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USING COGNITIVE TOOLS TO ENHANCE UNDERSTANDING IN DIFFERENTIAL EQUATIONS

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Abstract. This paper reports a research conducted to understand the development of concepts on differential equations in a learning environment supported with a cognitive tool developed known as DEgraf. The students' levels of understanding were analyzed via concept mapping as introduced by Novak. Their written solutions on various problems were examined and transformed to concept maps which were used as tools to understand their levels of understanding. Results showed the cognitive structures in terms of mathematical relationships and the number of concepts made by the students. These cognitive structures illustrare the students quality of relational understanding. The study showed that active learning environments incorporating the use of a cognitive tool may lead to the development of relational or conceptual understanding.

Keywords: Meaningful learning; concept maps; relational understanding; cognitive tools; differential equations

Abstrak. Artikel ini melaporkan satu kajian yang bertujuan untuk memahami perkembangan konsep persamaan pembeza dalam suasana pembelajaran yang disokong oleh alat kognitif yang telah dibangunkan dan dinamakan DEgraf.Tahap kefahaman pelajar dianalisis melalui peta konsep yang diperkenalkan oleh Novak. Kerja bertulis beberapa masalah yang dihasilkan pelajar diteliti dan diterjemahkan kepada peta konsep untuk digunakan bagi memahami tahap kefahaman. Kajian menghasilkan struktur kognitif yang terdiri daripada pelbagai konsep dan hubungan antara konsep. Struktur kognitif dapat menggambarkan kualiti kefahaman *relational* pelajar. Kajian menunjukkan penggunaan alat kognitif dalam suasana pembelajaran yang aktif boleh membantu perkembangan kefahaman *relational* atau konseptual pelajar.

Kata kunci: Pembelajaran bermakna; peta konsep; kefahaman *relational*; alat kognitif; persamaan pembeza

1.0 INTRODUCTION

In mathematics learning, the quality of an individual's thinking is represented by his understanding. Studies on student's thinking or understanding have been given much attention by many researchers (Skemp, 1971; Novak & Gowin, 1984; Hiebert & Carpenter, 1992; Pirie & Kieren, 1994; Godino, 1996; Kilpatrick, Hoyles &

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Skovsmose, 2005)). Some theories on understanding are classified as of organismic orientation such as Ausubel meaninful learning theory, Skemp's relational and instrumental understanding and Piaget's theory on cognitive development. Skemp (1976) suggested that learning should achieved relational understanding. However, there are some views that support both instrumental and relational understanding are important since they support one another (Heid, 1988).

Tall (1986) evaluated Skemp's theory on understanding as equivalent to Ausubel's meaningful and rote learning. According to Ausubel, meaningful learning occurs when new knowledge is assimilated to the existing cognitive structures. Ausubel's idea on meaningful learning is similar to relational understanding whereby in both cases learning occur by the development of schemas or cognitive structures in a student's mind. Knowledge that is acquired through memorization will not be assimilated to the existing cognitive structures. In other words new knowledge will not be properly linked to the concepts available in the cognitive structures. Moreover knowledge build from memorization cannot be retained for a long period of time compared to those learnt meaningfully. Rote learning occurs when the learner made no conscious effort to relate new knowledge with existing conceptual framework or with any element that has been previously developed in the cognitive structures. Ausuble views seriously the role of the existing cognitive structures in learning process as reflected in his suggestion to "Ascertain what your student knows and teach him accordingly." (Ausubel, 1968). Ausubel considers knowledge is stored in an organized manner. Cognitive structure plays a role in the development of relationship between new and old elements developing into hierarchical conceptions. In other words, a spesific knowledge is linked with a more general concept. Briefly, a cognitive structure represent a person's thinking on a certain body of knowledge.

Meaningful learning normally occurs through expository learning or discovery learning. Ausubel has been criticized for viewing that expository learning is more practical compared to discovery learning which is troublesome for many students as well as time consuming. In mathematics learning, an alternative to these learning methods is laboratory activities. A learning environment that can initiate the construction of knowledge can be fulfilled by providing appropriate software which act as a cognitive tool whereby students can explore and manipulate mathematical ideas (Dubinsky, 1991; Tall, 1991, Kirschner & Erkens, 2006; Kong, 2008). In a computer based learning environment students may involve in situations that need them to engage in cognitive processes such as conjecturing, interpreting information, making inferences, veryfying, testing, reflexing and trial and errors during problem solving. Routine jobs like numerical calculations, symbolic manipulations and graphing can be delegated to computers. Meanwhile, learners can give more attention to thinking and developing understanding on various mathematical concepts involved. The development of learner's thinking in a computer based learning can be studied by understanding the development of cognitive structures. Further more, these cognitive structures do not only provide information on how they think and what they know, most importantly, they will influence their future construction of knowledge. This paper reports a research which essentially investigate the development of students' understanding on differential equations in a computer based environment.

2.0 METHODOLOGY

The study involved 19 students majoring in mathematics, three males and sixteen females. They have completed college level calculus courses and during the study were enrolled in a two credit introductory differential equations course. This course was conducted both through conventional lectures and computer laboratory activities. The computer activities required the students to solve problems designed for computer-based activities developed by experienced lecturers. A software named DEgraf that was developed for the purpose of this study for helping the students to solve the problems. For the whole semester we had eight computer sessions, each lasted for about 100 minutes. The students worked in pairs or in groups of three depending on the availability of computers.

This qualitative research used multiple methods of collecting data. One method is document analysis based on students' written laboratory reports which were used for the purpose of eliciting the product of students' understanding in the form of cognitive structures. Since the solutions of differential equations are complex and lengthy, concept mapping is employed to analyze and interpret the data. Concept maps that were introduced by Novak and Gowin (1984) are considered practicle and effective to display concepts and their relationships (Noor Azlan, Zaleha & Hassan, 1995; Moreno & Azcarate, 1996; Canas, Novak & Gonzalez, 2004). A concept map is a network of concepts' labels joined by linking words to form propositions that express relations between concepts. A concept map can be constructed based on students' writing to evaluate their level of understanding by making inferences on how they organize and think about related concepts as stated by Greeno (1978), Romberg and Carpenter (1986). The type of concepts and their relations as well as the number of concepts and their relations on a concept map can represent an individual's cognitive structure on some knowledge.

3.0 UNDERSTANDING LINEAR MOTION PROBLEM

Every group involved in the study was requested to submit their written report on the problems they have solved using DEgraf. Alltogether, each group submitted five reports. These reports were thoroughly studied from various aspects, namely relations between concepts or mathematical elements in the form of answers, mathematical procedures, explanations or comments on the assigned problems. Some statements that are obscured and irrelevant are ignored during construction of concept maps.

Incorrect statements which led to false relationships between concepts are indicated with dotted lines.

One of the problems investigated is on linear motion system. This problem presented a system of differential equations in two different cases and is stated as follows:

A car moved on a plane with friction is modeled by the equation:

$$\frac{dx}{dt} = y \qquad \frac{dy}{dt} = -0.5\,y$$

In this problem, two initial values are given:

(a) *$x(0) = 2$ and $y(0) = 0$ (The car is released at initial position $x = 2$)*
(b) *$x(0) = 2$ and $y(0) = 4$ (The car is released at initial position $x = 2$ and initial velocity $y = 4$)*
   (i) *Without using DEgraf, plot the graph of yx, xt and yt for $-10 \le x \le 10$, $-10 \le y \le 10$, and $0 \le t \le 10$ at initial value (a). Repeat for initial value (b) using DEgraf.*
   (ii) *Base on the graphs, state the position of the car when it stops, the time it takes to stop and its velocity at t =2, 4, 6, and 8.*
   (iii) *Explain the motion pattern for each initial values.*
   (iv) *Derive the algebraic solutions for each case.*

The discussions on this problem by two groups of students, namely the high achievers group and low achievers group are displayed in two concept maps. All links that were made by the high achievers group are mathematically acceptable including the parts that are not solved with computers. On the other hand, low achievers group made four mistakes as shown by the dotted lines. Mistakes were related to interpreting the graphs of x-t and y-t in terms of the position and velocity for the linear system. Moreover this group only managed to analyze the first part of the problem using graphs, whereas the other group used graphs and algebraic solutions. Furthermore, the concept map of the high achievers indicates that they also explained the phenomenon of the system using "no initial velocity" and "friction." For this particular problem, the high achievers group came out with 23 concepts whereas the low achievers group used 19 concepts in their written lab reports. In terms of relations between concepts, the high achievers group made 40 compared to 22 made by the low achievers group.

4.0 DISCUSSION

In this research, the concept maps indicated that students were successful in developing cognitive structures which represent their understanding on a particular problem

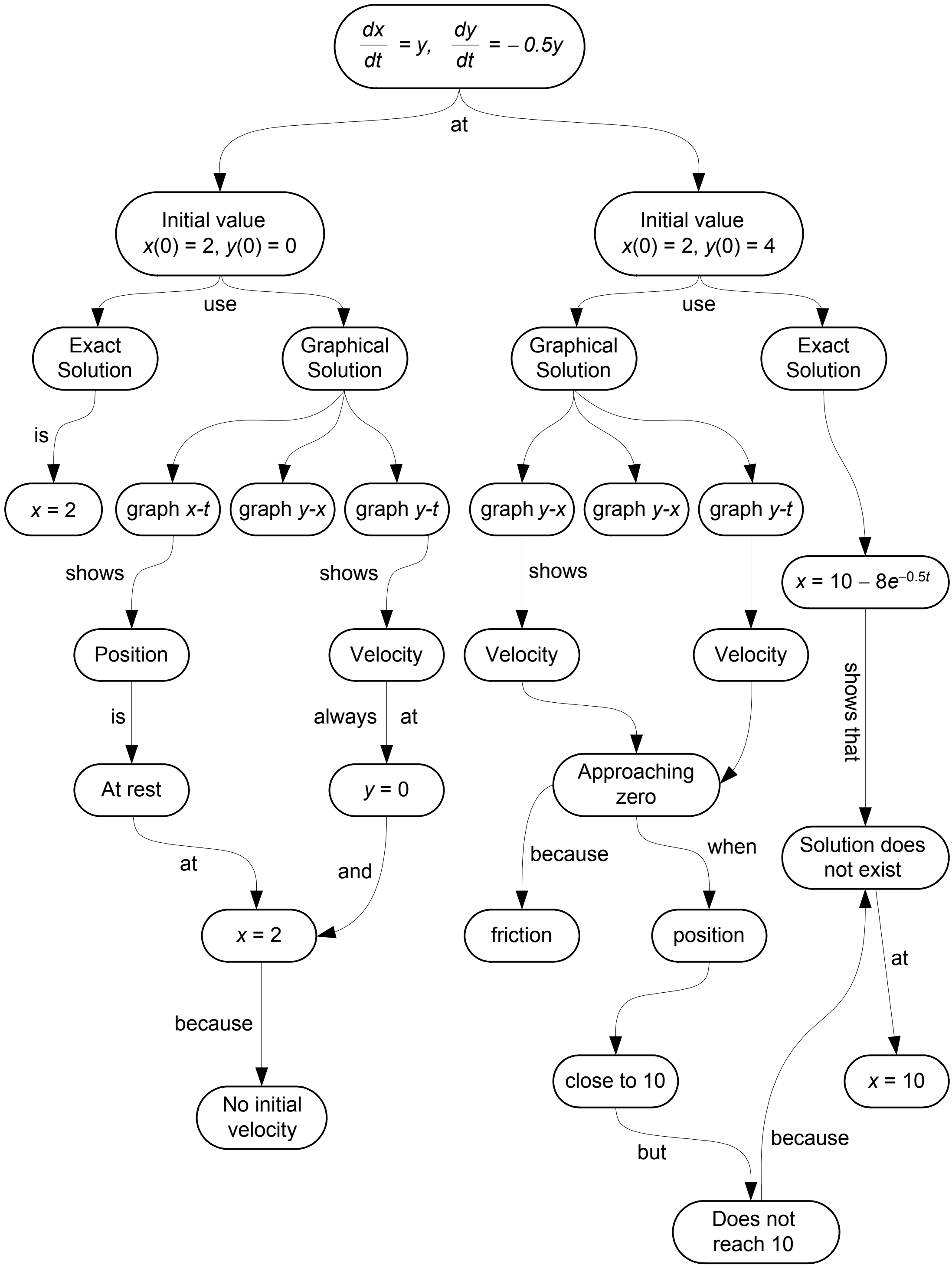

Figure 1 Concept map for linear motion problem, High-Achiever Group

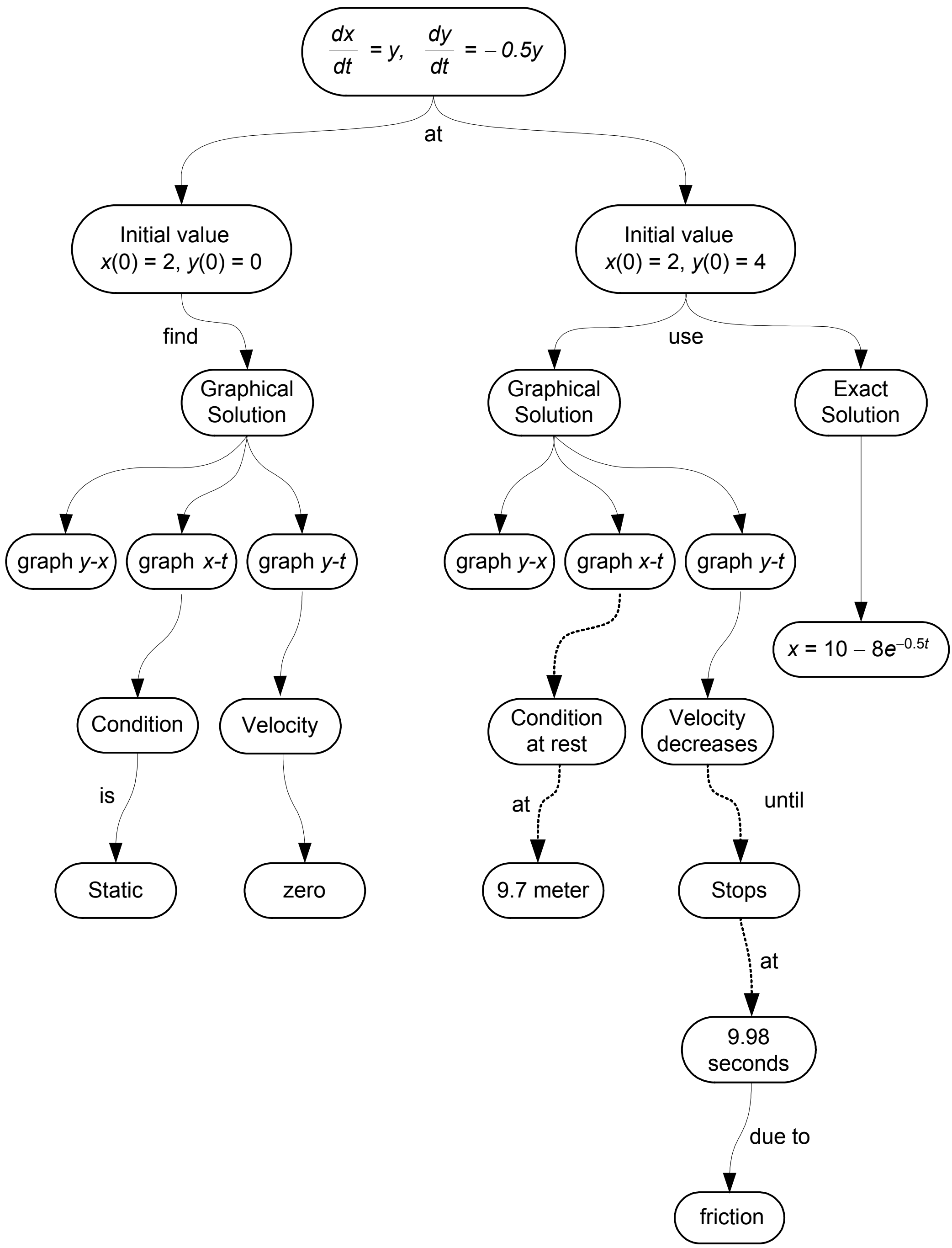

Figure 2 Concept map for linear motion problem, Low-Achiever Group

after they engaged in computer laboratory activies. Their abilities to solve non routine problems with a cognitive tool like DEgraf which is difficult to be solved manually suggest that the software played an important role in enhancing understanding. The concepts used also show their approach in solving the problem involve incorporating algebraic, graphical and numerical methods that were highlighted during lectures. The limitations of an algebraic solution as a technique to solve differential equations is obviously needed to be fasilitated with the use of computers.

Overall, the differences between number of concepts and relations made between the high achievers group and the low achievers group are significant. Concept maps showed that the number of concepts in every problem used by the high achievers group exceed those of the low achievers group. Concepts used by the high achievers group are clear, relevant and organized and therefore convenient for the researcher to construct the concept map. Their existing concepts have been used to develop new concepts or have been applied in new situations during problem solving. Interestingly, the high achievers frequently provide explanations on why the system or phenomena behaved that way with rational argument and not limited on explaining how things happened as portrayed by the low achievers. This suggest that in a computer based learning, the high achievers group achieved relational understanding as pointed by Skemp (1971).

The high achievers group explored the problems beyond what were needed in the problem statement. The low achievers group never made this effort. The high achievers group hardly made any mathematical errors. Some errors made by the low achievers group were resulted from analysing graphs in the static mode that have been printed earlier. The errors could be avoided if they had remembered the dynamical state when the graphs were plotted with DEgraf.

5.0 CONCLUSION

This study has contributed to two significant findings. First of all, the computer based learning environment benefit the students in terms of enhancing their understanding in differential equations. Secondly, it has proven the power of concept map as a tool for examining cognitive structures. However, it cannot be denied that the process of constructing concept maps is influenced by researcher's interpretations on the analysed document. In this study, the reliability of the concept maps are accepted since no necessary changes were made after repeating the process of construction for three times. For the purpose of strengthening the reliability of this research tool, this assumption can be further improved if the concept maps are validated by a panel of experts instead of depending on one researcher. Perhaps with minimum or no differences at all among the panel members in interpretating data on the related documents, accurate concept maps can be constructed to represent students' understanding.

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