Techniques for Analysis of Chaotic Pulse Trains Generated by Lightning: A Review

Chin-Leong Wooi Centre of Excellence for Renewable Energy School of Electrical System Engineering Universiti Malaysia Perlis (UniMAP), Perlis, Malaysia clwooi@unimap.edu.my

Syahrun Nizam Md Arshad, Centre of Excellence for Renewable Energy School of Electrical System Engineering Universiti Malaysia Perlis (UniMAP), Perlis, Malaysia syahrun@unimap.edu.my Zulkurnain Abdul-Malek Institute of High Voltage and High Current (IVAT) Faculty of Electrical Engineering, Universiti Teknologi Malaysia (UTM) Johor Bahru, Malaysia. zulkurnain@utm.my

Ahmad Muhyiddin Bin Yusof Department of Electrical Engineering Technology Faculty of Engineering Technology, Universiti Malaysia Perlis (Unimap) Perlis, Malaysia. muhyiddin@unimap.edu.my Mohamad Nur Khairul Hafizi Rohani, Centre of Excellence for Renewable Energy School of Electrical System Engineering Universiti Malaysia Perlis (UniMAP), Perlis, Malaysia khairulhafizi@unimap.edu.my

Abstract—This paper presents an overview of chaotic pulse trains (CPTs) analyzing techniques in recent years. Many techniques had been used such as time domain analysis, frequency analysis, power spectrum density, wavelet analysis, high-speed camera analysis, VHF interferometer technique, Xray radiation and the list goes on. It was found that a lot of researchers suggest to interpret CPTs as a types of pulse activity produced during the dart leader activities of subsequent leaders or the superposition of a few regular pulse train produced by dart leader activities. There are several previous studies in both regular and chaotic pulse trains agreed with this suggestion. The characteristic of chaotic pulse trains had also been reviewed in this study. Some authors suggest that when a "chaotic" component is observed during the subsequent leader activities, high frequency intensity would increase. It is also suggested that the downward dart leader gets a lot of luminosity when it goes together with chaotic pulse trains. The speed of the subsequent leaders (associated with CPTs) had also been discussed in this paper. The power spectrum density may be one of the methods to identify CPTs from another subsequent leaders. Several hypotheses on the origin of the CPTs were proposed. More research and analyses should be done in the future in order to clarify the physical phenomena behind chaotic pulse trains occur in between return strokes or in between cloud flashes.

Keywords— lightning, chaotic pulse trains, techniques, analysis, review

I. INTRODUCTION

Lightning detection and protection method had been discussed extensively in power system protection and high voltage engineering. Lightning is generally referring to positive and negative return strokes (RS) [1-3]. Other than the return strokes, other discharges also happen in lightning events, the events including stepped leader pulse [4, 5], dart-stepped leader pulse [5, 6], preliminary breakdown [7, 8] and narrow bipolar pulses [9, 10]. All these pulse trains consist of pulse regular patterns, characterized to each activity, which they are associated with. Other than normal pulses, pulses with irregular pulse shape, pulse width and pulse separation preceding subsequent return stroke, were also observed by Weidman [11], and Weidman [11] named them as subsequent strokes preceded by 'chaotic leaders' or 'chaotic subsequent strokes'. In most cases, chaotic leader is usually found to

precede natural subsequent return strokes (SRSs) [12-16]. The first observation of chaotic pulse trains has been made by Weidman [11]. In addition, another subsequent stroke was also recorded estimated 25 km from the striking place [17]. However, they did not analyze any the detail parts of chaotic pulse trains may be due to the doubts of aliasing problems [14].

'Chaotic subsequent strokes' are sometimes found to be mixed with cloud flashes activities [14, 18]. Recently, Hamzah and Gomes [19] also found that chaotic pulse trains happening in different places within the cloud-to-ground vertical electric field waveforms. Due to this reason, the term chaotic pulse trains (CPTs) is preferably used compared to chaotic leaders. In South Malaysia, Ahmad, et al. [18] found that the 41.1% of CPTs happened in between 1st and 2nd return strokes. In one sequence the maximum number of CPTs is 3, it is found between the 1st and 3rd return strokes. For cloud flashes activities, CPTs were observed to happen between cloud flash pulses.

In 2004, a statistically analysis were carried out by Gomes, et al. [14] about chaotic pulse trains with a sufficient samples collected from Sweden, Denmark and Sri Lanka. Gomes, et al. [14] observed that 25% of the CPTs occurred before subsequent return stroke (SRS). In 2011, Lan, et al. [16] performed broadband measurement of CPTs in Guangdong province and they recorded that CPTs were associated with 48% of subsequent strokes, while Mäkelä, et al. [15] by direct measurement found that 30% of subsequent stroke associated CPTs.

Some CPTs were found immediately preceded a subsequent stroke, but some it had a time delay before the return stroke. Lan, et al. [16] mentioned that the average delay duration is 18 ms and Gomes, et al. [14] mentioned that the average delay duration is 0.35 to 73 ms. Chaotic leader event is usually mixed with dart leader due to the reason that chaotic leader generally followed by dart leader or dart stepped leader. In addition, Davis [13] have found chaotic leader happened in intra-cloud (IC) flashes and it happened together with isolated breakdown pulses (IBPs) as well. This suggested that may be chaotic leader likely mixed with cloud activity compared to dart/dart stepped leader process.

In 2011, it is observed that 48% of the subsequent return strokes were mixed with chaotic characteristic [16], in other words almost half of the subsequent return strokes in negative cloud-to-ground lightning are preceded by CPTs. The values were found higher than the 26% found by Gomes, et al. [14] in year 2004 and 30% by Mäkelä, et al. [15] in year 2007. Other than the geographical factors, this phenomena may be partially attributed to the multiple types of leader process [16].

The relationship of CPTs with dart leader has been explained [14-16] and the possible origin of CPTs has been explained by Ismail, et al. [20] and Cooray [21]. The characteristic of chaotic pulse trains has been discussed extensively recently. Thus, it is motivated in conducting a review study in this regard. In this study, analyzing techniques of chaotic pulse trains included time domain analysis, frequency analysis, wavelet analysis, high speed camera, VHF interferometer and x-ray radiation analysis, power spectrum density and wavelet analysis on CPTs were studied.

II. ANALYZING TECHNIQUES

A. Time Domain Analysis on CPTs

Characteristic in lightning studies are usually related to temporal and spectral characteristic. According to Lan, et al. [16], the broadband electric field intensity improved a lot when a chaotic component is present. Lan, et al. [16] mentioned that the average duration of a CPTs was found to be roughly 472 µs within 95-1445 µs. In electric field signal, it is found that the individual CPTs pulse width was 1-2 µs, and lies within the range of 0.5-8 µs [16]. Average duration between the end of return stroke and CPTs pulse was reported to be 18 ms with the maximum value of 94 ms. 86% of CPTs were found immediately precede subsequent return stroke [16]. Other than this, they also found that there are a period of several tens of milliseconds low activity happens in between the ending of CPTs and RS pulse. They also reported that a period of several tens of milliseconds with relatively low activity in the broadband field occurs between the end of the CPTs and the RS pulse. However, the broadband electric field intensity increases dramatically whenever a chaotic component is present [16] disregard whether the CPTs immediately precedes a subsequent return stroke or not.

Possible origin of CPTs has been explained by Ismail, et al. [20]. Ismail, et al. [20] using time domain analysis reported that a typical CPTs could be produced by superimposing several regular pulse trains onto each other. Regular pulse trains (RPTs) are either produced by K-change or dart-stepped leader traveling within the thundercloud [22]. Every pulse train had a duration of 100-400 μ s with interval duration of about 5-6 μ s [22]. Single pulse was found to have a typical duration of 1-2 μ s with full width at half maximum (FWHM) of about 0.75 μ s [22]. Referring to Rakov, et al. [23], small pulses in RPTs is roughly two order of magnitude smaller compare to the return stroke pulses. Rakov, et al. [23] also mentioned that RPTs happen both in cloud flashes and also during cloud activity between return strokes. This observation was soon supported by Cooray [21] using wavelet analysis.

The average duration of the individual pulses of chaotic pulse trains are normally in the range of a few microseconds, with the lower limit of individual measurements extending into the sub-microsecond region. According to Gomes, et al. [14], individual pulse separation lies between $2-20 \ \mu$ s. While the pulse width lies with a consistent value of 400-500 μ s [14].

Several tens of microsecond to more than a millisecond (95–1445 μ s) [16] were found the total duration of CPTs but large amount of sample concentrated within 400–500 μ s, which is similar to the result obtained by Gomes, et al. [14]. In addition, Hill, et al. [24] showed that there are 4 types of leaders that showed "chaotic" electric field derivative (dE/dt) in ending of 10 to 12 ms. They exhibited special signatures that are not similar with the dE/dt waveforms found from dart and dart-stepped leaders in triggered lightning. Four "chaotic" dart leaders's vertical leader speeds were found within 2.0-4.3×10⁷ m/s [24].

B.Frequency Domain Analysis On CPTs

Other than the time domain analysis, another method to analyze the lightning characteristic will be frequency domain analysis. There are two types of frequencies usually discussed in lightning characteristic which is the low-frequency (LF) range below about 300 kHz and high-frequency (HF) range at about 3–30 MHz.

From a realistic point of view, the low-frequency (LF) influence is one of the most critical case since return stroke usually emit a lot of electromagnetic wave within those region [25]. Therefore, normally lightning-location devices also operate within this region. Compared to LF, high frequency (HF) was less studied. However, there are a few risks from HF impulses, although electromagnetic disturbances can be further induced in other structure such as planes or other structures in the size scale of 10-100 m, since this will correspond to resonant high frequency wavelengths [26]. Theoretically, high frequency can be used for remote detection of intense thunderstorm activity [27], but this method was not widely adopted. The is due to the reason that it is difficult to related any given narrowband HF signal to a specific lightning process, because there is a lot of process emit high frequency radiation. Practically, high frequency radiation can give a measure of energy emitted during a "composite" event including few lightning processes [28, 29].

There are two methods can be used to calculate the intensities of the high frequency signals [15]. One of the methods is using the rms voltage of the signal and using peak voltage in high frequency signal. These two methods give a little bit different results, indicating that the terms "high frequency intensity" is not clear enough. It was found that during close distance lightning flashes, return strokes emitted the most intensive high frequency signal [15]. On the other hand, regardless of the intensity metric used, the dart leaders mixed together with subsequent strokes are very weak emitters. High frequency activity rises when the dart leader shows some stepping behavior. However, when "chaotic" component is associated with subsequent leader, the high frequency magnitude increases a lot. This finding was similar with Lan, et al. [16], where high frequency give higher energies of CPTs are strong and continuous.

C. Power Spectrum Density and Wavelet Analysis of CPTs

In the frequency domain, Mäkelä, et al. [15] mentioned that the energy spectral density (ESD) perform weakly compared to power spectral density (PSD), and therefore PSD should be used to all subsequent leader processes. PSD spectrum of CPTs was studied and it was found that the spectrums was higher in radiation energy over the observed band. This is good enough to make a different compared to other subsequent leader processes [15]. It is suggested that this may be one of the method to identify CPTs from other subsequent leader.

The PSD of CPTs studied by Lan, et al. [16] also showed similar results with CPTs obtained by Willett, et al. [30]. With increasing frequency the spectral amplitude of CPTs decrease, at a rate of $f^{0.8}$ to $f^{1.2}$ up to 30 MHz [16]. Above 30 MHz, it decrease as f^2 . Non-chaotic subsequent leaders' spectra (300 µs window before subsequent RS pulse) have the tendency to range roughly 15–20 dB lower compared to CPTs over the observation band [16].

In previous literature, Fourier Transform (FT) is one of the favorable method used to transform the obtained time domain signal into frequency domain, but FT method cannot preserve the time information. Both time and frequency information can be preserved by using wavelet analysis as explained by Wooi, et al. [31]. In summary, the Type 2 CPTs have smaller electric field variations, lower frequency, and lower power radiated spectrum compared to Type 1 CPTs.

D. High Speed Camera, VHF Interferometer and X-Ray Radiation Analysis on CPTs

In year 2011, Hill, et al. [24] published the first of observations "chaotic" dart leaders during triggered-lightning discharges. Three "chaotic" dart leaders image were captured at a rate of 300 kilo-frames per second (exposure time of 3.33 ms) using high speed camera [24]. In this study, one image showed an upward positive leader prior to return stroke in one frame. The upward positive leader was 11.5 m in length traveling to negative leader above the streamer zone with the length of about 25 m. From the observation, four "chaotic" dart leaders were found unusual long duration and larger charge transfer. In addition, larger peak currents were also found in the following return strokes [24].

In addition, using high speed camera the downward dart leader gets very bright and travels to ground with "chaotic" leader behavior are found by Stolzenburg, et al. [32]. Wang, et al. [33] also observed chaotic pulse trains associated with dart-stepped leader using high speed camera. High speed camera observation seem to agree with the suggestion claimed by Ismail, et al. [34] where CPTs was can be developed by superpositing of several regular pulse burst produced by dart type discharge process in the cloud.

During the final 10-13 μ s of each "chaotic" dart leader [24], it was found that there is a continuous flux of energetic radiation (X-rays and gamma rays). The energies found were more than 1 MeV. From the study, it is found that the four "chaotic" dart leaders has vertical speed in between 2.0 to 4.3 × 10⁷ m/s [24]. This agreed with the speed reported by Lan, et al. [16] which is 2 × 10⁷ m/s.

Vertical leader speeds for the four "chaotic" dart leaders were estimated to range from 2.0 to 4.3×10^7 m/s [24] which is in common with the propagating speed of the downward leaders (associated with CPTs) is about 2×10^7 m/s reported by Lan, et al. [16], indicating a relationship between CPTs and the dart leaders.



Figure 1. Electric field derivative and LaBr3 energetic radiation measurement of (a) dart leader, (b) dart-stepped leader, and (c) "chaotic" dart leader preceding rocket triggered–lightning return strokes. [24]

In term of VHF observation, Lan, et al. [16] reported that there are three natural negative cloud-to-ground lightning flashes which contain CPTs and they were mapped using two-site VHF broadband interferometer. It is found that the stepped leader emits relatively weak radiation and speed of the leaders associated with the CPTs are similar to those of the speed of dart leaders. They conclude that the occurrence of the CPTs is associated with the development of dart phase leaders [16]. Pu, et al. [35] also found that there is a close relationship between the upper luminous leader segment and CPTs. They made a hypothesis that the CPTs originates from the luminous corona zone around the upper leader channel beyond the leader tip. The fast, sufficient supply of negative charge from the cloud can result in a net negative charge layer around the ionized channel surface [35]. Then, new diffuse discharge can make a corona zone outside the channel and radiates in a chaotic way [35]. The cloud charge reservoir and the speed of charge transfer, which can be indicated by the speed of the leader, are determinative to the generation of CPTs [35]. Using VHF location technique, they also estimated the speed evolution of the leader and link it with electric field change. Sun, et al. [36] also explained that the CPTs only occurred in the dart leader stage which propagated with a speed in an order of 10^7 m/s, accompanying strong and continuous VHF radiations.

III. CONCLUSION

There are many studies still going on currently, trying to explain the physical phenomena happen behind chaotic pulse trains that occurred in between return stroke or cloud flashes. Various technique had been used such as high speed camera analysis, VHF interferometer technique, X-ray radiation, electric field measurement, power spectrum density, wavelet analysis and the list goes on. Overview from this paper, it is found that a lot of researcher suggest to infer CPTs as a activity generated by the dart phase of subsequent leader or as one of the result of the superposition of a few regular pulse bursts produced in the thundercloud. The temporal characteristic of chaotic pulse trains also been reviewed in this study. It is found that chaotic pulse trains duration is within 100-400 µs with interval time between individual pulses of about 5-6 μ s. From frequency analysis, it can be suggested that the HF intensity increases dramatically when "chaotic" component was associated in a subsequent leader. In term of analysis method, power spectrum density and wavelet analysis may be one of the methods to identify CPTs from other subsequent leaders. In future, further research and analysis could be done in order to understand physical phenomena happen behind chaotic pulse trains and further improve the lightning protection system.

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