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Effect of inclusion lagrange interpolation method in mode shape curvature based damage detection

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Abstract. Mode shape curvature based damage detection capable to detect damage in structure with high sensitivity. Sparse and high density mode shape displacement data obtained experimentally pose difficulties for mode shape curvature algorithm to quantify damage size accurately. The objective of this study is to compare damage detection sensitivity of different mode shape curvature algorithm with the inclusion of Lagrange interpolation to enhance the algorithm damage detection sensitivity for sparse and high density curvature mode shape displacement data. Finite element analysis (FEA) model with free-free boundary condition of an aluminum beam has been carried out to investigate the feasibility of the proposed method. Undamaged curvature mode shape data from the damaged structure was estimated using Gapped Smoothing Method (GSM) and Savitzky-Golay (SG) filters with two different grid points of 149 and 74. Structural Irregularity Index (SII) and Damage Estimate Reliability (DER) were used to evaluate the effectiveness of the proposed algorithm. Numerical results show inclusion of Lagrange interpolation in mode shape curvature algorithm with Savitzky-Golay filter has better performance on estimate damage size by 2.81% of DER value for less dense (74 grid points) compared to GSM. The present method shows the inclusion of Lagrange interpolation has increased the sensitivity of mode shape curvature algorithm to identify damage size in beam-type structures compared to the previous method.

1. Introduction

Reliable non-destructive identification (NDI) method is important to ensure the effectiveness of structural health monitoring (SHM) in maintaining safety and integrity of structure [1, 2, 3, 4]. Because undetected damage in a structure by unreliable NDI may grow and reduce structural integrity which subsequently leads to catastrophic failure.

Vibration-based damage detection method based on physical changes in structures which manifested by changes in modal parameters (i.e. natural frequencies, mode shape, and damping coefficient). Therefore, the changes of modal characteristics in structures can be treated as the damage indicator. A study by [5] shows curvature mode shape as the parameter to detect damage in a structure by using the mode shape data from undamaged and damaged structures, and they are found curvature mode shape can be better indicators for damage identification compared to the natural frequency.

Gapped smoothing method (GSM) introduced by [6] is one of mode shape curvature method that has been studied extensively by researchers because it did not require data from the undamaged



structures. However, this method requires a sufficient number of measurement points to localize and identify damage size with decent accuracy in beam-type structures.

A study by [7] to detect damage in composite beams using mode shape curvature data measured with SLV found that the MSC data is contaminated with noise that 'hide' the actual damage signal. Modified Savitzky-Golay filter to smooth mode shape curvature data measured using SLV shows improvement in damage detection capability [8].

This paper will propose a method to detect damage in a structure using curvature mode shape data from a damaged structure and did not require data from an undamaged structure. This technique will use a small number of measurement data to estimate undamaged curvature mode shape data, ω_u using GSM and SG methods. The data subsequently used to calculate damage estimate reliability (DER) in order to localize and identify the size of the damage in structures. This method is expected to enable detection of damage without better resolution than the previous method.

2. Methodology

2.1 Gapped Smoothing Method (GSM)

The work in this study is based on Gapped Smoothing Method (GSM) by [6]. GSM estimated mode shape curvature, ϕ''^d with central finite difference equation from damaged structure:

$$\phi''^d = \frac{u_{i-1} - 2u_i + u_{i+1}}{\Delta x^2} \quad (1)$$

Where u_i is the transverse displacement mode shape data at point nodes i and Δx is the displacement between two consecutive nodes. In GSM, damage location identified from damage index (DI) plot that calculated from the difference between damaged and undamaged mode shape curvature.

$$\delta_i^m = |\phi''^d - \phi''^u| \quad (2)$$

Where, undamaged mode shape curvature was estimated using cubic polynomial regression using this equation.

$$\phi''^u = a_0 + a_1x + a_2x^2 + a_3x^3 \quad (3)$$

Although this method is sensitive to detect a small size of crack, however study by [6, 9] shows that GSM algorithm creates smeared noise near damage location for the wide size of damage because estimation of undamaged mode shape curvature using cubic polynomial that has localized effect which induced noise in the detection signal [9]. This causes this method unsuitable to identify size of damage with decent accuracy.

2.2 Savitzky-Golay Filters

A set of $2m+1$ consecutive samples is considered along with a local temporary coordinate system, i.e. $q \in \{-m, \dots, 0, \dots, +m\}$. The l th-order least-squares polynomial is represented by

$$f(q) = \sum_{r=0}^l b_r q^r \quad (4)$$

The Savitzky-Golay approach [10] applies Equation (4) at the midpoint only ($q=0$) whereas the value of the output at the next sample is obtained by shifting the analysis interval to the right by one sample and repeating the procedure at the new midpoint.

2.3 Lagrangian Interpolation Formula

Through any two points there is a unique line. Through any three points, a unique quadratic. etc. The interpolating polynomial of degree $n-1$ through the n points $y_1 = f(x_1)$, $y_2 = f(x_2)$, \dots , $y_n = f(x_n)$ is given explicitly by Lagrange's classical formula as follows [11]:

$$f(x) = \frac{(x-x_2)(x-x_3)\dots(x-x_n)}{(x_1-x_2)(x_1-x_3)\dots(x_1-x_n)} y_1 + \frac{(x-x_1)(x-x_3)\dots(x-x_n)}{(x_2-x_1)(x_2-x_3)\dots(x_2-x_n)} y_2 + \dots + \frac{(x-x_1)(x-x_2)\dots(x-x_{n-1})}{(x_n-x_1)(x_n-x_2)\dots(x_n-x_{n-1})} y_n \quad (5)$$

written in a compact form

$$f(x) = \sum_{i=0}^n y_i \prod_{\substack{j=0 \\ j \neq i}}^n \frac{x - x_j}{x_i - x_j} \quad (6)$$

There are n terms, each a polynomial of degree $n - 1$ and each constructed to be zero at all of the x_j except one, at which it is constructed to be y_i .

2.4 Structural Irregularity Index (SII)

Structural irregularity index (SII) is used to identify the location and size of damage by averaging DI over several modes. SII is calculated at each node by dividing them with their mean value over the grid points and averaged over the modes in order to improve damage detection performance as follows [12]:

$$\delta_i^A = \frac{N}{M} \sum_{m=1}^M \left(\frac{\delta_i^m}{\sum_{i=1}^N \delta_i^m} \right) \quad (7)$$

Where δ_i^A is the averaged SII at a grid point i , N is the number of all grid points and M is the number of all modes.

2.5 Damage Estimate Reliability (DER)

Damage Estimate Reliability (DER) is introduced to quantify the proposed damage identification method [13]. The whole interval along the axis of the beam (x axis) is split into 2 parts; the 1st part (a) is the one which does not contain damage, namely $0 \text{ mm} < x < 450 \text{ mm}$, $500 \text{ mm} < x < 750 \text{ mm}$ and $800 \text{ mm} < x < 1250 \text{ mm}$ (part a) and the 2nd part (b), containing damage, is $450 \text{ mm} < x < 500 \text{ mm}$ and $750 \text{ mm} < x < 800 \text{ mm}$.

In each of these parts standardized damage indices (SIIs) from equation (7) of a respective approximation function are summed and divided by the number of data points in this particular interval, giving the average amplitude of SDI (\overline{SDI}_i). DER is equal to average SDI in the area of damage (part b) divided by average SDI in all parts combined. It is expressed in percentage in Equation (8).

$$DER_i = \frac{\overline{SDI}_i(b)}{\overline{SDI}_i(a) + \overline{SDI}_i(b)} \times 100\% \quad (8)$$

In each of these parts standardized damage indices (SIIs) from equation (7) of a respective approximation function

2.6 Finite Element Analysis (FEA)

An aluminum beam with dimensions of $1250 \text{ mm} \times 50 \text{ mm} \times 5.25 \text{ mm}$ was modeled in ABAQUS. The beam has double damage at location $450\text{-}500 \text{ mm}$ and $750\text{-}800 \text{ mm}$ as shown in Figure 1. It has Young's Modulus, Poisson's ratio and density of 69.5 GPa , 0.31 and 2708 kg/m^3 respectively.

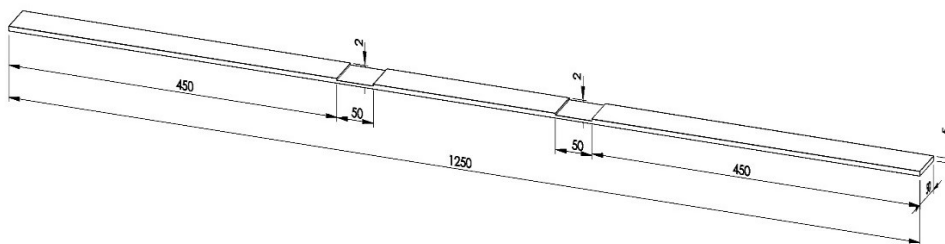


Figure 1. Geometry and dimensions of the tested aluminum beam

The beam was modeled using 1D beam elements (B32) that has 3 degrees of freedom, which are translations along the X and Y axes and rotation along the Z axis at each grid points. The model defined

with free-free boundary condition. Frequency analysis was performed to determine mode shape up to 20, this range included the first bending modes. The beam is constructed by means of 148 equal length elements ($i = 149$ grid points).

Four cases are considered in this study:

Case 1-Without Lagrange Interpolation: Analysis beam without Lagrange Interpolation act as validation for analysis with Lagrange Interpolation and to perform a comparative study between GSM and SG algorithm for damage detection.

Case 2-With Lagrange Interpolation: To study applicability the damage detection algorithm for real application, the effectiveness of the GSM and SG will be investigated by changing the number of measurement point from 149 to 74.

3. Result and Discussions

3.1 Case 1: Without Lagrange Interpolation

A general overview of the results confirms that GSM algorithm without Lagrange interpolation is less effective in determining damage size compared to Savitzky-Golay filter due to dispersed noise generated around damage area resulting from the local curve fitting as reported by [9]. On the other hand, Savitzky-Golay filter exhibit better performance compared to the GSM algorithm in detecting damage, in agreement with conclusions drawn by the previous study [8].

Table 1 shows DER value from numerical analysis without Lagrange interpolation for GSM algorithm and Savitzky-Golay filter. It was noted that for 149 grid points that use Savitzky-Golay filter has higher DER value by 2.81% than GSM algorithm. Figure 2 shows that damage detected using Savitzky-Golay method using 149 and 74 grid points capable to identify damage location with good accuracy with less dispersed noise compared to GSM algorithm.

It can be observed from Table 1 that for 74 grid points the value of DER value for Savitzky-Golay is reduced by 1.21% compared to the same method that uses 147 grid points. Reduction in DER value for Savitzky-Golay method because the medium number of grid points (74) is more sensitive to the presence of outlier (irregularity in MSC Data) in the data compared to Savitzky-Golay with the high number of grid points (149) as shown in Figure 3.

Table 1 DER from analysis without interpolation with 149 and 74 measurement points using different method

	GSM	Savitzky-Golay
149	91.89	94.48
74	84.25	93.34

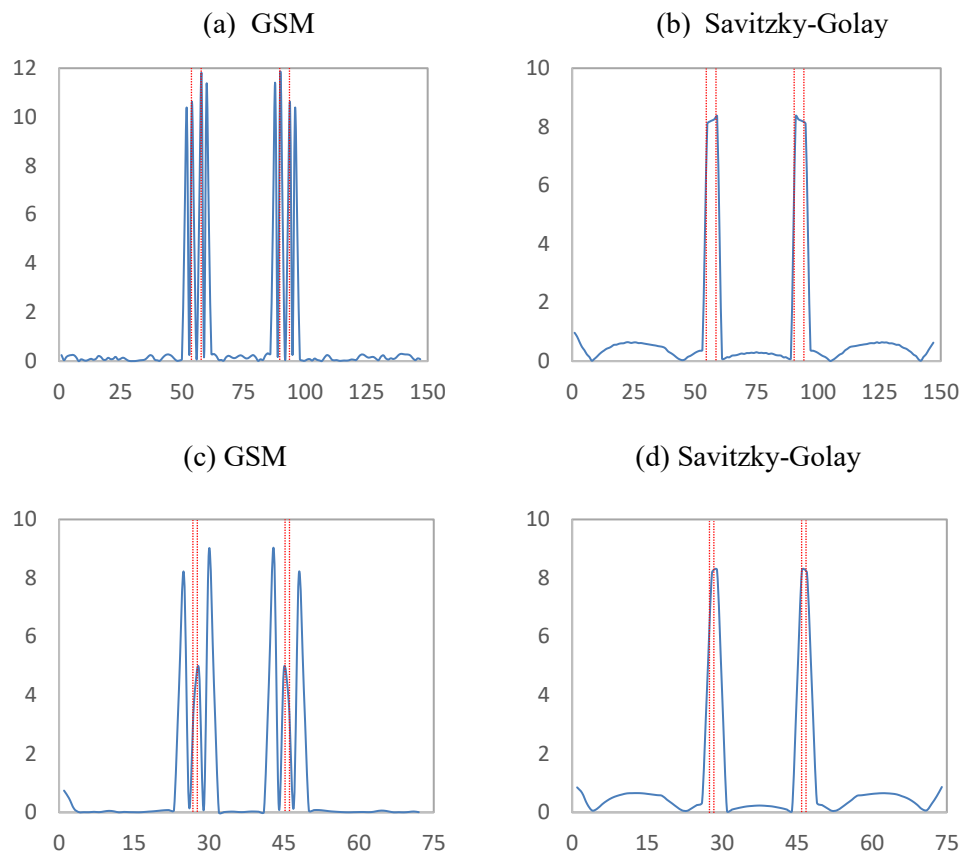


Figure 2. SII plot for analysis without interpolation with 149 GSM (a) Savitzky-Golay (b) and 74 GSM (c) Savitzky-Golay (d) grid points

Savitzky-Golay shows better performance compared to GSM algorithm for the high number of grid points (149) because the irregularity in damaged mode shape curvature has been filtered out to produce smooth undamaged mode shape curvature as shown in Figure 3, while GSM algorithm preserve the irregularity in damaged mode shape curvature then produce undamaged mode shape curvature with irregularity (unsmooth) as shown in Figure 4.

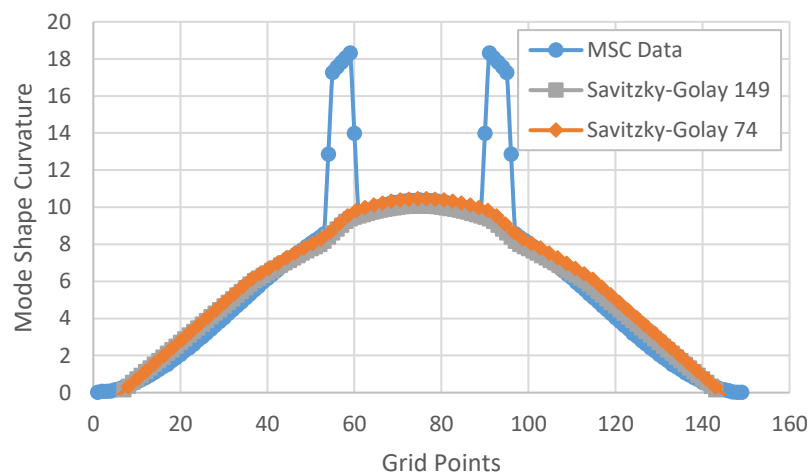


Figure 3. Mode Shape Curvature plot from numerical analysis without Lagrange interpolation using Savitzky-Golay filter with 149 and 74 grid points

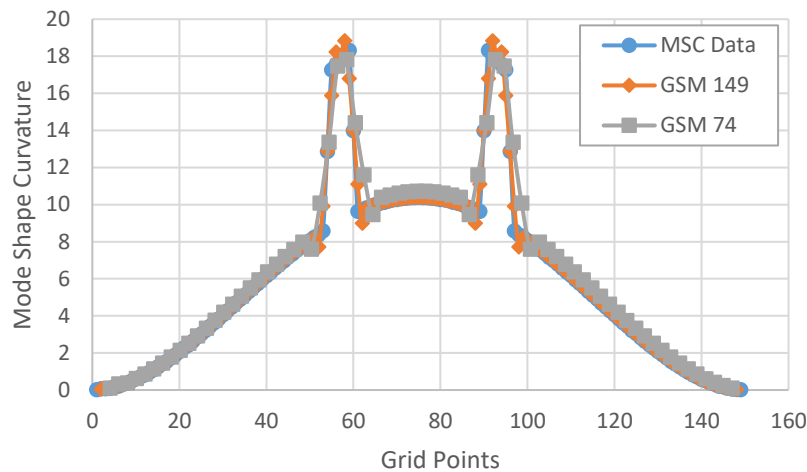


Figure 4. Mode Shape Curvature plot from numerical analysis without Lagrange interpolation using Gapped Smoothing Method (GSM) Algorithm method with 149 and 74 grid points

3.2 Case 2: With Lagrange Interpolation

Interpolation method is used to increase resolution of MSC damage detection with limited number of measurement points in real life application. A general overview of the results confirms that GSM algorithm with Lagrange interpolation is less effective in determining damage size compared to GSM algorithm without Lagrange interpolation due to more significant dispersed noise generated around damage area resulting from the local curve fitting as reported by [9].

Table 2 shows DER value from analysis with Lagrange interpolation for GSM and Savitzky-Golay method. It was noted that for 149 grid point Savitzky-Golay method has higher DER value by 16.37% compared to GSM algorithm and from SII plot in Figure 5 it can be see that damage detection using Savitzky-Golay method for 149 and 74 grid point shows damage location with good accuracy compared to GSM method. Savitzky-Golay has better performance than GSM, however it has lower performance by 0.49% compared to without Lagrange interpolation for 149 grid point. Which indicate that with Lagrange interpolation using less grid (74) point still less sensitive to the outlier (irregularity in MSC Data) in data.

Table 2 shows for 74 grid point the improvement in DER value for Savitzky-Golay is increased by 0.82% compared to same method that use 147 grid points. Reduction of in DER value for Savitzky-Golay method because medium number of grid points (74) is more sensitive to the presence of outlier (irregularity in MSC Data) in the data compared to Savitzky-Golay with high number if grid point (149) as shown in Figure 3.

It can be observed from Table 2 that for 74 grid point the value of DER value for Savitzky-Golay is reduced by 1.21% compared to same method that use 147 grid points. Reduction in DER value for Savitzky-Golay method because medium number of grid points (74) is more sensitive to the presence of outlier (irregularity in MSC Data) in the data compared to Savitzky-Golay with high number if grid point (149) as shown in Figure 3.

Table 2. DER from analysis with Lagrange interpolation with 149 and 74 measurement points using different method

	GSM	Savitzky-Golay
149	80.77	94.02
74	45.44	87.30

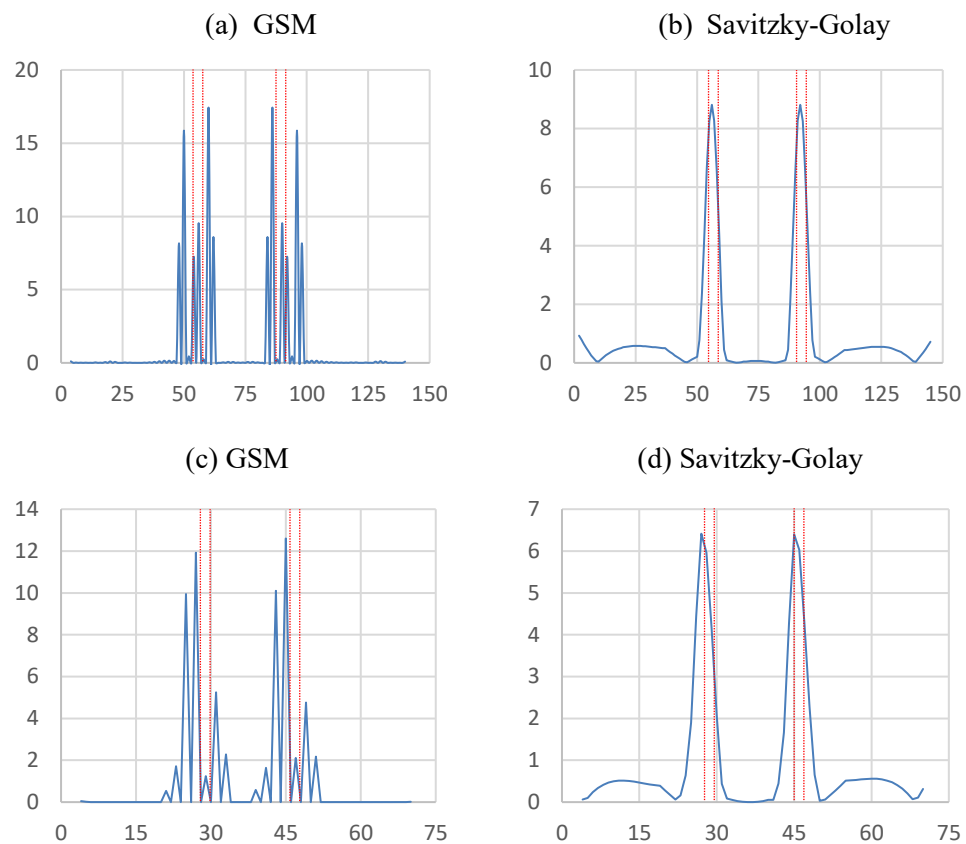


Figure 5. SII plot for analysis with Lagrange interpolation with 149 GSM (a) Savitzky-Golay (b) and 74 GSM (c) Savitzky-Golay (d) measurement points

4. Conclusion and future work

This paper addressed damage detection in beam structure using mode shape curvature data with GSM algorithm and Savitzky-Golay filter. Savitzky-Golay shows better performance for both number of grid points because SG has filtered irregularity in the damaged mode shape curvature to produce smooth undamaged mode shape curvature compared to GSM algorithm. Lagrange interpolation with Savitzky-Golay filter used to detect damage in reduced measurement point shows slightly lower detection capability (by comparing DER) compared Savitzky-Golay filter without Lagrange interpolation. Reduction in DER value can be related to fact that medium number of grid points (74) is more sensitive to the presence of irregularity in damaged mode shape curvature data compared to Savitzky-Golay filter with high number of grid point (149).

However, this study is only limited to numerical analysis. Hence, experimental works has to be done to validate the proposed MSC damage detection using Lagrange Interpolation is recommended for future study.

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