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On Determination of Input Parameters of the Mass Transfer Process In the Multiple Stages RDC Column by Fuzzy Approach: Inverse Problem

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Abstract Inverse problems are natural in many real world applications including industrial chemical engineering problems. In this paper, an inverse problem in determining the value of input parameters for a desired value of output parameters of mass transfer process in the 23 stages Rotating Disc Contactor (RDC) column is considered. Hence, an inverse model describing the process in obtaining the solution of the problem is developed. The model involves only the process of mass transfer of a single drop of size assumed to be smaller then the critical drop size, in multiple stages RDC column. This is an extension of the work done on the mass transfer process of a single drop in single stage RDC column. The algorithm is based on fuzzy approach and the assumptions made in mass transfer process as adopted in previous work are also being used. The presented algorithm yields results such that the norm between the actual outputs obtained from the approximate solution and the target output is less than 10%. The algorithm is also capable of determining the optimal values of the input parameters. This inverse model has successfully eliminated the trial and error aspect of the forward process in determining the correct inputs for the desired outputs.

Keywords Inverse problem, mass transfer, fuzzy algorithm.

1 Introduction

In the last twenty years, the field of inverse problems has undergone rapid development. This is due to the fact that the computing technology and the development of powerful numerical methods have enormously increased which made it possible to simulate the complex problem. In addition, there exist many problems in sciences and engineering which are ill-posed and in need of solutions. This leads to a growing appetite and stimulation of mathematical research particularly on the uniqueness solution and developing stable and efficient numerical methods for solving such problems.

Meanwhile, the RDC column is a mechanical device that is widely being used in the study of liquid-liquid extraction. In this column, the process of the extraction is brought about by dispersing one of the liquid phase into the other, the continuous phase. The counter current flow of the dispersed liquid, the drop phase, in the column is affected by the difference in densities of the two liquids. As the drops flow up the column, drops may break into smaller drops as they hit the rotating discs located along the column.

Even though the mathematical models for simulating the column have been developed successfully ([12], [4], [2]), the desired outputs (the exiting concentration of the continuous phase and the drop phase) are still hard to achieve as the accurate values of corresponding input parameters (the initial concentration of the continuous phase and the drop phase) are not known. The determination by trial and error of the input parameters value in order to produce the desired outputs need excessive computer time and it will be financially expensive if the actual processes are involved. Therefore, in this paper, an inverse problem in determining the value of input parameters for a desired value of output parameters of mass transfer process in the RDC column is considered. Hence, an inverse model describing the process in obtaining the solution of the problem is developed. The model involves only the process of mass transfer of a single non-solid drop of size assumed to be smaller then the critical drop size, in multiple stages RDC column. This is an extension of the work done on the mass transfer process of a single drop in single stage RDC column.

This paper is organized as follows. Section 2 states the forward equation, the inverse problem and the assumptions made in this paper. In section 3, we discuss the inverse modelling method including the detail of the algorithm and the results obtained in this work. Section 4 illustrates a numerical example, demonstrating that the technique is reasonable and effective. Lastly we conclude the work done in the last section of this paper.

2 Statement of the Problem

The forward modelling of mass transfer process in the RDC column had been developed successfully by previous researchers as can be found in [12], [4], [2]. Maan et.al[8] had also recently improved the model by considering the varied boundary condition, which is more realistic than a previous one. Due to the excessive computer time to produce simulation data, Maan et al.[9] had developed Neural Network models. These models are able to simulate the column faster than the previous mathematical models. But in this field, the major concern is the need of developing the inverse model and the technique of solving of this process. Therefore, for early development Maan et. al.[10] had developed an inverse model for mass transfer process in a single stage RDC column.

In this paper, the determination of input parameters for a certain value of output parameters for the mass transfer process in multiple stages RDC column is considered. The algorithm based on the fuzzy approach and assumptions made in mass transfer process as adopted in [10] are also being used in this work.

Here, a single drop, d which the size less than the critical drop size, d_{cr} that is

$$d < d_{cr} = 0.685 We_{D,\omega}^{-1.2} Re_{D,\omega}^{0.7} D_r,$$

where the disc angular Weber number and the Reynolds number are in the forms

$$We_{D,\omega} = \frac{\rho_c (2\pi N_r)^2 D_r^3}{\gamma},$$

$$Re_{D,\omega} = \frac{\rho_c 2\pi N_r D_r^2}{\mu_c},$$

is considered. Since the drop considered is smaller than the critical drop size, it moved upward the column without experiences the breaking process. Following this assumption we modelled the mass transfer process which is based on quadratic driving force as explained in [9].

The mass transfer model based on quadratic driving force is chosen since the interface of the drop in contact with the medium in the RDC column is assumed to be spherical in shape. The drop and medium concentration, y_s and x_s, at the surface can be found by solving the following non-linear equations,

$$\frac{2D_y\pi^2}{6d}(y_s - y_0)(\frac{1-F(t)^2}{F(t)}) = k_x(x_b - x_s) \quad (1)$$

$$y_s = f(x_s) \quad (2)$$

where $f(x_s) = x_s^{1.85}$ and $F(t)$ is the fractional approach to equilibrium. The detail explanation about this term can be found in [8]. x_b is bulk concentration in continuous phase and y_0 is the initial concentration of the drop. In this work we assumed that x_b and y_0 are the input parameters of the mass transfer process in the RDC column.

The average concentration of the drops can be obtained by equation

$$\frac{y_{av} - y_0}{y_s - y_0} = F(t), \quad (3)$$

where $F(t) = 1 - \frac{6}{\pi^2}\sum_1^\infty \frac{1}{n^2}(e^{\frac{Dn^2\pi^2 t}{a^2}})$. This average concentration of the drop is taken as the output parameter.

Then the concentration of the medium can be obtained by applying mass balance equation, that is

$$F_x(x_{in} - x_{out}) = F_y(y_{out} - y_{in}), \quad (4)$$

where F_x and F_y are the flow rate of continuous and the drop phase respectively. The concentration x_{in} and y_{in} are the uniform initial concentration of the continuous and drop phase. x_{out} and y_{out} are the exiting concentration of the continuous and drop phase. In this case, the value of x_b and y_0 are the same with x_{in} and y_{in} respectively where these are the input parameters of the system. Meanwhile x_{out} and y_{out} are the output parameters. In the RDC column, we assume y_{av} is the exiting concentration of the drop at first compartment and also this concentration is assumed to be the initial concentration of the drop in the next compartment.

In this forward process, the value of output parameters, x_{out} and y_{out} can be determined if the value of input parameters are given. Now, let consider the inverse problem of this system, which means to determine the input parameters, x_{in} and y_{in} for a desired value of output parameters, x_{out} and y_{out}. Since the direct solution of this problem is hard to achieved due to it's ill-posed characteristic, the approximation method has to be constructed. Hence the following section will describe the method used in gaining the solution of the problem.

3 Inverse Modelling Method

3.1 Preliminaries

The approach adopted in this work is based on the basic principles of fuzzy modelling which was laid down by Zadeh[3], where he stated indirectly that fuzzy modelling can provide an approximate and yet effective means of describing the behavior of system which are too complex or too ill-defined to admit use of precise mathematical analysis. The multivariate systems modelled by equations (2)-(4) can be simplified as the multiple input multiple output(MIMO) system of

$$\frac{D_y\pi^2}{3d}(y_{s_i} - y_{in_i})(\frac{1-F(t)^2}{F(t)}) = k_x(x_{in_i} - x_{s_i}) \quad (5)$$

$$y_{s_i} = f(x_{s_i}) \quad (6)$$

where equations (5) and (6) are used to obtain the the concentration of drop on the surface. The latter equation is known as the equilibrium equation. The value is then used in equation

$$\frac{y_{out_i} - y_{in_i}}{y_{s_i} - y_{in_i}} = F(t), \quad (7)$$

to obtain one of the output parameter, y_{out}. Then the other one of the output parameter, x_{out} is obtained by equation

$$F_x(x_{in_i} - x_{out_i}) = F_y(y_{out_i} - y_{in_i}), \quad (8)$$

which is known as mass balance equation. The subscript i refers to the number of stages in the RDC column. If $i = 1$ means at the first stage, the output parameters are y_{out_1} and x_{out_1}. These output parameters will become input parameters at the second stage which are y_{in_2} and x_{in_2}. These process continues until the end of the stage.

In our study, the values of input parameter are determined through experiments. These values can be modelled by the concept of fuzzy numbers. Basically, fuzzy number can be named as a fuzzy set satisfying some specific properties ([3]). For the current work, the concept of α-cut or α-level sets of fuzzy set is considered;

$$^{\alpha}A = \{x : \mu_A(x) \geq \alpha\} \quad \text{for } 0 < \alpha \leq 1 \quad (9)$$

where $\mu_A(x)$ denotes the membership function of fuzzy set A. Therefore, α-level sets correspond to an interval for each given value of α. In this work, triangular membership function is used due to its simple formula and computational efficiency. The definition of this function can be found in [3].

3.2 Fuzzy Algorithm

In order to introduce the fuzzy algorithm on this problem, it is necessary to identify the variables involved in the processes. These variables must be specified as input, output or performance parameters as mentioned in the previous section. Then, the fuzzy logic concept was introduced which finally can provide the desired outcomes. These requirements result in the three phases of a structure-based fuzzy system which are Fuzzification of the input variables, Fuzzy Environment and Defuzzification ([1],[2]).

In the first phase, the input parameters are fuzzified by membership function mentioned in the previous section. In the next phase, these values are employed for calculating the associated fuzzy sets for performance parameter. The input and fuzzified performance parameters are then processed by Zadeh's extension principle ([3]) to produce the most appropriate output data. In this study a triangular plane is used as the induced performance parameter which is different with the one used by Ahmad[1] and Ismail et. al.[7]. This idea was inspired by the result of the nonexistent intersection points between the induced triangular solution and the preferred performance parameter. Then the output data is defuzzified in order to obtain the best possible combination of the input parameters.

All the fuzzy sets F_{I_i} expressing preferences of all input parameters

$$X_i \in [a_i, b_i] \subset \Re^+ (i = 1, 2)$$

are determined, normalized and convex. Let $\mathbf{Y}_X$ be preferred performances parameter which takes all the input parameters as its variables and is presented by fuzzy set $F_{\mathbf{Y}_X}$. In this study $\mathbf{Y}_X$ refers to equations (6) to (8) in order to get the values of y_{out} and x_{out}.

The calculation may conveniently be performed using the following algorithm:

(i) Let $h_1 : I_1 \times I_2 \longrightarrow O_2$, $h_2 : I_1 \times I_2 \times O_2 \longrightarrow O_1$ and $\mathbf{Y}(h_1, h_2) = (O_2, O_1)$ where $O_2 \in Y_2$, $O_1 \in Y_1$. Let $\mathbf{Y} : h_1 \times h_2 \longrightarrow \Re^2$ is the performance parameter such that $\mathbf{r} = \mathbf{Y}(h_1, h_2)$.

(ii) Select appropriate value for α-cut, such that $\alpha_1, \alpha_2, \alpha_3, ...\alpha_k \in (0, 1]$.

(iii) For each X_i, determine the end points of all the α_k-cuts, F_{I_i} $(i = 1, 2)$.

(iv) Generate all 2^n combinations of all the endpoints of intervals representing α_k-cuts. Each combination is an n-tuple (in this problem $n = 2$).

(v) Determine $\mathbf{r}_j = \begin{pmatrix} r_1 \\ r_2 \end{pmatrix}_j = \mathbf{Y}(h_1, h_2) = \begin{pmatrix} O_2 \\ O_1 \end{pmatrix}$ for each 2-tuple $j \in 1, 2, ...2^n$.

(vi) For each α-cuts, determine the induced performance parameters, F_{ind} by taking the min value and max value of each element of $\mathbf{r}$ i.e let $u_1 = \min(r_1)_j$, $u_2 = \min(r_2)_j$ and let $v_1 = \max(r_1)_j$, $v_2 = \max(r_2)_j$, for all $j \in 1, 2, ...2^n$ and we define

$R_{\alpha_l} = \{(r_1, r_2, \mu(r_1, r_2)) : \alpha_l \leq \mu(r_1, r_2) \leq \alpha_{l+1}, u_1 \leq r_1 \leq v_1, u_2 \leq r_2 \leq v_2\}$, therefore $F_{ind} = \bigcup_{l \in k} R_{\alpha_l}$ and define α-cut of F_{ind} as

$$[\mathbf{r}]_\alpha = \left\{ \mathbf{r} = \begin{pmatrix} u_1 + (v_1 - u_1)t \\ u_2 + (v_2 - u_2)t \end{pmatrix}, 0 \leq t \leq 1, \mu(\mathbf{r}) \geq \alpha \right\}.$$

(vii) Repeat step 4, 5 and 6 by taking the output $\mathbf{r}_j$ in the first stage as the input X_i in the next stage. These process continues until the last stage.

(viii) Set $F_{\mathbf{Y}_X} \wedge F_{ind}$, where F_{ind} is the induced performance parameter at the last stage and find the fuzzy number of $f^\star = \sup(F_{\mathbf{Y}_X} \wedge F_{ind})$.

(ix) Find the α-cut of F_{I_i} for corresponding value of $f^\star$.

(x) Repeat step 4 and 5 for $\alpha = f^\star$ and denote the corresponding performance parameter as $\mathbf{r}_\mathbf{j}^\star$ for each 2-tuple $j \in 1, 2, ...2^n$.

(xi) Determine the optimal combination of input parameters.

The value determined in the final step of the algorithm is the approximate value of input parameter which will produce the desired value of output parameter. The value is determined by Theorem 3.3, which is an extension of Theorem of Optimized Defuzzified Value ([10],[7],[1]). However, we have to show that the induced solution for RDC column, F_{ind} is convex and normal before Theorem 3.3 works.

Definition 3.1 **([3])** *A fuzzy set $A \subset F(X)$ is convex if and only if*

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq min[\mu_A(x_1), \mu_A(x_2)],$$

$\forall\lambda \in [0,1]$ and $\forall x_1, x_2 \in F(X)$, where min denotes the minimum operator.

Lemma 3.1 *If $A \subseteq F(X)$ and $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq min[\mu_A(x_1), \mu_A(x_2)]$ $\forall\lambda \in [0,1]$ and $\forall x_1, x_2 \in F(X)$ then $[a]_{\alpha_2} \subseteq [a]_{\alpha_1}$ for all $\alpha_2 \geq \alpha_1$.*

Proof Assume $A \subseteq F(X)$ and

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min[\mu_A(x_1), \mu_A(x_2)], \forall\lambda \in [0,1]$$

and $\forall x_1, x_2 \in F(X)$. Let $\alpha_2 = \mu_A(\lambda x_1 + (1-\lambda)x_2)$ and

$$\min[\mu_A(x_1), \mu_A(x_2)] = \alpha_1.$$

By properties of alpha-cut([3]), if $\exists\ \alpha_2 \geq \alpha_1$, then $[a]_{\alpha_2} \subseteq [a]_{\alpha_1}$.

Definition 3.2 **([11])** *A fuzzy set $A \subset F(X)$ is normal if and only if*

$$\sup_{x \in A} \mu_A(x) = 1.$$

We state the following theorem which then proves by construction.

Theorem 3.1 *Let $h_1 : I_1 \times I_2 \longrightarrow O_2$, $h_2 : I_1 \times I_2 \times O_2 \longrightarrow O_1$ and*

$$\mathbf{Y}(h_1, h_2) = (O_2, O_1)$$

where $O_2 \in Y_2$, $O_1 \in Y_1$. Let $\mathbf{Y} : h_1 \times h_2 \longrightarrow \Re^2$ be the performance parameter such that $\mathbf{r} = \mathbf{Y}(h_1, h_2)$. Then if all the fuzzy set F_{I_i} expressing preferences of all input parameter $X_i \in I_i \subset \Re^+(i = 1, 2)$ is convex, it follows that the induced solution for RDC column, F_{ind} is also convex. $\square$

Proof (by construction) Given that I_1, I_2 is convex, i.e if $[I_i]_\alpha$ is a closed interval for each α and $\alpha_{k+1} \geq \alpha_k \Rightarrow [I_i]_{\alpha_{k+1}} \subseteq [I_i]_{\alpha_k}$ $\forall i = 1,2$ and $\alpha_k \in [0,1]$ for $k = 1,2,...,n$. Find all end points of $[I_i]_{\alpha_k}$ and denote as $\{I_{i_{min}}, I_{i_{max}}\}_{\alpha_k}$. Now, determine all the combination of end points for every $[I_i]$ of each α_k and write as $\{(I_1, I_2)\}_{\alpha_k}$.

Generate $h_1(I_1, I_2)_{\alpha_k}$, $h_2(I_1, I_2, O_2)_{\alpha_k}$ and $\mathbf{Y}(h_1, h_2)_{\alpha_k}$.

Determine $\mathbf{r_j} = \begin{pmatrix} r_1 \\ r_2 \end{pmatrix}_j = \mathbf{Y}(h_1, h_2) = \begin{pmatrix} O_2 \\ O_1 \end{pmatrix}$ for each 2-tuple $j \in 1, 2, ...2^n$.

Then for each α-cuts, determine the induced performance parameters, F_{ind} by taking the min value and max value of each element of $\mathbf{r}$ i.e if $u_1 = \min(r_1)_j$, $u_2 = \min(r_2)_j$ and let $v_1 = \max(r_1)_j$, $v_2 = \max(r_2)_j$, for all $j \in 1, 2, ...2^n$ and we define

$$R_{\alpha_l} = \{(r_1, r_2, \mu(r_1, r_2)) : \alpha_l \leq \mu(r_1, r_2) \leq \alpha_{l+1}, u_1 \leq r_1 \leq v_1, u_2 \leq r_2 \leq v_2\}.$$

Therefore $F_{ind} = \bigcup_{l \in k} R_{\alpha_l}$, and plot F_{ind}.

Now, we have to prove that if $\alpha_{k+1} \geq \alpha_k$ and all α-cut of F_{ind} is a closed domain $\Rightarrow$ $[\mathbf{r}]_{\alpha_{k+1}} \subseteq [\mathbf{r}]_{\alpha_k}$ $\forall i = 1,2$ and $\alpha_k \in [0,1]$ for $k = 1,2,...,n$. Obviously α-cut of F_{ind} which is defined as $[\mathbf{r}]_\alpha = \left\{ \mathbf{r} = \begin{pmatrix} u_1 + (v_1 - u_1)t \\ u_2 + (v_2 - u_2)t \end{pmatrix}, 0 \leq t \leq 1, \mu(\mathbf{r}) \geq \alpha \right\}$ is a closed domain $\forall\ \alpha = [0,1]$. Take $\mathbf{r} \in [\mathbf{r}]_{\alpha_{k+1}}$, therefore $\mathbf{r} \in \bigcup_{l=k+1,...n} R_{\alpha_l}$ and $\mathbf{r} = (r_1, r_2) = \mathbf{Y}(h_1, h_2)_{\alpha_{k+1}}$ where

$$h_1(I_1^\star, I_2^\star)_{\alpha_{k+1}} = r_2 \text{ and } h_2(I_1^\star, I_2^\star, r_2)_{\alpha_{k+1}} = r_1$$

for some $(I_1^\star, I_2^\star)$ where $I_i^\star \in [I_i]_{\alpha_{k+1}} \forall i = 1,2$.

Since I_i is convex i.e $I_i^\star \in [I_i]_{\alpha_{k+1}} \subseteq [I_i]_{\alpha_k}$ $\forall i = 1,2.$, which implies that

$$\mathbf{r} \in \bigcup_{l=k,k+1,...n} R_{\alpha_l}$$

i.e $\mathbf{r} \in [\mathbf{r}]_{\alpha_k}$. Therefore $[\mathbf{r}]_{\alpha_{k+1}} \subseteq [\mathbf{r}]_{\alpha_k}$. Hence the induced solution for RDC column, F_{ind} is convex.□

Following Theorem 3.1, we state this corollary for the multi-stage RDC column.

Corollary 3.1 *If all the fuzzy set F_{I_i} expressing preferences of all input parameter $X_i \in I_i \subset \Re^+ (i = 1,2)$ is convex at any stage , then the induced solution for respective stage RDC column, F_{ind} is also convex.* □

Proof The proof is repetitive of Theorem 3.1 for any stage of RDC column. □

For 23 stages RDC column, we have the following corollary.

Corollary 3.2 *If all the fuzzy set F_{I_i} expressing preferences of all input parameter $X_i \in I_i \subset \Re^+ (i = 1,2)$ is convex, then the induced solution for 23 stages RDC column, F_{ind} is also convex.* □

Theorem 3.2 *Let $h_1 : I_1 \times I_2 \longrightarrow O_2$, $h_2 : I_1 \times I_2 \times O_2 \longrightarrow O_1$ and*

$$\mathbf{Y}(h_1, h_2) = (O_2, O_1)$$

where $O_2 \in Y_2$, $O_1 \in Y_1$. Let $\mathbf{Y} : h_1 \times h_2 \longrightarrow \Re^2$ be the performance parameter such that $\mathbf{r} = \mathbf{Y}(h_1, h_2)$. Then if all the fuzzy set F_{I_i} expressing preferences of all input parameter $X_i \in I_i \subset \Re^+ (i = 1, 2)$ is normal, it follows that the induced solution for RDC column, F_{ind} is also normal. □

Proof Since all the fuzzy sets F_{I_i}, expressing preferences of all input parameters $X_i \in I_i \subset \Re^+ (i = 1, 2)$ are normal, then there exist $i \in I_1$ and $i \in I_2$, such that $\mu(i_1) = \mu(i_2) = 1$. Look, $h_1(i_1, i_2) = O_2$, $h_1(i_1, i_2, O_2) = O_1$, and $\mathbf{Y}(h_1, h_2) = (O_2, O_1) = \mathbf{r}$ such that $\mu(\mathbf{r}) = 1$. Hence F_{ind} is normal.

Following Theorem 3.2, we state this corollary for multiple stages RDC column.

Corollary 3.3 *If all the fuzzy set F_{I_i} expressing preferences of all input parameter $X_i \in I_i \subset \Re^+ (i = 1, 2)$ is normal at any stage , then the induced solution for respective stage RDC column, F_{ind} is also normal.* □

Proof The proof is repetitive of Theorem 3.2 for any stage of RDC column. □

For 23 stages RDC column, we have the following corollary.

Corollary 3.4 *If all the fuzzy set F_{I_i} expressing preferences of all input parameter $X_i \in I_i \subset \Re^+ (i = 1, 2)$ is normal, then the induced solution for 23 stages RDC column, F_{ind} is also normal.* □

By Corollary 3.1, 3.2, 3.3 and 3.4, we use the following Theorem 3.3 to determine the optimal combination of input parameters.

Theorem 3.3 *Let $P = \{(\mathbf{r}_j, \mu(\mathbf{r}_j)), \|\mathbf{a}\| \leq \|\mathbf{r}_j\| \leq \|\mathbf{c}\| : \mu(\mathbf{a}) = 0, \mu(\mathbf{c}) = 0, \|\mathbf{a}\| \leq \|\mathbf{b}\| \leq \|\mathbf{c}\|, j \in Z^+\}$ where $(\mathbf{r}_j, \mu(\mathbf{r}_j)) \in F_{ind}$ and let $S = \{(\mathbf{r}_l, \mu(\mathbf{r}_l)), \|\mathbf{a}\| \leq \|\mathbf{r}_l\| \leq \|\mathbf{b}\| : \mu(\mathbf{a}) = 0, \mu(\mathbf{b}) = 1\}$ be max side of induced plane, where $(\mathbf{r}_l, \mu(\mathbf{r}_l)) \in F_{ind}$ and $S \subset P$. If there exist $Y_X^\star = \mathbf{r}_l^\star$ such that $\mu(\mathbf{r}_l^\star) = f^\star$ and $(\mathbf{r}_l^\star, f^\star) \in S$ where $f^\star = sup(F_{\mathbf{Y}_X} \cap F_{ind})$ for some $(\mathbf{r}_l, \mu(\mathbf{r}_l)) \in F_{ind}$, then $\mathbf{r}_l^\star = Y_X^\star = max\|Y_X^\star(X_1^\star, X_2^\star)\|$ where $\mu(X_i) = f^\star$.* □

Proof Suppose $Y_X^\star = \mathbf{r}_l^\star \in S$ such that $\mu(\mathbf{r}_l^\star) = f^\star$ where $f^\star = \sup(F_{\mathbf{Y}_X} \cap F_{ind})$ for some $(\mathbf{r}_l, \mu(\mathbf{r}_l)) \in F_{ind}$. Determine all the $f^\star$-cuts of all F_{I_i} to create all 2-tuples of $(X_1^\star, X_2^\star)$ such that $\mu(X_i^\star) = f^\star$ and $(X_i^\star, f^\star) \in F_{I_i}$. Set $\mathbf{r}_l^\star = max\|Y_X^\star(X_1^\star, X_2^\star)\|$, therefore $(\mathbf{r}_l, f^\star) \in F_{ind}$. However, since F_{ind} is normal and convex, which is given by Corollary 2 and 4, this imply that $\mathbf{r}_l = \mathbf{r}_l^\star$.

Theorem 3.3 indicates that if the preferred fuzzy intersects on the maximum side of the fuzzy induced plane, then the set of optimized parameters is the set for the maximum norm of the induced values. The theorem enables the decision maker to identify the best optimized value from the predicted results in the final phase of the algorithm.

4 Experimental Result and Discussion

To study the capability of the proposed method, a numerical example is given by considering a system with two input parameters and two output parameters of mass transfer process of a single drop in the 23 stages RDC column. As mentioned previously, the aim here is to predict the values of input parameters in order to produce certain desired output parameters. The input parameters involved are the initial concentration of continuous and the drop phase while the output parameters are the the exiting concentration of continuous and the drop phase. The domain and preferred input and output parameters are given in Table 1 and Table 2.

Table 1: The domain and the preferred input parameters.

Input parameters	Min	Preferred	Max
CC_{in}	32.8	45.48	65
Cd_{in}	7.0	9.0	12.5

Table 2: The domain and the preferred output parameters.

Output parameters	Min	Preferred	Max
CC_{out}	25	38	40
Cd_{out}	22	25	28

These figures are simply fuzzy numbers where the preferred and the domain are set to have the highest and lowest membership values respectively. The α-cuts of all input parameters, with increment of 0.2 which are obtained from triangle membership function are calculated. Then these values are used to calculate the fuzzified induced plane, F_{ind} as mentioned in step 6 of the algorithm. Similarly for the output parameters given in Table 2, are fuzzified with increment of 0.2. These values are used to calculate the preferred output parameters.

Since the system is a multistage system, we have to repeat the forward process as in step 6 until the end of the stage is reached. The process of defuzzification begin by applying step 7. The intersection of preferred and induced plane of output parameters is shown in Figure 1. Finally the desired approximate values of input parameters are determined using the remaining steps of the algorithm.

The fuzzy algorithm has suggested the best combination of input parameters of 45.2919 g/l and 8.9703 g/l (for the initial concentration of continuous and dispersed phase respectively) with $f^{\star} = 0.9852$. The percentage relative errors are approximately 3.7307% and 7.0845% respectively.

In this work, the simulations for different number of stages are also being carry out in order to see the effect of the relative error with respect to the height of the column. The results of the simulations can be found in Table 3.

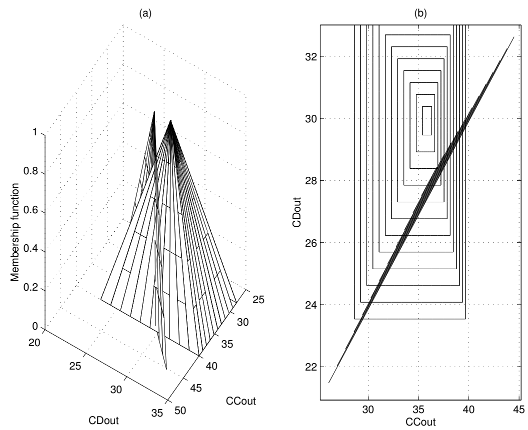

Figure 1: (a)- The intersection between the fuzzified preferred output and the induced plane of the output parameters. (b)- The level curve of (a) .

From the table, we can see that there is no intersection between the induced and the preferred output for one up to four stage of the column. This indicates that the corresponding stage is insufficient for the purpose of the system. The table also shows that after stage 13 there is only a slight change in the relative error. This phenomena is explained by the fact that the interface of the drop is in equilibrium with the interface of the medium as time increases. In other word, the process of mass transfer from the medium to the drop is accomplished at this stage. This conclude that the increase of stage is not giving much effect on the relative error. The relationship of the error of the two outputs with respect to the preference outputs and the height of the column is best shown in the Figure 2.

5 Conclusion

This paper has described the new inverse modelling method for the mass transfer process in multiple stages RDC column. The essential feature of this method is the fuzzy algorithm which requires three phases of a structure-based fuzzy system, those are fuzzification of the input variables, fuzzy environment and defuzzification.

In this algorithm, all the input parameters were fuzzified to create fuzzy environment. This is then processed to produce the induced output parameters. The best output parameters were extracted through defuzzification.

This study proved that if all the fuzzy set F_{I_i} expressing preferences of all input parameters $X_i \in I_i \subset \Re^+(i = 1, 2)$ are convex and normal, then the induced solutions for 23 stages RDC column, F_{ind} are also convex and normal. This study also showed that the presented method is able to solve the inverse problem of MIMO system and capable of determining the optimal value of output parameters. The corresponding input parameters are then chosen which are the best suggested values.

In addition, the presented algorithm yields results such that the percentage of relative error between the actual outputs obtained from the approximate solution and the target

Table 3: The simulation result at different stage of RDC column.

No of Stages	Fuzzy no.	Output 1	Output 2	Error 1	Error 2
1	-	-	-	-	-
4	-	-	-	-	-
5	0.2067	32.44.86	16.4224	14.6089	34.3103
7	0.7548	37.5935	22.9865	1.0698	8.0541
8	0.8802	38.6593	24.8248	1.7350	0.7007
10	0.9917	39.5623	26.5976	4.1112	6.3904
13	0.9872	39.4466	26.7659	3.8069	7.0635
15	0.9856	39.4233	26.7701	3.7456	7.0802
16	0.9853	39.4200	26.7707	3.7369	7.0827
17	0.9852	39.4186	26.7709	3.7332	7.0837
19	0.9852	39.4178	26.7711	3.7310	7.0844
23	0.9852	39.4177	26.7711	3.7307	7.0845

output is less than 10%. This inverse model has successfully eliminates the trial and error aspect of the forward process in determining the correct inputs for the desired outputs. The result of the simulation of different number of stages also showed that after stage 13, the increase of stage is not giving much effect on the relative error.

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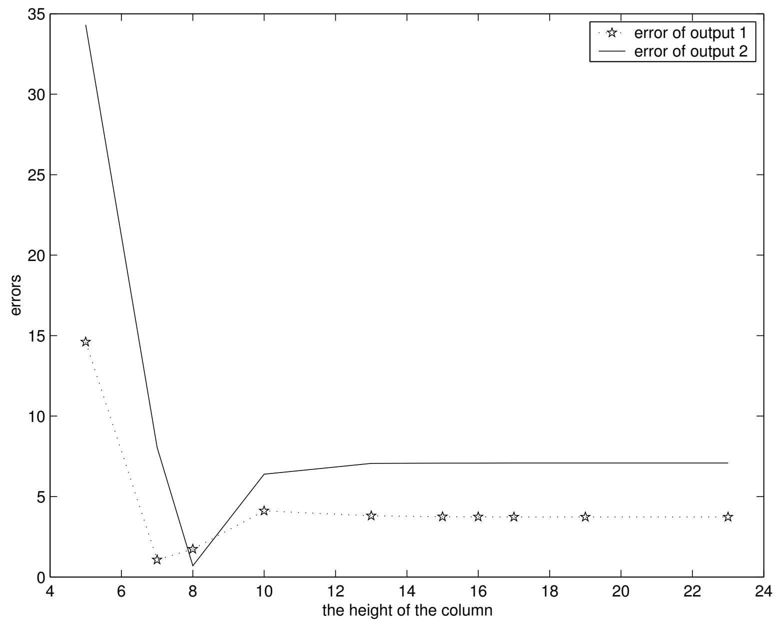

Figure 2: The relationship between errors of output parameters and the height of the column.

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