The Effect of Numerical Integration Stiffness in Ship Motion Simulation

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Abstract In ship motion stability, generally, capsizing occurs due to the following effects: loss in directional control such as in broaching-to; loss in stability (pure loss of stability) and transient effect like parametric excitation. It is generally accepted that hydrodynamic forces due to waves are dominant that cause vessels' capsizing. It is generally accepted that numerical simulation using computers are reliable to study the ship motions. As a result, the 6 degrees-of-freedom time domain simulation will be used to study the ship motions especially the large amplitude motions. In this paper, we adopt three classes of numerical approach namely explicit Runge-Kutta, implicit Runge-Kutta and Rosenbrock-type Runge-Kutta methods in order to verify the effect of stiffness in ship motion simulation.

Keywords Time Domain Ship Simulation, Explicit Runge-Kutta method, Implicit Runge-Kutta method, Rosenbrock-type Runge-Kutta method. Stiff Ordinary Differential Equations.

***Abstrak** Dalam aspek kestabilan pergerakan kapal, secara amnya, kapal tersebut terbalik disebabkan oleh keadaan berikut: kehilangan kawalan arah seperti "broaching-to"; kehilangan keseimbangan (kehilangan keseimbangan tulen) dan keadaan ketidaktetapan seperti rangsangan parametrik. Daya hidrodinamik merupakan daya dominan yang menyebabkan kapal terbalik. Pada masa kini, penggunaan simulasi berpandukan komputer telah menjadi terkenal dan boleh dipercayai untuk mengkaji pergerakan kapal. Seterusnya, simulasi enam darjah kebebasan domain masa akan digunakan untuk mengkaji pergerakan kapal khususnya pergerakan beramplitud besar. Dalam penyelidikan ini, kita menggu-*

nakan tiga kelas pendekatan berangka iaitu kaedah Runge-Kutta jenis tak tersirat, kaedah Runge-Kutta jenis tersirat dan kaedah Runge-Kutta jenis Rosenbrock untuk mengkaji kesan kaku dalam simulasi pergerakan kapal.

Katakunci *Simulasi kapal berlandas domain masa, kaedah Runge-Kutta jenis tak tersirat, kaedah Runge-Kutta jenis tersirat, kaedah Runge-Kutta jenis Rosenbrock. Sistem Persamaan Terbitan Biasa kaku.*

1 Background

The study of stability in following seas has been conducted by many distinguished researchers including W. Froude, Wendel and Paulling [10]. The prediction of capsizing in following/quartering seas was studied through the research carried out in the University of California by Paulling and his team of researchers (Paulling et al, [11]). Subsequently, Paulling et al [11] and Hamamoto & Akiyoshi, [7] also use the technique of simulating capsizing in astern seas situation using a time domain simulation approach. This technique becomes popular because of its capability for incorporating non-linear forces and its ability to simulate non-linear modes of motion.

The presently developed mathematical model follows the ideas introduced by Paulling et al, [11]. In his model, Paulling emphasized the dominant effect of Froude-Krylov forces on the motions and capsizing characteristics of a vessel in quartering/following seas. However, according to Umeda et al [13], the diffraction effect is inevitable since it will affect the wave-induced sway force, yaw moment and roll moment. Subsequently, this research adopts a six-degrees-of-freedom system which incorporates Froude-Krylov forces and diffraction effect. In the present model, the main interest is on the detection of stiffness effect of ship motions.

2 Equations of Motion

A vessel is assumed as a rigid body having six degrees of freedom. For this vessel, Euler equations of motion can be described as follows:

$$m(\dot{V} + \omega \times V) = F$$
$$\dot{H} + \omega \times H = G \tag{1}$$

where m is the mass and H is the angular momentum of the vessel. G and F are the external force and moment vectors acting on the vessel. V and ω are the linear and angular velocity vectors. Then V and ω can be resolved into u, v and w along the respective x, y, z axis and p, q and r about each axis.

For six-degrees-of-freedom, the equations of motion referred to body axes can be written as follows :

Linear motions and forces
$Surge(x)$
$$m(\dot{u} + wq - vr) = F_x \tag{2}$$

$Sway(y)$

$$m(\dot{v} + ur - wp) = F_y \tag{3}$$

$Heave(z)$

$$m(\dot{w} + vp - uq) = F_z \tag{4}$$

   Angular motions and moments

$Roll(\phi)$

$$I_x\dot{p} + (I_z - I_y)qr = K \tag{5}$$

$Pitch(\theta)$

$$I_y\dot{q} + (I_x - I_z)rp = M \tag{6}$$

$Yaw(\psi)$

$$I_z\dot{r} + (I_y - I_x)pq = N \tag{7}$$

where I_x, I_y and I_z are the principal moments of inertia about the x, y and z axes respectively. The corresponding external forces and moments acting are F_x, F_y, F_z, K, M and N. In addition, u, v and w are the velocity vectors along the x, y and z axes.

2.1   Treatment of Forces/Moments

Usually, the fluid forces comprise of Froude-Krylov and diffraction forces or moments acting on the vessel. In ship simulation program, the vessel heading is controlled by rudder moment which is activated by autopilot system as mentioned by Clarke et al, [2].
As indicated by Equations (1), the fluid force, F and moment, G on the vessel can be deduced from the integration of the pressure acting over the total wetted surface area:

$$F = -\iint\limits_{S} p \cdot n \mathrm{d}S \tag{8}$$

$$G = -\iint\limits_{S} p \cdot (n \times r)\mathrm{d}S \tag{9}$$

where,

   S is the wetted surface of the body,

   n is the unit normal vector directed out of the body surface,

   r is the position vector from the C.G. of the vessel, and

   p is the pressure at a point on the surface at the instantaneous time, t.

Based on the potential flow theory, the pressure, p acting on the surface element can be described by Bernoulli's equation in the following manner:

$$P = -\rho g z - \rho \frac{\partial \Phi}{\partial t} - \frac{1}{2}\rho|\nabla\Phi|^2 \tag{10}$$

As indicated by Equations (8) to (10), the fluid-dynamic force and moment can be deduced by solving the total velocity potential. The total velocity potential can be expressed as follows:

$$\Phi(x, y, z, t) = [-Ux + \phi_s(x, y, z)] + \phi_T e^{i\omega_e t} \tag{11}$$

The term $(-Ux + \phi_s)$ is a time independent steady part due to the steady flow (Ux) and steady wave pattern (ϕ_s) surrounding the vessel. The last term is the time dependent velocity potential associated with unsteady body motions, incident and diffracted waves. This velocity potential, ϕ_T can be further expressed as follows:

$$\phi_T = \phi_I + \phi_D + \sum_{j=1}^{6} \eta_j \phi_{Rj} \tag{12}$$

where,

  ϕ_I is the incident wave potential,
  ϕ_D is the diffracted wave potential, and
  ϕ_{Rj} is the generated/radiation wave potential due to unit motion in j_{th} direction.

The forces and moments derived from ϕ_I and ϕ_D are usually described as wave excitation forces/moments. The forces/moments obtained from the incident wave potential ϕ_I are then termed Froude-Krylov forces and moments.

However, the diffraction potential must satisfy five conditions namely: Laplace equation, linearized free surface condition, kinematics condition on the ship hull, radiation condition and condition at infinity (∞).

The present method adopts Ohkusu method [9] in following oblique waves and it is assumed that the encounter frequency is very low. For simplicity, it is assumed that the ship's beam and draft are small compared to the length $L(\varepsilon = B/L)$ by factors of order $\varepsilon << 1$. Subsequently, the diffraction potential will be analyzed in inner and outer regions. Finally, the diffracted wave profile will be included in order to modify the current wave profile.

In order to simplify the calculation of force and moment in Equations (8) and (9), the surface integrals could be replaced by volume integrals using Gauss's divergence theorem as below:

$$F = - \iiint_V \nabla p \, \mathrm{d}V \tag{13}$$

$$G = - \iiint_V (\nabla p \times r) \mathrm{d}V \tag{14}$$

According to Equation (13) and (14), the Froude-Krylov forces and moments can be obtained by integrating the pressure p derived from incident wave over the entire submerged volume V of the vesesl.

2.2   Simulation Conditons

The ship simulation also includes the smith effect, current effect and auto-pilot system. For ship motion stability, the parameters that affect ship stability can be divided into four groups namely: (1) the environment, (2) vessel geometry, (3) loading condition and (4) vessel/environment interaction.

In the time domain simulation, we incorporate auto-pilot system in order to study the ship stability as mentioned by D. Clarke, et. al. [2]. According to an auto-pilot system, the rudder motion is governed by the equation below:

$$\delta + t\dot{\delta} = k_1 \psi + k_2 \dot{\psi} \tag{15}$$

where,

  - δ and $\dot{\delta}$ are rudder angle and velocity respectively,
  - ψ and $\dot{\psi}$ are yaw angle and yaw velocity respectively,
  - k_1 and k_2 are the coefficient related to the yaw amplitude and velocity respectively.

2.3   Time Domain Integration

According to Equations (2) to (7), the motions of the vessel can be derived by solving the Euler's equations of motion using numerical integration. The present methods provide three methods of numerical integration namely explicit Runge-Kutta, implicit Runge-Kutta and Rosenbrock-type Runge-Kutta methods for a system of ordinary differential equations. A

computer program is written in Borland C++ version 5 and executed using Pentium personal computer (PC) with Windows 95 platform. The final form of the system of equations used is as follows:

$$a_j\ddot{x} + b_j\dot{x} + c_jx + \sum_{k\neq j}^{6} a_{jk}\ddot{x}_k + \sum_{k=1}^{6} b_{jk}\dot{x}_k + \sum_{k=1}^{6} c_{jk}x_k = F_j \quad , \qquad j = 1,\ldots,6 \qquad (16)$$

To derive velocities and displacements of motions, the set of accelerations need to be integrated as below:

$$\ddot{x} = \left(F_j - b_j\dot{x} - c_jx - \sum_{k\neq j}^{6} a_{jk}\ddot{x}_k - \sum_{k=1}^{6} b_{jk}\dot{x}_k - \sum_{k=1}^{6} c_{jk}x_k \right) / a_j \quad , \qquad j = 1,\ldots,6 \qquad (17)$$

To integrate the above system of equations, the second - order term is transformed into first - order system of differential equations in the following way:

Let,

$$y_1 = x, \qquad y_2 = \dot{x} \qquad (18)$$

Finally, the first-order system of ordinary differential equations become:

$$\dot{y}_1 = y_2 \qquad (19)$$

$$\dot{y}_2 = \left(F_j - b_jy_2 - c_jy_1 - \sum_{k\neq j}^{6} a_{jk}\dot{y}_{2k} - \sum_{k=1}^{6} b_{jk}y_{2k} - \sum_{k=1}^{6} c_{jk}y_{1k} \right) / a_j \quad , \qquad j = 1,\ldots,6 \,(20)$$

After substituting Equation (18) into Equation (19) & (20), the motions of the vessel in time domain can be obtained.

3   Numerical Integration Method

In this paper, we use one-step Runge-Kutta method to calculate the ship motion. The present method use three approaches namely explicit, implicit & Rosenbrock-type Runge-Kutta methods. The explicit Runge-Kutta method is not applicable for stiff problem. Implicit, even Rosenbrock-type (semi-implicit) methods, are very expensive to implement and cannot rival explicit method in efficiency when the problem is not stiff. They are more restricted to stiff systems, in which their superior stability properties justify the high cost of implementation.

For explicit and implicit Runge-Kutta method, the following scheme is used:
For the system of ordinary differential equations as below:

$$\dot{y}(t) = f(t, y(t)), \qquad 0 \leq t \leq T, \quad y(0) = y_0, \qquad (21)$$

Here $y(t)$ is a real vector of m elements and f a real-valued vector function, possibly non-linear in the dependent and independent variables.
All the one-step Runge-Kutta formula has the form

$$y_{n+1} = y_n + h \sum_{i=1}^{s} b_ik_i, \qquad (22)$$

where

$$k_i = f(t_n + c_ih, y_n + h \sum_{j=1}^{s} a_{ij}k_j), \quad h = t_{n+1} - t_n, \qquad (1 \leq i \leq s). \qquad (23)$$

The real parameters b_i, c_i and a_{ij} define the method. The formula above is called an s-stage formula because it is based on s evaluations of the derivative function f. If $a_{ij} = 0$ for $j \geq i$ and all i, then the k_i can be computed in an explicit way from $k_1, \ldots, k_{i-1}$. Such formulas

are therefore called explicit. If $a_{ij} = 0$ for $j > i$ and $a_{ii} \neq 0$, such Runge-Kutta methods are called diagonally implicit. If the method is neither explicit, nor diagonally implicit, it is just called implicit; then all k_i must be computed simultaneously.

For Rosenbrock-type Runge-Kutta method, the following non-autonomous system is adopted:

$$\left(\frac{1}{h\gamma_{ii}}I - \frac{\partial f}{\partial y}(t_0, y_0)\right)u_i = f(t_0 + \alpha_i h, y_0 + \sum_{j=1}^{i-1} a_{ij}u_j) + \sum_{j=1}^{i-1}(\frac{c_{ij}}{h})u_j + \gamma_i h \frac{\partial f}{\partial t}(t_0, y_0). \qquad (24)$$

If $\gamma_{ii} \neq 0$ for all i, the matrix $\Gamma = (\gamma_{ij})$ is invertible, where

$$\alpha_i = \sum_{j=1}^{i-1} \alpha_{ij}, \qquad \gamma_i = \sum_{j=1}^{i} \gamma_{ij}. \qquad (25)$$

$$(a_{ij}) = (\alpha_{ij})\Gamma^{-1}, \qquad (m_1, \ldots, m_s) = (b_1, \ldots, b_s)\Gamma^{-1}. \qquad (26)$$

$$C = diag(\gamma_{11}^{-1}, \ldots, \gamma_{ss}^{-1}) - \Gamma^{-1}. \qquad \text{(where } c_{ij} \text{ elements of matrix } C) \qquad (27)$$

For simplification of the order conditions, the following assumption is made:

$$\gamma_{ii} = \gamma, \qquad \text{for all } i, \qquad (28)$$

The solution of Equation (21) is given by

$$y_{n+1} = y_n + \sum_{j=1}^{s} m_j u_j, \qquad (29)$$

3.1   Explicit Runge-Kutta Method

In this paper, we adopt two explicit Runge-Kutta methods namely: 5-stages explicit Runge-Kutta-Merson method & 7-stage Step-Control Stable Explicit Runge-Kutta[1]. The former is 5-stage, order 4(5) (for linear equations) or 4(3), (for nonlinear equations) which produces a second approximation $\widetilde{y_1}$ by using embedded method, E.Hairer et.al[6]. The latter is order 5(4), introduced by D.J. Higham & G. Hall [6]. Step-control stability will cause the step sizes h_n behave smoothly and less step rejections.

3.2   Implicit Runge-Kutta Method

Presently, we use four type of implicit Runge-Kutta methods for stiffness verification. 2-Stages Gauss-Legendre Runge-Kutta method with 4th order is used. It is an implicit Runge-Kutta method with the characteristic of B-stable, algebraically stable and B-convergent behaviour. We adopt this code since it has fewer stages as well as of higher order. The second method is 3-stages 4th order Diagonally Implicit Runge-Kutta method. We also adopt 3-stages Radau IIA method which is A-stable, L-stable, stiffly accurate and high order with order 5. Finally, we adopt a 4th order 5-stages singly diagonal implicit Runge-Kutta method with L-stable characteristic. It also satisfies the stiffly accurate behaviour.

3.3   Semi-Implicit Runge-Kutta Methods

In this research, two approaches are adopted by using accurate Jacobian or inaccurate Jacobian matrices as mentioned below :

Approach I : 3rd order linearly implicit L-stable Method

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[1]The coefficients of this method are used for stiffness detection.

We adopt Type A1 method for stiff ordinary differential equation which was introduced by R.E. Scraton (for autonomous problem, R.E. Scraton [12]).
We consider a system of differential equations:

$$\dot{y} = f(y) \tag{30}$$

where,

y is a vector,

f is a vector-valued function, and

$\dot{y}$ denotes differentiation with respect to the independent variable t.

The values of x, y and $f(y)$ at the beginning and end of a step are denoted by x_0, y_0, f_0 and x_1, y_1, f_1 respectively, the interval $(x_1 - x_0)$ being denoted by h.
We get,

$z_0 = (I - 0.55hK)^{-1} h f_0$
$z_1 = (I - 0.55hK)^{-1} z_0$
$y^{(2)} = y_0 + 1.0490950708 z_0 - 0.1857285894 z_1$
$z_2 = (I - 0.55hK)^{-1}(z_1 - 1.1601352857 h f^{(2)})$
$y^{(3)} = y_0 + 0.5286554427 z_0 - 0.1943049474 z_1 - 0.3114735159 z_2$
$z_3 = (I - 0.55hK)^{-1}(z_2 + 0.6876300395 h f^{(2)} + 1.0701819460 h f^{(3)})$
$y_1 = y_0 + 1.0460771164 z_0 - 0.9233005966 z_1 - 0.4975710601 z_2 + 0.4991903534 z_3 \tag{31}$
$E = -0.1195847349(y_1 - y_0) + 0.0148758403 h f_0 + 0.0408237989 h f^{(2)} + 0.0638850956 h f^{(3)} \tag{32}$

where E is the approximation error. The z_1, z_2, z_3 are the interim values and $y^{(i)}$ is an intermediate value of y.

Approach II : Runge-Kutta Kaps-Rentrop Method

This is a 4th order, 4-stages Rosenbrock-type method which requires accurate Jacobian evaluation. It involves the evaluation of $\frac{\partial f(t,y)}{\partial t}$ for non-autonomous system. It is $A(\alpha) -$ stable for $\alpha \le 89.3^o$ and $|R(\infty)| = 0.454$.
The value of coefficients is not unique and is calculated by solving a system of nonlinear equation which is the whole set of order conditions.
For Kaps-Rentrop method with $\gamma = 0.231$, we derived the following coefficients values:

$(m_1, m_2, m_3, m_4) = (4.78507293, 7.506159769, 0.543508635, 0.489547912), \tag{33}$
$(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = (0.0, 0.462, 1.128542758, 1.128542758), \tag{34}$

$$(a_{ij}) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 2.0 & 0 & 0 & 0 \\ 6.345080376 & 8.508038521 & 0 & 0 \\ 6.345080376 & 8.508038521 & 0.0 & 0 \end{pmatrix} \tag{35}$$

$$(c_{ij}) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ -5.071675325 & 0 & 0 & 0 \\ -0.1468358363 & -16.17294805 & 0 & 0 \\ -2.537290077 & -32.42414949 & -4.80617153 & 0 \end{pmatrix} \tag{36}$$

$$\gamma_i = \sum_{j=1}^{i} \gamma_{ij} = \begin{pmatrix} \gamma \\ -0.270629667 + \gamma \\ 1.003223868 - 0.863004681 + \gamma \\ 0.777813216 - 0.772055131 - 0.256462119 + \gamma \end{pmatrix} \qquad (37)$$

4 Rate of Convergence

According to Butcher [1], if the iterated approximations to y (where $\dot{y} = f(t,y)$) are $y^{[0]}$, $y^{[1]}$, $y^{[2]}$, ..., then the convergence rate, r can be estimated as

$$r \approx \|y^{[k]} - y^{[k-1]}\| / \|y^{[k-1]} - y^{[k-2]}\| \text{ for } k= \textit{2, 3, ...} \qquad (38)$$

By adjusting the value of h, the convergence rate can be set between 0 and 1. If r is too close to 0, the number of steps will be excessive, whereas if r is close to 1, the convergence will be slow. Since the convergence rate is subjected to change, we use the average convergence rate. Butcher pointed out that the optimal value of r is approximated to 0.366 (e^{-1}). Butcher further point out that the value of r can be used to determine an appropriate stepsize. However, in ship motion simulation, it is required to calculate the motion results in a certain time which is requested by the user/customer. Consequently, the steps size control mechanism is not required/allowed.

5 Automatic Stiffness Detection

In order to detect the stiffness of a system of ordinary differential equation, we adopt the ideas of Shampine & Hiebert [E.Hairer,5] by introducing two error estimators namely : $err_1 = y_1 - \widehat{y}_1$ and second error estimator $\widetilde{err_1}$ where $\widetilde{err_1} \approx \widetilde{E}(hJ)d_0$ which we defined $E(z) = R(z) - \widehat{R}(z)$ where $R(z)$ is stability function. The second error estimator must satisfy the following conditions:

(i) $|\widetilde{E}(z)| \leq \theta |E(z)|$ on $\partial S \cap C^-$ with a small $\theta < 1$;

(ii) $\widetilde{err_1} = O(h^2)$ for h $\rightarrow 0$.

The condition (ii) leads to $\|\widetilde{err_1}\| \gg \|err_1\|$ when the problem is not stiff. The above conditions imply that if
$$\| \widetilde{err_1} \| < \|err_1\| \qquad (39)$$
occurs several times, β (say 15 times as mentioned by E.Hairer,[5]) in succesion, then a stiff code might be more efficient near the y_{n+1}.
For the construction of $\widetilde{err_1}$ we put
$$\widetilde{err_1} = h(d_1k_1 + d_2k_2 + \ldots + d_sk_s) \qquad (40)$$
where $k_i = f(x_0 + c_i h, g_i)$. In order to fulfil the above two condition, (i) & (ii), the coefficients d_i need to satisfy the following conditions:
$$\sum_{i=1}^{s} d_i = 0, \qquad \sum_{i=1}^{s} d_i c_i = 0.02. \qquad (41)$$
The solution of Equations (41) is not unique and one of the solutions for 5-stage SDIRK is:

$$(d_1, d_2, d_3, d_4, d_5) = (-0.54, 0.44, 0.1, 0.1, -0.1) \tag{42}$$

The second possibility for detecting stiffness is to estimate directly the dominant eigenvalue of the Jacobian of the problem. The eigenvalue can be approximated as below:

$$|\lambda| \approx \frac{\|f(x,y+v)-f(x,y)\|}{\|v\|} \tag{43}$$

where v denotes an approximation to the corresponding eigenvector with $\|v\|$ sufficiently small. Whenever the stiffness is detected, the product of the step size with the dominant eigenvalue of the Jacobian lies near the border of the stability domain.

We apply the second method in SC-stable explicit method introduced by D.J. Higham & G. Hall where $c_6 = c_7 = 1$. The stability domain of SC-stable is approximated in order to detect the stiffness as:

$$h|\lambda| > 4.0 \tag{44}.$$

If the inequality (44) is satisfied, then it is stiff near the solution of y_n.

6 Results and Discussion

In variable sea environment, the analytical solution is not available. In order to validate the solution, all the generated results will be compared to a more reliable solution which is produced by L-Stable implicit Runge-Kutta method. Basically, the effect of stiffness will be measured in two different sea environment namely: beam sea and capsize mode. For the following graphs, 'stiffness' refers to Equation (44) (1 means stiffness is detected) and β refers to inequality (39) (number of occurrence).

6.1 Beam Sea

Refering to the Figures 1, 2 and 3, it is shown that ship simulation suffers from numerical integration stiffness in beam sea. The effect of stiffness is more obvious in implicit Runge-Kutta methods as indicated in Figures 1, 2 and 4. However, for explicit and semi-explicit methods, the speed of calculation is more stable especially for Kaps Rentrop method as shown in Figure 3. Figure 4 showed that the stiffness induced a significant changing of time consumption and convergence rate. Obviously, the convergence rate affects the computation time not proportionally since the convergence rate is measured at the beginning step of y_n. Figures 2 and 4 showed some discrepency of stiffness detection when the simulation is runned using L-stable 5-stage SDIRK (Fig.2) and L-stable 3-stage RadauIIA (Fig.4). In terms of simulated motions, two categories of methods are compared, namely explicit Merson Runge-Kutta and L-stable Implicit RadauIIA. As shown in Figures 5, 6, 7 and 8, the dominant forces like roll & yaw motion are promising. The recessive component like yaw velocity is seem slightly different in phase, however it is still acceptable as shown in Table 1.

The average time consumption of explicit and semi explicit methods in beam sea are:

Explicit Merson method = 0.65 sec.

SC stable explicit method = 0.85 sec.

Linear implicit method = 4.43 sec.

Kaps Rentrop method = 5.52 sec.

6.2   Capsizing mode

In capsizing mode, Table 2 showed that only four methods can successfully detect the stiffness at the same time ($t = 6$). Basically, only the capsizing time calculated by Gauss Legendre method is obviously deviated from common capsizing range ($6.4 \leq t \leq 6.5$). The result produced by Linear Implicit method is slightly deviated from this acceptable range. As shown in Figure 9, obviously the stiffness detected using inequality (44) is more promising than β (number of occurrence) which was based on inequality (39).

The average time consumption of explicit and semi explicit method in capsizing mode are:

Explicit Merson method = 0.5583 sec.
SC stable explicit method = 0.7417 sec.
Linear implicit method = 4.3667 sec.
Kaps Rentrop method = 4.9717 sec.

7   Conclusion

Based on the results of the research presently undertaken, the following conclusion can be drawn:

1. Numerical stiffness is detected randomly which directly increases the computation time.

2. Numerical stiffness is also detected before the capsizing of ship.

3. Implicit Gauss Legendre method is not suitable especially in capsize mode.

4. For the development of real time ship simulation system, the explicit Runge-Kutta method especially 5-stage Merson and 7-stage SC stable Runge-Kutta method are suitable, despite some discrepancy in recessive motion components.

8   References

[1]   J. C. Butcher, *Order, Stepsize and Stiffness Switching*, Computing, 44(1990), No.3, 209-220.

[2]   D. Clarke, P. Gedling and G. Hine, *The Application of Manoeuvring Criteria in Hull Design Using Linear Theory*, Trans RINA, Vol. 125, 1983.

[3]   K. Dekker and J. G. Verwer, *Stability of Runge-Kutta methods for stiff nonlinear differential equations*, Elsevier Science Publishers, Netherlands, 1984.

[4]   S. Grochowalski, *Investigation into the Physics of Ship Capsizing by Combined Captive and Free-Running Model Tests*, The Society of Naval Architects and Marine Engineers, Annual Meeting, N.Y., Nov.15-18, 1989.

[5]   E. Hairer & G.Wanner, *Solving Ordinary Differential Equations II*, Springer-Verlag, Berlin, 1991.

[6]   E. Hairer, S. P. Nϕrsett & G.Wanner, *Solving Ordinary Differential Equations I*, Second Revised Edition, Springer-Verlag, Berlin, 1993.

[7]   M. Hamamoto and T. Akiyoshi, *Study on Ship Motions and Capsizing in Following Seas (1st Report)-Equations of Motion for Numerical Simulation*, Spring Meeting of the Society of Naval Architects of Japan, May, 1988.

[8]   Maimun, A., *Stability of Fishing Vessels in an Astern Sea-Shallow Water Environment*, Ph.D Thesis, Dept. of Ship and Marine Technology, University of Strathclyde, 1993, Glasgow.

[9]   M. Ohkusu, *Prediction of Wave Forces on a Ship Running in Following Waves with Low Encounter Frequency*, Naval Arch. and Ocean Engineering,Vol. 25, 1987.

[10]   J. R. Paulling, *The Transverse Stability of a Ship in a Longitudinal Seaway*, Jour. of Ship Research, Vol. IV, No. 4, 1961.

[11]   J. R. Paulling, O. H. Oakley and P. D. Wood, *Ship Capsizing in Heavy Seas: The Correlation of Theory and Experiments*, Intl. Conf. on Stability of Ships and Ocean Vehicles, Glasgow, 1975.

[12]   R. E. Scraton, *Third-Order Linearly Implicit L-Stable Methods for Stiff Differential Equations*, Intern. J. Computer Math, 28(1989), 195-205.

[13]   N. Umeda, Y. Yamakoshi and S. Suzuki, *Experimental Study for Wave Forces on a Ship running in Quartering Seas with Very Low Encounter Frequency*, The Sevastianov Symposium, International Symposium ship Safety in a Seaway, Kaliningrad, Russia, 15-19 May, 1995.

[14]   S. H. Yeak, *The Effect of Diffraction on Large Amplitude Simulation of A Fishing Vessel*, M.Sc Thesis, Dept. of Mechanical Engineering, Universiti Teknologi Malaysia, 1998.

Rajah 1: Implicit Methods versus Time Con-
sumption (Gauss Legendre method)

Rajah 2: Implicit Methods versus Time Con-
sumption

Rajah 3: Explicit Methods versus Time Con-
sumption

Rajah 4: Convergence Rate versus Time Con-
sumption

Rajah 5: Simulation result of RadauIIA Runge-
Kutta method

Rajah 6: Simulation result of RadauIIA Runge-
Kutta method (surge & yaw)

Rajah 7: Simulation result of Explicit Merson Runge-Kutta method

Rajah 8: Simulation result of Explicit Merson Runge-Kutta method (surge & yaw)

Rajah 9: Numerical Method versus Time Consumption (capsize mode)

Jadual 1: Average of absolute different percentages compared with 3-stage RadauIIA method

Force/ moment components	5-stage Merson Runge-Kutta	7-stage SC stable Runge-Kutta
Roll (dominant)	0.096	0.128
Yaw velocity (recessive)	1.5	1.88

Jadual 2: Capsizing time and stiffness detection

Runge-Kutta Method	Capsize time (sec.)	Stiffness detection
3-stage DIRK	6.5069	no
Gauss Legendre	5.9	no
3-stage RadauIIA	6.4645	yes
5-stage SDIRK	6.475	yes
Merson	6.45	yes
SC-stable	6.45	yes
Linear Implicit	6.6	no
Kaps Rentrop	6.4462	no