



Influence of Initial Conditions on Unsaturated Groundwater Flow Models

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Abstract

Slope failure in unsaturated soils is common in tropical countries due to the seasonal pattern of abundant rainfall preceding a period of prolonged drying. Much research has been undertaken to understand the behaviour of unsaturated slopes and reduce the number of catastrophic failures. The initial conditions are important factors in numerical modelling of the groundwater flow yet rarely considered in detail in the literature. This paper presents a parametric study of the initial conditions at a slope in Bukit Timah, Singapore. The intensity and duration of rainfall are varied to assess the effect on the pore-water pressure in the slope. The generated pore-water pressure profile is compared with field measurements and previous numerical studies. It is discovered that a low rainfall, with an intensity of 1×10^{-7} m/s over a period of 62 days, results in initial pore-water pressure which is consistent with data recorded at the field. Unlike the duration, changes in the rainfall intensity are shown to have a significant effect on the pore-water pressure in the slope. This study, therefore, demonstrates the importance of determining appropriate initial conditions in unsaturated groundwater flow analysis.

Keywords: rainfall; groundwater flow; unsaturated soils;

1. Introduction

Rainfall has been recognized as the dominant triggering event for landslides in Singapore [1]. These natural disasters cause loss of life and economic losses [2] and have drawn the attention of many researchers to investigate slope behaviour in unsaturated soils by undertaking physical and numerical studies. Several research works were conducted in physical modelling, but many were modelled numerically due to cost and time constraints such as the research work by Oh et al. [3] and Zhang et al. [4]. Nevertheless, there are also many challenges in numerical modelling for example to determine the appropriate initial conditions. Numerous numerical studies have been conducted using assumed or common values for the input parameters due to the lack of field data. By executing these options, the calculation may reduce the accuracy in predicting realistic soil behaviour.

In numerical modelling, the initial condition is an important phase as it generates the initial pore-water pressure of the slope profile. There is limited information in the literature about the procedures to develop the initial conditions. The method is crucial for certain geotechnical software packages that do not provide pre-setting conditions (i.e. initial pore-water pressure) and rely on generating the pore-water pressure manually. One recognised method which is normally used is to apply a small amount of rainfall. In addition, the major factors that can be taken into account in generating the pore-water pressure are the intensity and duration of rainfall.

To model the infiltration of rainfall corresponding to soil behaviour, the interaction of soil and water has to be carefully determined. In unsaturated soil mechanics, the soil hydraulic properties can be measured as the volumetric water content and the coefficient of permeability with respect to matric suction, using the soil-water characteristic curve (SWCC) and permeability function respectively. The SWCC is a relationship between the water content and suction in the soil while the permeability function is the relationship between the coefficient of permeability and soil suction [5]. Many equations of these curves have been introduced over the decades and the most implemented ones are those by Van Genuchten [6] and Fredlund et al. [7].

2. Unsaturated groundwater flow

The investigation of unsaturated groundwater flow is important to understand the behaviour of the unsaturated soils with the effects of rainfall. Over the decades, the behaviour of partially saturated soils has been widely modelled using the SWCC and the permeability function. As there are many equations available in the literature, it is important to select a suitable equation in order to predict realistic soil behaviour. The governing equation applied in modelling the groundwater flow is presented below. In general, the fundamental theory of flow by Darcy's Law is widely adopted in many groundwater flow analyses. Many other theories were later developed for more complex soil profiles. Hence, the two theories of SWCC and permeability functions are essential in capturing the groundwater flow in partially saturated soils.

2.1. Unsaturated soil

Unsaturated soil is formed above the phreatic level due to the cyclic weathering process of prolonged drying and abundant rainfall. The SWCC is used to describe the water extraction for drying and the permeability function for the rate of water seepage. The curve as proposed by Fredlund et al. [7] consists of the water content at specific suction, θ_w , and at saturation, θ_s , degree of saturation, S , suction given as the difference of pore-air pressure and pore-water pressure, $(u_a - u_w)$, e as a natural number with value of 2.71828, $C(\psi)$ as the correction factor and the three shape-parameters ; a , n and m that refers to the air-entry value (AEV), pore size distribution and the slope of the SWCC respectively. The SWCC is given as:

$$\theta_w = C(\psi) \frac{\theta_s}{\{ \ln[e + (u_a - u_w/a)^n] \}^m} \tag{1}$$

They further suggested an additional parameter which is the residual water content, θ_r as it will plot a sigmoidal curve. Leong et al. [8] concluded that sigmoidal curve is more versatile and perform better fit of the SWCC. Fredlund et al. [7] also discovered that when $C(\psi)$ is taken as 1.0, the equation can be used with less complexity. Therefore, the equation is given by Equation (2) and it is used in this study.

$$\theta_w = \theta_r \frac{\theta_s - \theta_r}{\{ \ln[e + (\psi/a)^n] \}^m} \tag{2}$$

Moreover, the permeability equation defines the coefficient of permeability with respect to water, k_w as a function of both the void ratio and water content [9]. By using k_w which can be derived from the SWCC corresponding at any value of the water content, Childs et al. [10] suggested the transformation of the SWCC, $\theta_w(\psi)$ into $\theta_w(r)$ where r is the pore radius. The transformation requires tedious work thus Marshall [11] improved the permeability function and recommended the application of identical water content intervals leading to the following equation:

$$k_w(\theta_w) = \frac{T_s^2}{2\rho_w g \mu} \frac{n^2}{m^2} \sum_{i=1}^l \frac{2(l-i)-1}{\psi_l^2} \tag{3}$$

Where T_s is the surface tension of water, ρ_w is the density of water, μ the dynamic viscosity of water, n the porosity of soil, m as the total number of intervals given by $= (\frac{\theta_s}{\Delta\theta_w})$, l as the number of intervals corresponding to θ_w given by $= (\frac{\theta_w}{\Delta\theta_w})$, ψ_l as the matric suction corresponding to the midpoint of the i th interval of the SWCC. This permeability function has been adopted to calculate the saturated coefficient of permeability.

2.2. Groundwater flow

The groundwater flow analysis can be performed using finite element codes program with an initial phase of steady state and a series

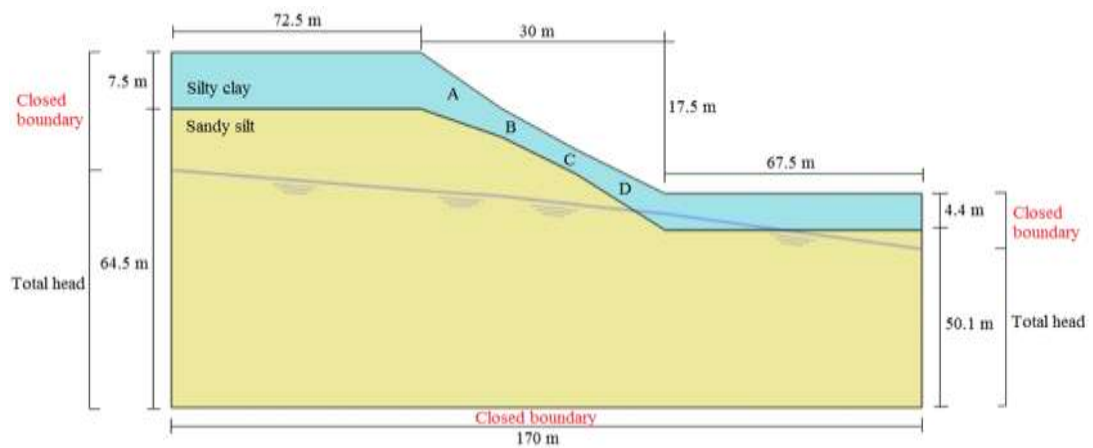


Fig. 1: Slope geometry and boundary conditions

of transient seepage phases. Commonly, groundwater flow is governed by the fundamental Darcy's law and the continuity equations. In three dimensions the flow equation can be written as in Equation (4) [12].

$$\underline{q} = \frac{k}{\rho_w g} (\nabla p_w + \rho_w \underline{g}) \tag{4}$$

Where $\underline{\nabla} = \begin{bmatrix} \delta/\delta x \\ \delta/\delta y \\ \delta/\delta z \end{bmatrix}$ and $\underline{g} = \begin{bmatrix} 0 \\ -g \\ 0 \end{bmatrix}$

The terms q , k , g and ρ_w are the specific discharge (fluid velocity), permeability tensor, acceleration vector due to the gravity and the density of water, respectively. The groundwater flow is the resultant of the pore-water pressure gradient presented as ∇p_w . However, when hydrostatic conditions are assumed, the term $\rho_w g$ will be used because the flow is no longer affected by the gradient of pore-water pressure in vertical direction. In unsaturated soil mechanics, the coefficient of permeability, \underline{k} is related to soil saturation by:

$$\underline{k} = k_{rel} \times \underline{k}^{sat} \quad (5)$$

$$\text{Where } \underline{k}^{sat} = \begin{bmatrix} k_x^{sat} & 0 & 0 \\ 0 & k_y^{sat} & 0 \\ 0 & 0 & k_z^{sat} \end{bmatrix}$$

The k_{rel} is the ratio of permeability at a specific saturation of the permeability in saturated state, \underline{k}^{sat} . For the continuity equations, the simplified transient groundwater flow by neglecting the displacement is given as:

$$\nabla^T \cdot \left[\frac{k_{rel}}{\rho_w g} \underline{k}^{sat} (\nabla p_w + \rho_w g) \right] - n \left(\frac{s}{K_w} - \frac{\partial s}{\partial p_w} \right) \frac{\partial p_w}{\partial t} = 0 \quad (6)$$

In steady-state conditions, where $\frac{\partial p_w}{\partial t} = 0$, the formula is given as in Equation (7). The parameters n and S are the porosity and the degree of saturation respectively.

$$\nabla^T \cdot \left[\frac{k_{rel}}{\rho_w g} \underline{k}^{sat} (\nabla p_w + \rho_w g) \right] = 0 \quad (7)$$

The saturation and permeability with respect to pressure head determine the unsaturated groundwater flow in the Plaxis2D software [13], as explained in Plaxis [14].

3. Numerical modelling

A parametric study is undertaken to investigate the effect of rainfall intensity and duration on the initial pore-water pressure. The geometry of the slope and the flux boundary conditions applied are shown in Fig. 1. The lateral boundaries are assigned with impermeable layers above the groundwater table and total head below the phreatic level. Furthermore, the slope surface is modelled with precipitation while the bottom boundary is assumed impermeable. Based on the slope profile, the impermeable layers (no water flow) are indicated by the closed boundaries. There are two different soil layers with different soil parameters as shown in Table 1 for mechanical and hydraulic properties of the soil. The slope geometry also shows the groundwater table as presented by the phreatic line. The height of end to end phreatic line are given at 64.5 m and 50.1 m with a distance of 170 m.

The calibrated SWCC and permeability function are shown in Fig. 2 and the typical rainfall intensity used in the simulation is plotted in Fig. 3. The SWCC were developed using curve-fitting based on the curve parameters against the Van Genuchten [6] model. The field data of the pore-water pressure are recorded by the tensiometers placed at locations indicated in Fig. 1 at the crest, mid-slope and toe of the slope as A, B, C and D. All locations are calculated at same depth of 2.08 m.

Table 1: Mechanical soil properties used in the simulation [15]

Description	Silty sand	Sandy silt
<i>Mechanical properties</i>		
Effective cohesion, c' (kPa)	9	0
Effective angle of friction, ϕ' ($^\circ$)	34	33
Total density, ρ_s (Mg/m ³)	2.03	1.88
Specific gravity, G_s	2.66	2.58
Void ratio, e	0.8	0.86
<i>Hydrological properties</i>		
Saturated coefficient of permeability, k_s (m/s)	6×10^{-6}	3.3×10^{-5}
Saturated volumetric water content, θ_s	0.6	0.5306
Fitting parameter, a (kPa)	1.1	7
Fitting parameter, n	0.55	5
Fitting parameter, m	1.33	0.7
Air-entry value, (AEV)	15	5
Residual volumetric water content, θ_r	0.15	0.15
Residual suction ψ_r (kPa)	6000	22

4. Result and discussion

4.1. Effect of rainfall intensity

The pore-water pressure calculated using groundwater flow analysis in Plaxis was compared with the results obtained using Seep/W [16]. The results of pore-water pressure are calculated at locations A, B, C and D (see Fig. 1) as presented in this paper. Five different intensities have been applied to the slope model with one control value. Based on the report from the MET Office [17], Singapore has two major periods of abundant rainfall which is in the mid-year in between June to August and another at the end of the year in between November and January. The heaviest rainfall is recorded at 200 mm per month on average. Hence, the control value is selected based on the heavy rainfall with an amount of 8.64 mm/day. In this study, the rainfall intensity is converted to 1×10^{-7} m/s for a period of 62 days. From the

results shown in Fig. 4, the pore-water pressure has changed more at the toe, represented at location D. At the toe, a lower negative pore-water pressure was captured at the earlier duration compared to other locations but suction was eliminated when larger rainfall intensity (i.e. 5.5×10^{-7} m/s) was used. It can be said that slow infiltration occurred at the toe due to higher saturation at the surface which causing ponding to occur. Moreover, Kristo et al. [17] suggested that there is a maximum amount of rainfall to destabilise a slope which the additional water will continue to flow down the slope as runoff. They also added that this scenario indicates the marginal effect of slope stability. When a lower rainfall intensity of 5.5×10^{-8} m/s was applied, it can be observed in Fig. 4 that the negative pore-water pressure reduced very minimal compared to other rainfall intensities. Zhang et al. [4] stated that when the water flux is less than the saturated coefficient of permeability, the matric suction can decrease but not disappear entirely which explains the low reduction of suction in this case.

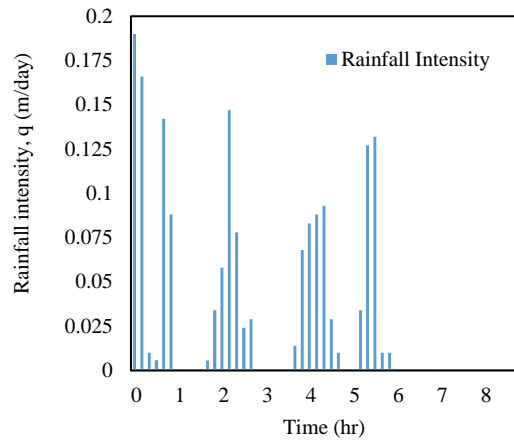


Fig. 1: Typical rainfall in Singapore [15]

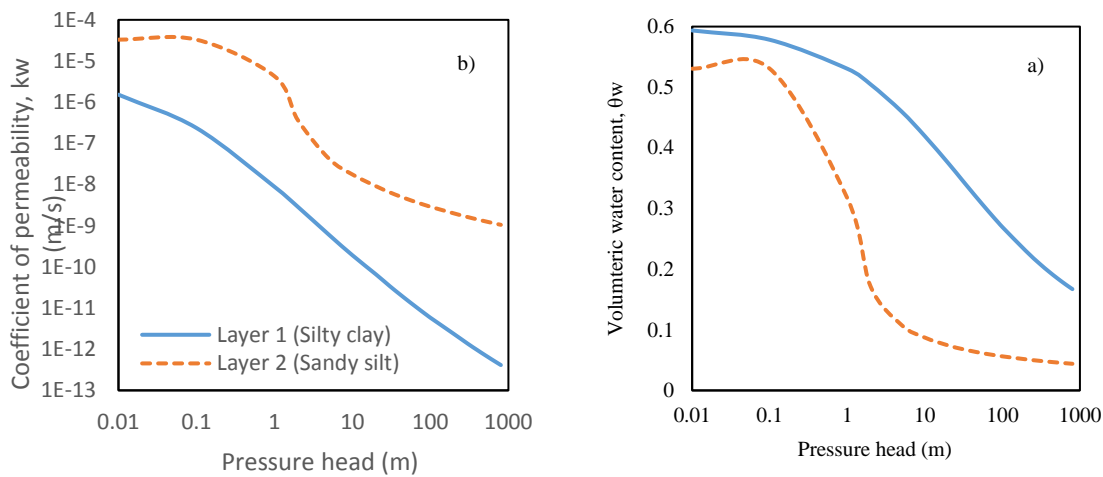


Fig. 2: (a) SWCC (b) permeability function

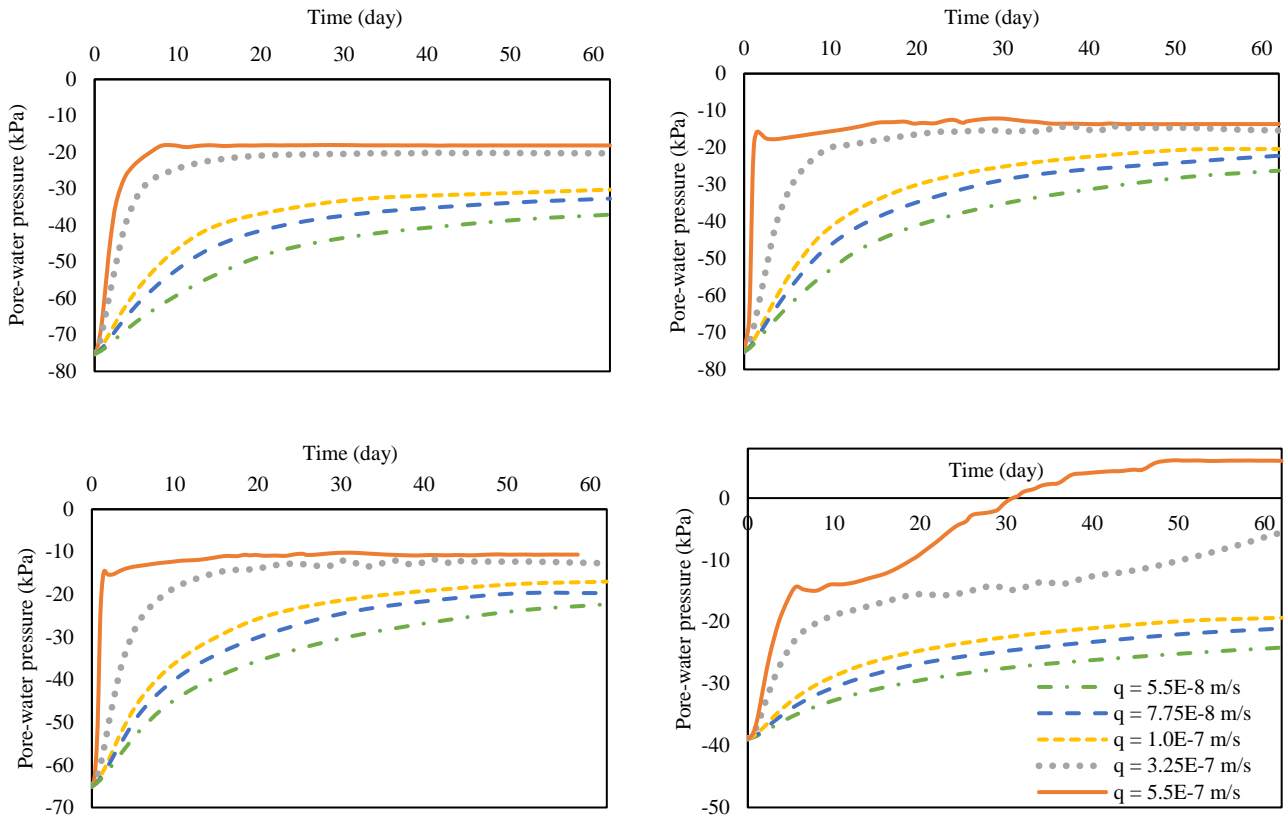


Fig. 4: Initial PWP for different rainfall intensity at depth of 2.08 m for location: (a) A, (b) B, (c) C and (d) D

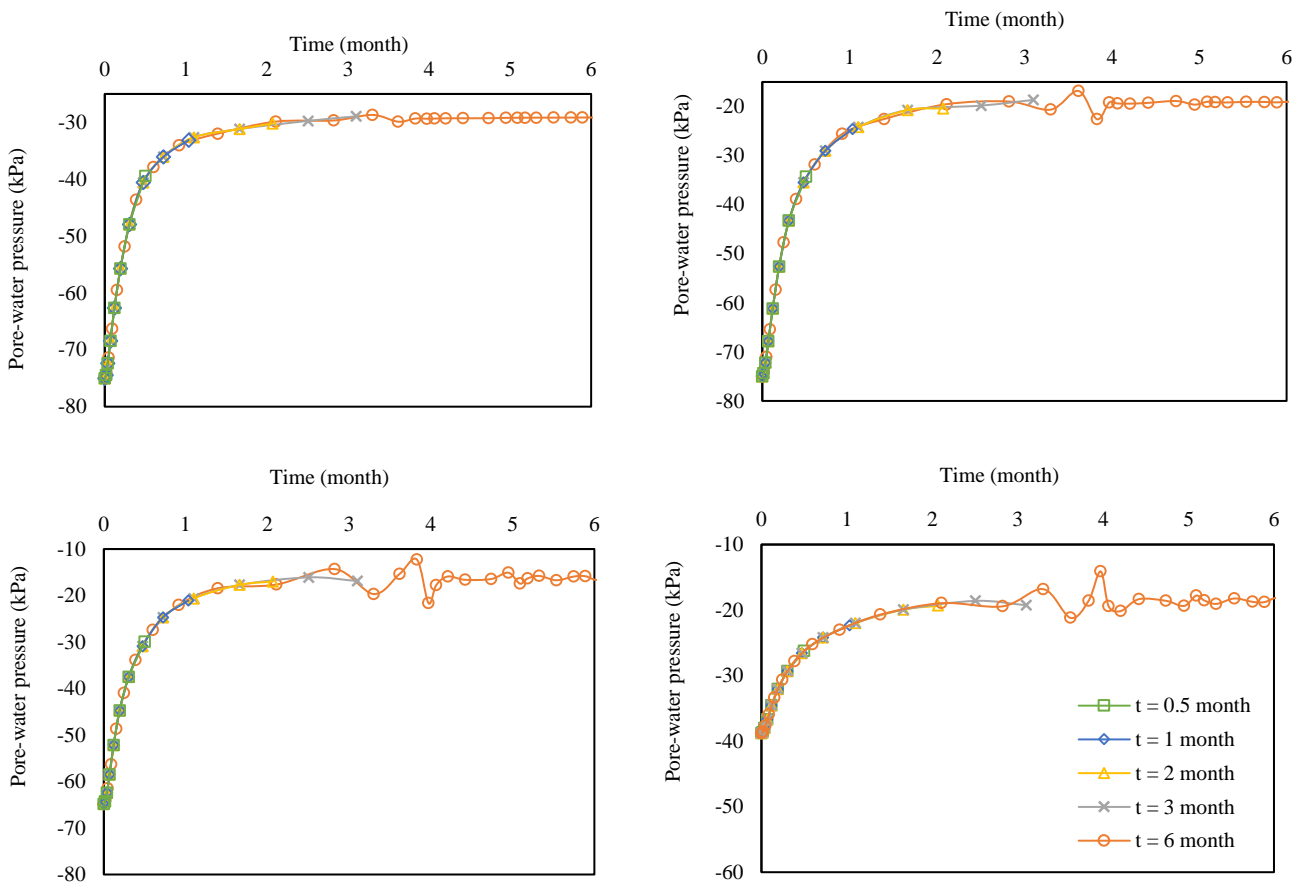


Fig. 5: Initial PWP for different durations at depth of 2.08 m for location (a) A, (b) B, (c) C and (d) D

When a higher rainfall intensity had been applied, the negative pore-water pressure continues to reduce but at a faster rate. The largest rainfall intensity applied in the parametric study shows that the negative pore-water pressure reduced at the highest rate and it remains constant over the period of rainfall. This can be explained by the large amount of rain-water infiltrated into the soil and once the soil has reached saturation, the negative pore-water pressure becomes steady. Thus, a ponded surface is identified. The formation of ponding, however, only occurs for larger amount of rainfall intensity which is in this case, $q = 5.5 \times 10^{-7}$ m/s. Although, other intensities applied did not show ponding, the result from the second largest amount of rainfall is indicating a formation of ponded surface in progress. With longer duration, the wetting front might may form and creates the ponded surface on top. The effect of wetting front is important in unsaturated soils and can be studied extensively for example Jeong et al. [18] whom investigated the wetting front suction of different percentage of fines material. They found that only up to 15% of fines may considerably affects the unsaturated slope stability. In this scenario, the process of ponding affected the development of pore-water pressure particularly at shallow depth of the slope.

4.2. Effect of rainfall duration

The duration applied for the small rainfall is also an important variable in order to determine the most suitable initial pore-water pressure. Five durations were used to investigate the behaviour of the slope with short and prolonged rainfall selected at 14 days and 31, 62, 93 and 186 days respectively. The amount of rainfall used is 1×10^{-7} m/s as an average value of applied rainfall amount in Singapore. This is conducted to evaluate the infiltration rate and amount of the rainwater allowed to percolate into the soil. From the results shown in Fig. 5, although the pore-water pressure recorded at Location D is lower at the beginning of the duration, the pore-water pressure increases and reaches saturation at a similar rate with other locations for the first two months. Due to the fact that the difference is not large, it can be said that the duration does not affect the generation of initial pore-water pressure significantly when a constant and low rainfall intensity is used (i.e. 1×10^{-7} m/s) for shorter durations. However, it can be seen that a fluctuation of pore-water pressure is generated during the third and fourth months. This fluctuation is not shown in earlier or later duration and only happened from Location B and lower. One of the factors that may result in this changes can be due to a considerably higher infiltration rate of rainwater on the slope surface by the runoff coming from the top of the slope. Constant runoff flowing downwards may induce constant infiltration thus the fluctuations of pore-water pressure recorded.

Other factors that can be related to this changing pressure could be the effect of the groundwater table as the depth increases. The presence of groundwater table creates the effect of capillary fringe and therefore, influences the changing pressure. In addition, Ng et al. [19] further mentioned that a long duration of rainfall influences greater changes in deeper groundwater. In this scenario, the depth does not change because all locations of the pore-water pressure were calculated at a similar depth of 2.08 m. Nevertheless, the location which indicated by the crest, mid-by the crest, mid-slope and toe differentiates the generation of the pore-water pressure. It can be said the fluctuations of the pore-water pressure are caused by the different locations with different height of the slope. Higher locations are less affected from the water accumulated from the rainfall unlike the toe which faces larger amount of water for infiltration. Therefore, the duration of the initial pore-water pressure is only affected after a period of two months depending on the intensity as well as the locations of the pore-water pressure calculated.

5. Conclusion

The effect of the initial condition in modelling unsaturated groundwater flow was investigated. The initial pore-water pressure generated due to the rainfall applied was modelled using seepage analysis. Initially, a small amount of rainfall was applied to generate the initial pore-water pressure where the intensity and duration of rainfall were varied. A case study was used to validate the unsaturated flow model with the field and numerical results by [Rahardjo et al. [15]]. From the parametric study, it can be concluded that the intensity affects the initial pore-water pressure. The duration, however, does not have a major impact depending on the rainfall intensity and location or depth of the calculated pressure. Moreover, saturation also showed an influence on the pore-water pressure when small rainfall intensity was used. Further improvement can be achieved to develop an extensive model by modifying the magnitude of permeability, adjusting the infiltration boundary and taking into account the different depth of the calculated pressure.

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References

- [1] Toll DG (2001), Rainfall-induced landslides in Singapore. *Geotechnical Engineering*, 149(4), 211-216.
- [2] Tsaparas I, Rahardjo H, Toll DG, and Leong EC (2002), Controlling parameters for rainfall-induced landslides. *Computers and Geotechnics*, 29(1), 1-27.
- [3] Oh S and Lu N (2015), Slope stability analysis under unsaturated conditions: Case studies of rainfall-induced failure of cut slopes. *Engineering Geology*, 184, 96-103.
- [4] Zhang LL, Fredlund DG, Fredlund MD, and Wilson GW (2014), Modeling the unsaturated soil zone in slope stability analysis. *Canadian Geotechnical Journal*, 51(12), 1384-1398.
- [5] Zhang LL, Fredlund DG, Zhang LM, and Tang WH (2004), Numerical study of soil conditions under which matric suction can be maintained. *Canadian Geotechnical Journal*, 41(4), 569-582.
- [6] Van Genuchten MT (1980), A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. *Soil Science Society of America Journal*, 44(5), 892-898.
- [7] Fredlund DG & Xing A (1994), Equations for the soil-water characteristic curve. *Canadian Geotechnical Journal*, 31(3), 521-532.
- [8] Leong EC & Rahardjo H (1997), Review of soil-water characteristic curve equations. *Journal of geotechnical and geoenvironmental engineering*, 123(12), 1106-1117.

- [9] Leong EC and Rahardjo H, 1997. Permeability functions for unsaturated soils. *Journal of Geotechnical and Geoenvironmental Engineering*, 123(12), 1118-1126.
- [10] Childs EC & Collis-George N, "The permeability of porous materials", *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, (1950), 201(1066), 392-405.
- [11] Marshall TJ (1958), A relation between permeability and size distribution of pores. *Journal of Soil Science*, 9(1), 1-8.
- [12] Galavi V, Groundwater flow, fully coupled flow deformation and undrained analyses in PLAXIS 2D and 3D. 2010, Plaxis BV.
- [13] Plaxis, Scientific manual. 2012.
- [14] Plaxis, Material models manual. 2012.
- [15] Rahardjo H, Santoso VA, Leong EC, Ng YS, and Hua CJ (2011), Numerical analyses and monitoring performance of residual soil slopes. *Soils and foundations*, 51(3), 471-482.
- [16] GeoSlope IL, SEEP/W user's guide for finite element seepage analysis, (2004), GEO-SLOPE International Ltd, Calgary, Alta.
- [17] Kristo C, Rahardjo H, and Satyanaga A (2017), Effect of variations in rainfall intensity on slope stability in Singapore. *International Soil and Water Conservation Research*, 5(4), 258-264.
- [18] Jeong S, Kim J, and Lee K, Effect of clay content on well-graded sands due to infiltration. Vol. 102. 2008. 74-81.
- [19] Ng CW, Wang B, and Tung YK (2001), Three-dimensional numerical investigations of groundwater responses in an unsaturated slope subjected to various rainfall patterns. *Canadian Geotechnical Journal*, 38(5), 1049-1062.