step and linear approach when solving a mathematical problem. Logical thinking, on the other hand, refers to the reasoning applied before students make any conclusion (Ison, 2010). Mathematics is a subject that requires systematic measures when solving a problem. It also generates methodological and logical thinking that requires a thorough check in order to determine the rationale of a solution (Salleh, 2006). Kohn (2009) states that an increasing number of teachers only coach their students on answering examination questions, thus reducing the number of students who can think critically. In this situation, students demonstrate lower-order thinking skills that require them to remember and memorise only. Based on the TIMSS results in 2007, Faridah and Effandi (2010) claim that Malaysian students only solve problems by writing sentences with a one-step solution and using their basic mathematical knowledge in easy situations. In addition, Malaysian students only understand simple algebraic relationships. Their proficiency is, therefore, limited to basic concepts of geometry. This clearly shows that they are not exposed to HOTS questions that seem appropriate for application in their real lives.

HOTS in problem-solving

Higher-order thinking is a widely accepted application that requires a student to interpret, analyse or manipulate information in order to solve a problem (Onosko & Newmann, 1994). Thompson (2008) states that higher-order thinking is a problem-solving technique that requires students to provide an explanation and a clarification in certain steps. No algorithm is taught in their learning and only a few steps are required to obtain a final answer.

Besides being proficient in mathematical concepts, applying HOTS is important in solving mathematical problems. Kailani and Ismail @ Nawi (2009) assert that most students are not proficient in answering questions that require problem-solving, which is due to their inability to meet the problem-solving phases set by the Model of Polya. According to Wiederhold (1997) teachers should emphasise appropriate teaching methods and focus on a cognitive hierarchy that requires students to apply, analyse, assess and create a solution when applying HOTS in the classroom. Critical and creative thinking processes are closely related to the process of higher-order thinking. Wiederhold (1997) states that the process of higher-order thinking begins when students critically analyse knowledge, information or situations given by their teachers. In this way, they use creative thinking skills to plan the next steps and ultimately make new decisions, instructions, results or values.

The teacher's role is very important – especially in planning lessons that incorporate the elements and applications of thinking skills (King, Goodson & Rohani, 1998). This is because different types of learning require different thinking strategies. A study related to the Malaysian experience reports that HOTS may help students develop their capabilities as enshrined in the National Education Philosophy.

Problem-solving models

De Corte (2003), Schoenfeld (1985) and many academicians have explained that the pattern of problem-solving is an essential element in solving a mathematical problem. Students should always assess their abilities to solve complex tasks and think of alternative ways when the method used is less productive or effective. Yap (2013) says that having the knowledge in problem-solving, one is potentially very skilled in a problem-solving process. The person will be able to identify and define a problem, delegate related issues in mental representation, plan strategically, set a clear objective, select and implement a strategy, oversee the implementation process, use feedback, and evaluate the work systematically. Given these measures, a person will be able to implement a problem-solving process more effectively.

Many academicians have submitted various models that address problem-solving skills. Polya (1957) has submitted a model that consists of four main steps: (1) understanding the problem; (2) planning the solution strategies; (3) implementing the strategies; and (4) reviewing them. Besides this model, many researchers have also proposed other models to solve mathematical problems, such as the Model of Lester (1975), the Model of Mayer (1983) and the Model of Schoenfeld (1985). In addition, a new model was proposed in 2003 called the Model of De Corte. This model involves five phases: (1) developing mental representation of the problems; (2) deciding on how to solve the problems; (3) implementing the required calculations; (4) interpreting the results; and (5) formulating the answers and evaluating the solutions (Tan & Mohammed, 2003).

Methodology

This study was conducted in the form of a case study and classified as qualitative research. The HOTS patterns in mathematical problem-solving were identified among Form-two students. This method is known as a purposive sampling method. The samples of 10 students were selected from Form-three students who were candidates for the PT3 examination in 2015. The samples were randomly selected from the same population. The sample profile is shown in Table 2.

Table 2 The profile of the samples in this study

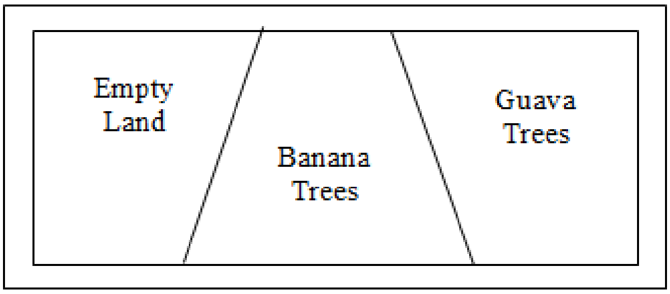
		Frequency	Percentage (%)
Gender	Male	6	60
	Female	4	40
Race	Malay	-	0
	Chinese	9	90
	Indian	1	10
	Other	-	0

Various ways exist to determine the patterns of students' thinking when answering HOTS questions. These include a product analysis (examining the result of problem-solving), an observation of behaviour (action protocols), an interview, a self-report questionnaire and a verbal report (Van Someren, Barnard & Sandberg, 1994). According to Van Someren, a verbal report requires an individual to translate what he thinks either when he is doing the task or after the task has been completed. A thinking aloud protocol (TAP) is a form of a verbal report in which respondents are requested to state out loud what they are thinking, feeling, how they are reacting, and what comes to mind during the problem-solving process. It provides the opportunity for researchers to observe the respondents' thinking process without disrupting them with questions normally asked during an interview.

In this study, TAP was employed to collect the data. The students answered two PT3 2014 questions individually. The researchers video recorded every step of the students' work using a video camera. The respondents' behaviour and thinking when answering HOTS mathematics problems were verbalised and recorded. The data obtained was transcribed individually and labelled according to the steps in Polya's mathematical problem-solving model namely understanding, planning, implementation and review.

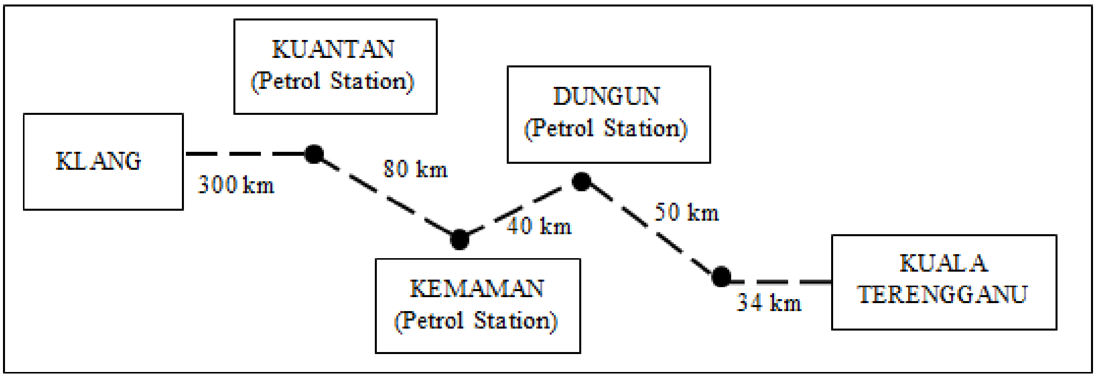
Instruments

In this study the students completed a test to determine their level of HOTS in solving mathematical problems. The test comprised two HOTS mathematical questions as shown in Figure 1 and Figure 2. The students were required to answer all questions. The use of a scientific calculator was allowed in order to ensure more accurate results. The questions were part of an actual set of the PT3 questions from the MOE (2014). As these questions were developed by the Malaysian Examination Board, their validity and reliability were ensured and high.



The above figure shows a piece of Pak Ali's land which is divided into three parts. The ratio for the number of banana trees to guava trees that have been planted is 3:5. The total number of trees planted is 96. Pak Ali intends to plant banana trees in the open spaces. Find the minimum number of banana trees that need to be added so that the total number of banana trees exceeds the total number of guava trees (MOE 2014).

Figure 1 First question – A piece of Pak Ali's land (MOE, 2014)



The above figure shows Fahmi's journey from Klang to Kuala Terengganu. His car's fuel tank can fill up to 45 litres of petrol. When he starts his journey, the car's fuel tank is $\frac{3}{4}$ full. The car is using 5 litres of fuel for a distance of 60 km. Fahmi does not plan to stop as long as there is enough petrol car. Determine the most distant gas stations he should stop before his car runs out of fuel. Justify your comment with calculations (MOE, 2014).

Figure 2 Second question – Fahmi's journey from Klang to Kuala Terengganu (MOE, 2014)

Each student's answers were accepted whether in the form of calculations, explanations or diagrams. Every step of the students' work and conversations was recorded.

Findings

The findings obtained were transcribed. As mentioned earlier, each sentence in the transcript was labelled using the steps in the Model of Polya. The labels were also validated by three subject-matter experts using inter-rater reliability. The data analysis (labelled transcripts) produced HOTS patterns for mathematical problem-solving. Table 3 and Table 4 show a sample of labelled transcripts of the students who successfully answered the first and second questions. The students who successfully solved these HOTS questions produced the same pattern beginning with understanding followed by planning, implementation and ended with the final answer. The findings showed that five students successfully answered the first question. The students started the mathematical problem-solving process by understanding the problem first. Before they began with the calculations, they first planned the most appropriate strategy to solve the problem. The planning and implementation processes of the calculation were conducted consecutively before obtaining the final answer. The students repeated the planning processes in order to ensure that their plan was organised and proceeded smoothly until the completion step. Before obtaining the final answer, the students summarised all the calculation results to ensure that the answers were accurate and convincing. Figure 3 summarises the general pattern of mathematical problem-solving of the students who successfully solved the HOTS questions.

Table 3 Samples of the problem-solving analysis of students who successfully answered the first question

Sample 1			Sample 2		
Behaviour	Transcript/description	Step	Behaviour	Transcript/description	Step
The students read the question out load and extract important information	The ratio of the number of banana trees to guava trees is 3:5.	Understanding	*The students read the question out load and extract important information*	The ratio of the number of banana trees to guava trees is 3:5.	Understanding
	The total number of trees planted is 96.	Understanding		The total number of trees planted is 96.	Understanding
	Pak Ali wants to plant banana trees on the empty land.	Understanding		Pak Ali wants to plant banana trees on the empty land.	Understanding
	Find the minimum number of banana trees that needs to be added so that the total number of banana trees is more than the total number of guava trees.	Understanding		Find the minimum number of banana trees that needs to be added so that the total number of banana trees is more than the total number of guava trees.	Understanding
Student writes and verbalises thoughts 5 + 3 = 8	First we must plus ratio 3 and 5 … and we get 8.	Planning	*Student writes and verbalises thoughts* $96 \times \frac{5}{8} = 60$	Number of guava tree $96 \times \frac{5}{8}$	Planning
	And the total number is 96 so we want to divide.	Planning		Number of guava tree is 60.	Implementation
Student writes and verbalises thoughts 96 ÷ 8 = 12	With 8 equal to 12.	Implementation	*Student writes and verbalises thoughts* $96 \times \frac{3}{8} = 36$	Now, find the number of banana trees $96 \times \frac{3}{8}$	Planning
Student writes and verbalises thoughts 12 × 3 = 36	The banana tree is 12 × 3 = 36.	Implementation	*Students verbalises thoughts*	Number of banana tree is 36. More than the total number.	Implementation
Student writes and verbalises thoughts 5 × 12 = 60	Then guava tree is 5 × 12 = 60.	Implementation	*Student's written and think aloud* 61 – 36 = 25	61 – 36 = 25.	Planning
Students verbalises thoughts	If we want to know the banana tree ...	Planning	*Students verbalises thoughts*	Because it says that the number of banana trees is higher than the number of guava trees.	Final answer
Students verbalises thoughts	How much banana tree more than the guava tree we need to minus some.	Planning	*Students verbalises thoughts*	So 61 – 36 = 25.	Final answer

Sample 1			Sample 2		
Behaviour	Transcript/description	Step	Behaviour	Transcript/description	Step
Student writes and verbalises thoughts 60 – 36 = 24	60 – 36 = 24.	Implementation			
Student writes and verbalises thoughts 24 + 1 = 25	And 24 + 1 = 25.	Implementation			
Students verbalises thoughts	So we need to find more than 25.	Final answer			

Table 4 Samples of the problem-solving analysis of students who successfully answered the second question

Sample 1			Sample 2		
Behaviour	Transcript/description	Step	Behaviour	Transcript/description	Step
The students read the question out load and extract important information	Diagram shows Fahmi's journey from Klang to Kuala Terengganu.	Understanding	*The students read the question out load and extract important information*	Diagram shows Fahmi's journey from Klang to Kuala Terengganu.	Understanding
	The capacity of petrol tank in his car is 45 litres.	Understanding		The capacity of petrol tank in his car is 45 litres.	Understanding
	When he starts off his journey, the petrol tank is $\frac{3}{4}$ full.	Understanding		When he starts off his journey, the petrol tank is $\frac{3}{4}$ full.	Understanding
	His car consumes 5 litres of petrol for a distance of 60 km.	Understanding		His car consumes 5 litres of petrol for a distance of 60 km.	Understanding
	Fahmi does not intend to stop as long as his car does not run out of petrol.	Understanding		Fahmi does not intend to stop as long as his car does not run out of petrol.	Understanding
	Determine the farthest petrol station he should visit before the petrol tank is empty.	Understanding		Give your reason with calculation.	Understanding
	Give your reason with calculation.	Understanding	*Student writes and verbalises thoughts* 45 × ¾ = 33.75	Hmmmm ... first we must use 45 litre × ¾ = 33.75.	Planning
Students verbalises thoughts	First we want to know how much his car has petrol.	Planning	*Students verbalises thoughts*	This sentences also said that 5 litres = 60 km.	Planning
Student writes and verbalises thoughts 45 × ¾ = 33.75 ℓ	So we need to find 45 × ¾ = 33.75 litres … than we know his car has 33.75 litres of petrol.	Planning	*Students verbalises thoughts*	Now we must find 1 litre for what km.	Planning

Sample 1			Sample 2		
Behaviour	Transcript/description	Step	Behaviour	Transcript/description	Step
	Then first he goes to the Kuantan petrol station which is 300 kilometres away.	Planning	*Student writes and verbalises thoughts* $60 \text{ km} \div 5 = 12$	So we must use 60 km ÷ 5 = 12.	Implementation
Student writes and verbalises thoughts $300 \div 60 = 5$	So we divide 60 = 5.	Implementation	*Students verbalises thoughts*	That means, for 1 litre, he will drive for 12 km.	Implementation
Student writes and verbalises thoughts $5 \times 5\ \ell = 25\ \ell$	5 × 5 = 25 … he already uses 25 litres.	Implementation	*Student writes and verbalises thoughts* $33.75 \times 12 = 405$	Hmmmm ... next, we must use 33.75 × 12 km = 405 km.	Implementation
	Then, we subtracted the petrol	Implementation	*Students verbalises thoughts*	So Klang to Kuantan = 300 km.	Planning
Student writes and verbalises thoughts $33.75 - 25 = 8.75\ \ell$	We obtained … he has 8.75 litres left.	Implementation	*Students verbalises thoughts*	Kuantan to Kemaman = 80 km.	Planning
	Then he goes to the Kemaman petrol station which is 80 kilometres away.	Planning	*Student writes and verbalises thoughts* $300 \text{ km} + 80 \text{ km} =$	We can use 300 km + 80 km = 380 km.	Implementation
Student writes and verbalises thoughts $80 - 20 = 60 \text{ km}$	We first know 60 km is equal to … we need 5 litres of petrol to go about the distance of 60 km.	Planning	*Student writes and verbalises thoughts* 380 km	So 380 km is the farthest petrol station he should visit.	Implementation
	So, first, we subtracted 80 km to 20 km.	Planning	*Student writes and verbalises thoughts* Kemaman	So Fahmi can go to the farthest petrol station, which is the Kemaman petrol station before the tank is empty.	Final answer
Student writes and verbalises thoughts $60 \text{ km} = 5\ \ell$	We get 60 km = 5 litres.	Planning			
Student writes and verbalises thoughts $8.75 - 5 = 3.75\ \ell$	Then we subtracted 8.75 litres – 5 litres = 3.75 litres left.	Implementation			
	Fahmi can’t go to Dungun because he need 5 more litres of petrol.	Implementation			
Student writes and verbalises thoughts Kemaman	So he goes to the Kemaman petrol station.	Final answer			

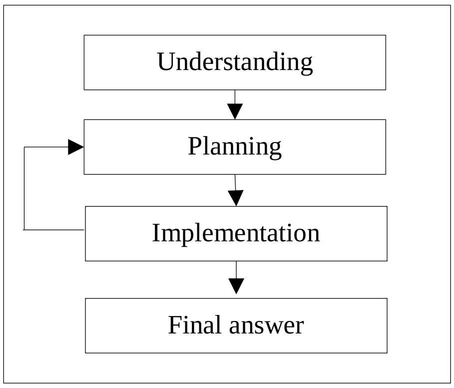


Figure 3 The pattern of mathematical problem-solving of successful students

Table 5 and Table 6 show samples of the labelled transcripts of the students who were unsuccessful in answering the first and second questions respectively. The students who failed to answer the HOTS questions went through several beginning phases in the mathematical problem-solving process. This was due to their failure to understand the given problem. In addition, they also seemed hesitant in their planning, which resulted in their failure to proceed to the implementation phase of the calculation. Their failure to plan properly disrupted the calculation process and the final answer given was, therefore, inaccurate. Figure 4 shows the general pattern of mathematical problem-solving of the students who were unsuccessful to solve the HOTS questions.

Table 5 Samples of the problem-solving analysis of students who failed to answer the first question

Sample 1			Sample 2		
Behaviour	Transcript/description	Step	Behaviour	Transcript/description	Step
The students read the question out load and extract important information	Find the minimum number of banana trees that needs to be added so that the total number of banana trees is more than the total number of guava trees.	Understanding	*The students read the question out load and extract important information*	The ratio of the number of banana trees to guava trees is 3:5.	Understanding
	First write down all solutions at the question.	Planning		The total number of trees planted is 96.	Understanding
	Banana tree to guava tree is 3:5 … 3:5 … banana tree, guava tree … the total number of trees planted is 96.	Understanding		Pak Ali wants to plant banana trees on the empty land.	Understanding
	But empty land is not … want to find banana tree ... total number is 96.	Understanding		Find the minimum number of banana trees that needs to be added so that the total number of banana trees is more than the total number of guava trees.	Understanding
Student writes and verbalises thoughts $3 + 5 + x = 96$	So … 3 + 5 total is 96.	Planning		The total of banana tree plant is 96.	Understanding
Student writes and verbalises thoughts $8 = 96$	3 + 5 is 8 = 96.	Planning	*Student writes and verbalises thoughts* $96 \div 3 = 32$	So divide 3 … equal to find the number of banana trees … and divide 5 in order to find the number of guava trees.	Planning
Student writes and verbalises thoughts $1 = 12$	1 of part is 12.	Implementation	*Student writes and verbalises thoughts* $96 \div 5 = 19.2$	$96 \div 3 = 32 \ldots 96 \div 5 = 19.2$	Implementation
Students verbalises thoughts	So they want the minimum number of banana trees.	Planning	*Students verbalises thoughts*	Then want to find the ... want to find banana tree on the empty land	Planning
Students verbalises thoughts	Then we get 12, we multiply all at once.	Planning	*Student writes and verbalises thoughts* $96 - 32 - 19 = 45$	Use $96 - 32 - 19 = 45$	Implementation
$12 \times 3 = 36$	$12 \times 3 = 36$.	Implementation			
$12 \times 5 = 60$	$12 \times 5 = 60$.	Implementation			
Students verbalises thoughts	Haaaah ... it wants to find the minimum number of banana trees that need to be added.	Planning			

Table 6 Samples of the problem-solving analysis of students who failed to answer the second question

Sample 1			Sample 2		
Behaviour	Transcript/description	Behaviour	Transcript/description	Behaviour	Transcript/description
The students read the question out load and extract important information	Diagram shows Fahmi's journey from Klang to Kuala Terengganu.	Understanding	*The students read the question out load and extract important information*	Diagram shows Fahmi's journey from Klang to Kuala Terengganu.	Understanding
	The capacity of petrol tank in his car is 45 litres.	Understanding		The capacity of petrol tank in his car is 45 litres.	Understanding
	When he starts off his journey, the petrol tank is $\frac{3}{4}$ full.	Understanding		When he starts off his journey, the petrol tank is $\frac{3}{4}$ full.	Understanding
	His car consumes 5 litres of petrol for a distance of 60 km.	Understanding		His car consumes 5 litres of petrol for a distance of 60 km.	Understanding
	Fahmi does not intend to stop as long as his car does not run out of petrol.	Understanding		Fahmi does not intend to stop as long as his car does not run out of petrol.	Understanding
	Determine the farthest petrol station he should visit before the petrol tank is empty.	Understanding		Determine the farthest petrol station he should visit before the petrol tank is empty.	Understanding
Student writes and verbalises thoughts ¾ × 45 = 33.75	Give your reason with calculation.	Understanding		Give your reason with calculation.	Understanding
Students verbalises thoughts	First the capacity of petrol tank is 45 litres.	Understanding	*Student writes and verbalises thoughts* ¾ = 0.75	Ehem … The tank is empty ... so ¾ = 0.75.	Planning
Student writes and verbalises thoughts 5 litres of petrol is for the distance of 60 km	And we must put ¾ × 45 litres = 33.75.	Planning	*Student writes and verbalises thoughts* 45 + 0.75 = 45.75	So add with 45 = 45.75.	Implementation
	And 5 litres of petrol is for the distance of 60 km.	Planning	*Student's think aloud*	Then, he can go as farthest as 9 km is 'wrong.'	Implementation
	I have reached here, teacher.		*Student writes and verbalises thoughts* $\frac{45}{5} = 9$	The farthest petrol station he can visit is the Kuantan petrol station.	Implementation
			Students verbalises thoughts	Because oooo 60 km ... 45 km ... 5 litre … 60 km ... 5 litres.	Planning
				How many 60 in 'wrong.'	Planning
				How many 5 in 45, that 9.	Planning
			Student writes and verbalises thoughts	So … oooo … I know ... I know.	

Sample 1			Sample 2		
Behaviour	Transcript/description	Behaviour	Transcript/description	Behaviour	Transcript/description
			$9 \times 60 = 540$ *Student writes and verbalises thoughts*	$9 \times 60 = 540$.	Implementation
			$300 + 80 + 40 + 50$		

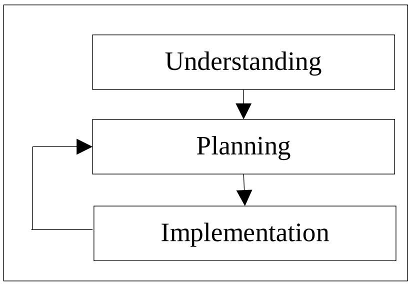


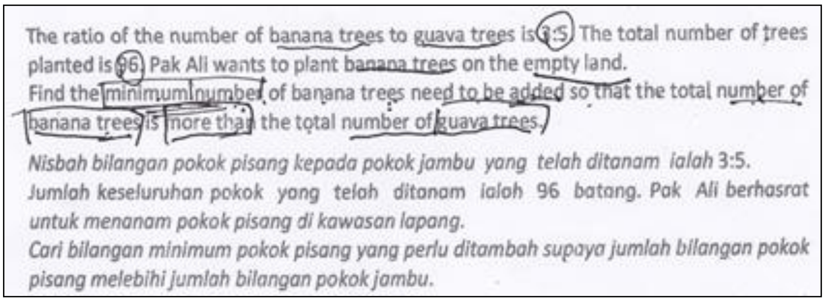
Figure 4 The pattern of mathematical problem-solving of unsuccessful students

The researchers compared the students' pattern of problem-solving with Polya's existing model. As shown in Table 7, it was found that all of the students who successfully answered the questions showed the same pattern of mathematical problem-solving as suggested in the model. However, these students failed to undertake the fourth phase of the Model of Polya (i.e. reviewing). They did not attempt to review their work after obtaining the final answer. Based on the interviews with the students, it was generally found that the students felt that the HOTS questions were at moderate and difficult levels. They claimed that the HOTS questions were challenging and required longer times for them to think. Not all students agreed that the HOTS questions were suitably assessed in PT3. Two out of five students were of the view that the HOTS questions were inappropriate for PT3, claiming that the questions might potentially affect their overall score.

Table 7 The comparison between students using the Model of Polya

Model of Polya	Successfully answered	Failed to answer
Understanding the problem	Read and underline the essential facts	Read the questions
Plan the strategy	Plan the strategy thoroughly	Plan the strategy
Implement the strategy	Carry out calculations	Carry out calculations (planned strategy is not complete)
Review	-	-

With regard to the problems in solving HOTS questions in PT3, the results show that, in the understanding phase the students read the questions carefully and outlined important information, as shown in Figure 5. Then, they read the question several times to understand the problem more clearly. Subsequently, they began planning the best strategy to solve the problem. However, students seemed to take a long time to understand the problem.


The ratio of the number of banana trees to guava trees is 3:5. The total number of trees planted is 96. Pak Ali wants to plant banana trees on the empty land.
Find the minimum number of banana trees need to be added so that the total number of banana trees is more than the total number of guava trees.

Nisbah bilangan pokok pisang kepada pokok jambu yang telah ditanam ialah 3:5. Jumlah keseluruhan pokok yang telah ditanam ialah 96 batang. Pak Ali berhasrat untuk menanam pokok pisang di kawasan lapang.
Cari bilangan minimum pokok pisang yang perlu ditambah supaya jumlah bilangan pokok pisang melebihi jumlah bilangan pokok jambu.

Figure 5 Students' outlining strategy in understanding the problem

The results show that the students did not fully understand the context of the question, which led them to a wrong answer. This pattern is in line with Maizan's (2001) claim that the students' difficulty in solving mathematical problems may be due to the difficulties in understanding a question, extracting relevant information and selecting appropriate solution operations. According to Ahmad and Halim (2013) students usually encounter difficulties when it comes to answering a question that requires them to extract information and find an appropriate strategy to obtain an answer.

Two problems arose in strategy planning. In the first instance the students failed to extract the information from the question and were therefore unable to translate the question into a simpler form. Secondly, the students' choice of a problem-solving strategy was limited. As shown in Figure 6, the students used a common mathematical procedure to solve the problem. This proves that the students in Malaysia are at a level where they can only solve problems with a single solution and use basic mathematical knowledge in simple situations.

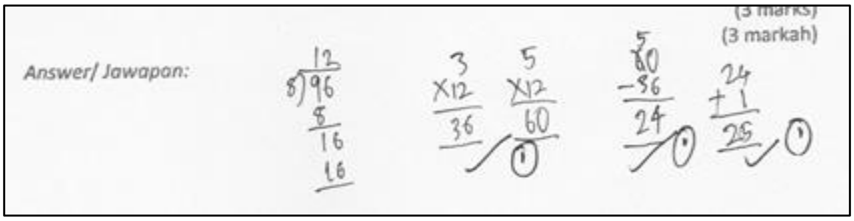


Figure 6 Sample of calculations made by the student

According to Tambychik and Meerah (2010) low visual-spatial skills may cause difficulty among Malaysian students to differentiate, link and compile the information contained in mathematical questions. As a result, students are unable to visualise the mathematical problems that can help them solve the problem.

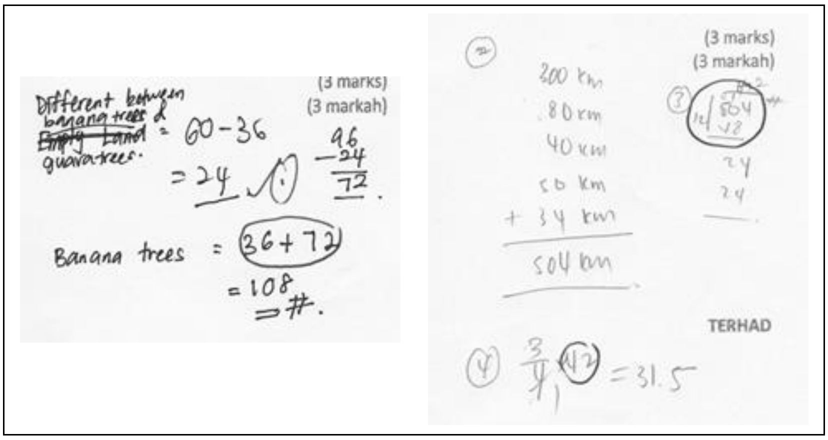


Figure 7 Sample of errors made by the students in the calculation process

When trying to implement the strategy, the students seemed to face difficulties proceeding with the problem-solving process. They did not understand the exact requirements of the given problem resulting in disorganised planning and poor execution. In addition the students made errors in the calculation process, wrongly produced the information, and subsequently entered the wrong value (see Figure 7). Prior to providing the final answer the students should first review each of the solution steps to avoid any mistakes in each phase of the problem-solving process. However, the students did not perform the review process. As a result, their final answers were incorrect.

Discussion

The findings show that the HOTS mathematical problem-solving process followed the Model of Polya. It started with the understanding phase, followed by the phases of implementation, calculation and ended with providing the final answer. The students who successfully answered the questions followed all the phases in a systematic manner. There were some repetitions in the planning and implementation phases before the students decided on the operations and provided the final answer. Mathematics requires systematic measures when solving a problem (Hamza & Griffith, 2006). The process should thus be repeated regularly to ensure an organised planning process.

This study has shown that the participating students had the skills to solve mathematical problems by using the process of understanding, planning, implementation and providing the correct answer. The results also show that five of ten students successfully answered the first question and eight of them successfully answered the second question. Osman (2001) argues that highly capable students can only demonstrate a problem-solving process that involves four phases: (1) selecting information; (2) formulating side issues; (3) selecting appropriate solution strategies; and (4) implementing these solutions strategies on a regular basis in an orderly and systematic manner. High-achieving students are more likely to successfully answer questions using a problem-solving process than low-achieving students, which is due to high-achieving students' ability to understand the questions better. The results of the current study show that all students went through the understanding phase, as they could understand the questions correctly. However, a small number of students could not complete the planning phase, resulting in their failure to answer the final question correctly. According to Saad, Mohd, Rahma, Musa, Mat and Zamzamir (2004) students who obtain outstanding achievements have a more positive perception than average or weak students. In addition, they can also find the solution to a problem in a better way than average and weak students.

A problem-solving process is a condition in which students use their existing knowledge, skills and understanding to identify different and unusual methods as compared to normal circumstances (Goldstein & Levin, 1987). Krulik and Rudnick (1989) state that this process contains a series of tasks and thinking processes associated with the formation of a set of heuristics. Students are expected to develop new knowledge and skills through the application of various strategies in the problem-solving process. Through students' experiences to answer previous questions, they will be familiar with problem-solving questions. Consistent guidance from teachers is needed to improve the students' capability to solve HOTS mathematical problems. Various strategies and teaching methods applied in the classroom are critical to assist students in evaluating and selecting an effective strategy to solve a problem.

The results also show that the students read the questions carefully and underlined the important contents. Subsequently, they repeatedly read the questions in order to be clearer and to take more details into account. Once they were certain that they understood the questions, they continued planning the best strategy. After having planned the best strategy, the students carefully completed the calculation process. Finally, before answering the questions, the students summarised the results of their calculations. The results of the study are in line with those found by Webb (1979) who claims that the first step taken by the students during problem-solving is to read the questions carefully, followed by planning appropriate strategies such as drawing, writing equations, using algorithms and verifying the answers. The findings of the current study indicate that the students who failed to provide the correct answers initially faced difficulties in the problem-solving process as they failed to understand the questions properly. A number of students also quit after having read the questions. Among the difficulties faced by the students were (1) identifying the main idea in the questions, (2) associating the information in the questions with the diagram, (3) associating relevant information and distinguishing such information in planning steps, and (4) translating the main ideas into algebraic terms and expressions (Saad et al., 2004). In addition, this study we also identified some of the problems that emerged when students answered the HOTS questions. The suggestions and problems are summarised in Table 8.

The results show that the errors made by the students are similar to those observed by Kaur (1997a). In her study, Kaur (1997b) found that the students' failure to solve mathematical problems was due to several factors such as a lack of knowledge to plan the solution strategies, a lack of understanding the problems, and making mistakes when changing the problem into mathematical sentences. Peter (2003) also found five sources of errors made by students: (1) failure to make the transformation (i.e. changing the story into mathematical sentences); (2) carelessness when answering the questions; (3) misunderstanding the questions; (4) a lack of self-motivation (i.e. the desire to answer mathematical questions with sentences); and (5) computational mistakes (i.e. errors in calculations). The errors made by the students in this research are consistent with those in a study by Brijlall and Ndlovu (2013) who found that having some difficulties in modelling problems and preferring rules and formulas were among the common errors made by the students while solving mathematical problems. The study conducted by Fatimah (2005) also supports the studies above. Fatimah (2005) managed to trace the causes of students' failure in solving non-routine mathematical problems. Some of the causes are a lack of understanding the questions, failure to change the questions into mathematical sentences, and a failure to properly plan the right strategies. The results of the current study clearly show that the students often made the same mistakes when solving HOTS mathematical problems. This also proves that the students' proficiencies on HOTS questions are low and should be improved.

Table 8 The suggestions for improvement to overcome the problems that emerged in the process of HOTS mathematical problem-solving

Phase	Problems emerged	Suggestions for improvement
Understanding	• Required more time to understand what the questions required	• Familiarise the students with questions in problem-solving formats
	• Students easily gave up in understanding the requirements of the questions	• Read the question over and over again
	• Wrong interpretation of the information in the questions	• Guide the students to draw diagrams or charts in order to analyse such information in the problems
	• No students used the charts or diagrams that could help them better understand the requirements of the questions	• Guide the students on the basic concepts of ratio and distance
	• No students mentioned the basic concepts in words	• Guide the students to underline and produce the essential facts in the questions
	• No students wrote down or underlined the important question content	• Familiarise the students with the questions in problem-solving formats
Planning	• Selected the wrong strategy	• Guide the students to check each result in their work
	• Confused and was not clear about the method of mathematical problem-solving	• Provide the students with various strategies and methods of problem-solving
	• Lack of confidence in selecting the method for problem-solving	• Multiply the number of HOTS questions
	• No students had detailed plans to achieve the problem-solving goal	• Suggest that students write down the relationship between the concept and the problem-solving steps
Implementation	• Errors in the calculation process	• Advise the students to always be careful during the calculation process
	• Wrongly produced the information and subsequently entered the wrong value	• Guide the students to always be aware of each step in the calculation and always check the work at every level
	• Students did not emphasise the units in each calculation	• Guide the students to be aware of the units in the questions as well as the correct method of exchanging the units
Final answer	• Students were in a hurry to get the final answer	• Guide the students in the right time-management techniques when answering the questions
	• Weakness in checking the different steps from time to time	• Familiarise the students with the methods of revision of all results
	• No students rechecked the calculations	

Conclusion

Malaysian students' poor performance in TIMSS and PISA has an impact on the content of the Malaysia Education Blueprint 2013–2025 (MOE, 2012b). One of the emphases is the incorporation of mathematics HOTS questions in PT3. However, studies on mathematical problem-solving in the PT3 exams have not yet been conducted comprehensively in Malaysia. The current study has shown that most students are capable of solving HOTS mathematical problems in PT3. However, they are still less skilled in selecting and using appropriate strategies to solve the given problems. In order to master this skill, the students need guidance from their educators. Therefore, an appreciation of knowledge culture must be developed, and problem-solving skills should be the main agenda in the process of teaching and learning. It is hoped that the findings of this study will help teachers understand the emerging patterns of problem-solving of HOTS questions by students. This study also discusses some suggestions to overcome student's problems that might emerge in the process of HOTS mathematical problem-solving for each step in the Model of Polya. However, it would be very informative if future researchers could conduct interviews with students and ask metacognitive-based questions in the process of solving mathematics HOTS problems.

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Authors' Contributions

Abdul Halim Abdullah wrote the manuscript, Siti Safuraa Fadil provided data for this study, Lokman Mohd Tahir analysed the qualitative data, Sharifah Nurarfah S. Abd Rahman did the formatting and Mohd Hilmi Hamzah proofread the manuscript. All authors reviewed the final manuscript.

Notes

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