

DEVELOPMENT OF A DOUBLE INVERTED PENDULUM MODEL FOR
HUMAN GAIT ANALYSIS

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DEDICATION

This thesis is dedicated to my father, who taught me that the best kind of knowledge to have is that which is learned for its own sake. It is also dedicated to my mother, who taught me that even the largest task can be accomplished if it is done one step at a time.

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ABSTRACT

Gait analysis is an essential tool in clinical rehabilitation. It facilitates the diagnosis, treatment, monitoring and implementation of methodologies that mitigate the effect of some pathologies associated with the movement. This analysis uses a systematic quantification, follow up and interpretation of the temporal sequence of movements that characterize human locomotion. It also requires a collection of kinematic and kinetic data that describes the displacements, angles and forces on the lower limbs and its joints during a gait cycle. However experimental procedures are difficult to carry out, time consuming and experienced people are required. The aim of this project is formulation and implementation of a double inverted pendulum (DIP) model to describe the dynamics of human gait. The model consists the physiological extension of a head-arms-trunk (HAT) and leg segment. LaGrange Euler mechanics used for mathematical modelling. Simulations were performed, using Simulink and MATLAB software. The hip angle, ankle angle and ground reaction forces were compared against experimental data. It was found that experimental hip angle and model hip angle matched against each other from 20% to 75% of gait cycle while the experimental ankle angle deviated considerably from model ankle angle from 50% gait cycle onwards. In addition, GRF calculations showed that both model and experiment horizontal GRF reached the same maximum value of 15% BW while the minimum values deviated considerably. On the other hand, vertical GRF of experiment and model followed a similar pattern although significant deviations were observed at mid cycle. As a result, the computed Root Mean Square Error (RMSE) for the modelled and experimental results were considerably higher which could be attributable to uncertainties in the model input parameters.

ABSTRAK

Analisis gaya berjalan adalah salah satu kaedah yang digunakan dalam rehabilitasi klinikal. Analisis gaya berjalan membantu dalam proses diagnosis, rawatan, pemantauan kesihatan serta kaedah-kaedah lain dengan mengurangkan kesan patologi yang berkaitan dengan pergerakan. Analisis ini menggunakan kaedah kuantifikasi secara sistematik, diikuti sistem susulan dan sistem tafsiran keatas siri pergerakan manusia. Analisis ini juga memerlukan data kinematik dan kinetik bagi menerangkan anjakan, sudut dan kuasa yang dihasilkan bahagian bawah anggota badan dan sendi semasa kitaran berjalan yang berlaku. Walaubagaimanapun, kaedah eksperimen ini agak sukar untuk dijalankan, mengambil masa yang panjang dan memerlukan kepakaran tinggi manusia. Oleh itu, tujuan kajian projek ini adalah untuk menghasilkan formulasi dan pelaksanaan model pendulum berganda terbalik bagi menerangkan kesan dinamik gaya berjalan manusia. Model ini terdiri daripada beberapa sambungan yang mewakili komponen badan manusia seperti kepala-lengan-badan dan bahagian kaki. Kaedah persamaan LaGrange Euler digunakan dalam melakukan pemodelan matematik. Simulasi model ini dilakukan menggunakan perisian Simulink dan MATLAB, serta perbandingan data eksperimen dilakukan secara kuantitatif. Hasil kajian mendapati sudut pinggang yang diambil dari eksperimen mencapai keserasian dengan hasil model iaitu pada 20% sehingga 75% daripada keseluruhan kitaran gait, manakala sudut buku lali yang diambil secara eksperimen menyimpang dari hasil model pada 50% dan keatas daripada keseluruhan kitaran gait. Tambahan itu juga, pengiraan komponen kinetik iaitu daya tindak balas permukaan atau Ground Reaction force (GRF) untuk component mendatar bagi model and eksperimen mencapai nilai maksimum yang sama iaitu sebanyak 15% dari berat badan (BW) manakala nilai maksimum adalah menyimpang. Selain itu, GRF bagi komponen menegak bagi model dan eksperimen menunjukkan corak yang sama walaupun terdapat penyimpangan nilai yang ketara semasa pertengahan kitaran berjalan. Hasil akhir kajian menunjukkan bahawa analysis Root Mean Square Error (RMSE) untuk kedua-dua hasil samada secara model dan eksperimen adalah agak tinggi. Oleh itu, penyelidikan berterusan perlu dijalankan untuk mengkurangkan jurang antara hasil model dan eksperimen sebenar.

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LIST OF ABBREVIATIONS

HAT	-	Head Arm Trunk
ROM	-	Range of Motion
GRF	-	Ground Reaction Force
DIP	-	Double Inverted Pendulum
RMSE	-	Room Mean Square Error
PID	-	Proportional Integral Derivative
GRF	-	Ground Reaction Force
vGRF	-	Vertical Ground Reaction Force
hGRF	-	Horizontal Ground Reaction Force
COM	-	Centre of Mass
BW	-	Body Weight
LQR	-	Linear Quadratic Regulator
IP	-	Inverted Pendulum

LIST OF SYMBOLS

$\dot{\theta}_1, \dot{\theta}_2$	-	Angular velocities
$\ddot{\theta}_1, \ddot{\theta}_2$	-	Angular accelerations
y	-	Vertical displacement
x	-	Horizontal displacement
\dot{x}_1, \dot{x}_2	-	Linear velocities in horizontal direction
\dot{y}_1, \dot{y}_2	-	Linear velocities in vertical direction
v	-	Velocity
T	-	Kinetic energy
V	-	Potential energy
I	-	Moment of Inertia
m	-	mass
M	-	Moment
g	-	Gravitational force
d	-	Distance from centre of mass
L	-	Lagrange
l	-	Length

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CHAPTER 1

INTRODUCTION

1.1 Problem Background

Gait is the method of human locomotion using limbs. Normal gait sequences are cyclic and demonstrate almost perfect periodic behaviour (Runge et al., 1999). Recently, the analysis of human motion has received a large amount of research attention. The majority of this research focuses on analysis of human gait. Human gait contains a wide variety of information that can be used in biometrics, disease diagnosis, load determination and injury rehabilitation (Komura et al., 2005).

Modelling of the gait provides understanding of the underlying processes that determine why humans walk the way they do. It can help identify specific impairments affecting people with pathologies that inhibit their ability to walk in a natural manner and hence the appropriate treatment or orthotic can be prescribed. In industry, where the health and safety of workers is paramount, it can be used to investigate actions that may cause potential injury (Yang et al., 2015)

Since using the model doctors can check their assumptions about a disease or condition of the lower limbs without conducting experimentation on a patient. Also, modelling of human gait would greatly simplify the process of testing artificial limbs and orthotic devices, which are now checked empirically, despite the fact that this approach is expensive and ineffective. This research aims to develop the simplest modular human gait simulation circuit which can be used to compare the simulation data and the data obtained by the motion capture cameras (Vicon) and force plate.

For years, conceptual modelling was the standard in the gait analysis field (Saunders et al., 1953) i.e. models based on observation and theories rather than measurable evidence. Using simplified approximations of the geometries relevant to

walking and broadly based on observation, gait was defined as having six different mechanisms, or 'gait determinants. These were pelvic rotation and obliquity, stance phase knee flexion, ankle mechanisms, foot mechanisms and lateral body displacement. These mechanisms were said to smooth the trajectory of a person's centre of mass (CM) and therefore reduce energy dissipation during walking. However, a number of experimental and mathematical based studies have since brought into question the validity of some of these determinants (Zhang et al., 2013).

Research that tried to formally describe upright posture is often applied to sagittal-plane single inverted pendulum (SIP) biomechanics with ankle joint actuation (Suzuki et al., 2012). Such SIP models have successfully been used to describe human responses to external disturbances. Some of these SIP based models use continuous sensory feedback control with realistic neural time delays (e.g. (Millard et al., 2011)). However, humans often involve in addition to the ankle joints also the hip joints. Thereby they improve their balancing, increasing its flexibility in face of the multitude of external disturbance scenarios and its robustness in terms of fail safe. To better understand these skills, it is desirable to use double inverted pendulum (DIP) biomechanics in the modelling.

As gait analysis progressed, simple mathematical models of walking became more important. Unlike those concepts developed purely from observation, these studies provided mathematical evidence to justify their claims, which consequently carried more weight. The body would be approximated into a number of rigid body segments, joined together, that were assumed to have point masses and each was given appropriate geometric and inertial properties. These could then be used to investigate numerous aspects of gait analysis but due to the many assumptions made, simple models were often purpose designed to only investigate one or two specific aspects of walking at a time (Yang et al., 2015).

In this research DIP model is developed in order to analyse human gait using kinetic and kinematic approach. The model was implemented and developed using MATLAB and Simulink software. As for the data validation root mean square error (RMSE) was calculated. In over all the model has a very similar pattern of normal

gait even though high RMSE resulted due to phase difference and time delay including some deviations.

1.2 Problem Statement

The gait analysis aims to interpret the complex combination of several motion patterns generated by the interaction of different systems. In the clinical routine, the gait examination is considered as the most important tool for identifying motion disorders, and it is also used as a biomarker of some neuromuscular illnesses like the cerebral palsy or the Parkinson disease, supporting thereby diagnosis and follow up. Interestingly, during this analysis it is possible to evaluate the effectiveness of a specific treatment and the particular response of the multiple gait subsystems. However, these analyses are not actually carried out in the clinical practice, among others because this is very time consuming and examiner dependent. Moreover, measures are contaminated by noise during the capture or, by invasive devices like markers, which inevitably alter the natural motion gestures.

Mobility impairment have greatly reduced the quality of life of many patients suffering from various injuries and diseases. During physical therapy patient's gait is usually assessed by clinicians through observation. These diagnostic methods are highly dependent on the clinicians' knowledge and experience. The gold standard for clinical gait analysis uses floor-mounted force plates and optical motion capture systems in gait laboratories to obtain spatial-temporal parameters of the gait, including stride length, walking speed, and lower limb kinetics. For interpretation and assessment of gait performance high-dimensional gait data experienced clinicians are also required. Since gait analysis requires both professional staff, and expensive equipment results in limited availability to patients (Ma and Liao, 2017).

In addition to this, robotic assistive devices for patients with impaired mobility, such as exoskeletons, powered lower limb orthoses, and smart prostheses, have been widely developed in recent years (Herr, 2009). Therefore, to provide appropriate and user-adaptive assistance, a patient's gait pattern and performance

need to be modelled before this information can be used for device control. For these applications, a gait analysis method needs to be developed based on real-time gait data from wearable sensors. However, this is not an easy task due to the reasons mentioned before.

Furthermore, human gait data measured for gait analysis mainly include lower limb kinematics and ground reaction forces (GRFs). Inertia Measurement Units (IMUs), including accelerometers, magnetometers and gyroscopes, are the most widely used wearable sensors in clinical studies, and are often used instead of professional optical motion capture systems (Bejarano et al., 2015). However, capturing accurate lower limb kinematics requires strict calibration before use and multiple IMUs worn on different parts of the human body. These restrictions are not convenient for patients (Kong et al., 2009).

Modelling the human dynamic motion of gait is challenging from analytical and computational point of view. Several studies have attempted in the literature to develop realistic and natural human walking using mechanical models such as single inverted pendulum (SIP) model. SIP models have only one segment representing stance leg, pivoting about its distal end representing ankle joint (Suzuki et al., 2012). Although such SIP models have successfully been used to describe human responses to external disturbances, human dynamic movement often involves hip joint in addition to ankle joint for improving the balance and flexibility. Therefore, it is more desirable to use double inverted pendulum (DIP) biomechanics in the modelling

1.3 Objectives

The objectives of the thesis are given below.

- (a) To develop a double inverted pendulum (DIP) model for human gait analysis.
- (b) To optimize the simulated model by including a sensory feedback model.
- (c) To compare the simulated model with actual experimental data.

1.4 Scope of the Study

In this project an extensive literature was done and a double inverted pendulum (DIP) model consisting two rigid and connected segments representing ankle and hip angle was developed. Lagrange dynamics was obtained for mathematical modelling. MATLAB and Simulink software were used for implementing the model. Finally, I PID controller was used as a feedback control system.

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