

Position Estimation Comparison of a 3-D Linear Lateration Algorithm with a Reference Selection Technique

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ABSTRACT

Multilateration system estimates emitter position using time difference of arrival (TDOA) measurement with a lateration algorithm. The position estimation (PE) accuracy of the system depends on several factors such as the number of ground receiving station (GRS)s deployed, the reference station used and the type of lateration algorithm. In this paper, the 3-D PE accuracy of a four-GRS linear lateration algorithm combined with a GRS reference pair selection (GREPS) technique is determined and compared with two different five-GRS linear lateration algorithms. The two five-GRS linear lateration algorithms chosen for comparison are based on single reference total least square (SF-TLS) and multiple reference least square (MF-LS) approaches. Monte Carlo simulation result comparison shows that the four-GRS linear lateration algorithm with the GREPS technique outperformed the SF-TLS and MF-LS with a reduction in horizontal coordinate PE error of about 50% and 30% respectively, and with a 90% reduction in the altitude error.

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1. INTRODUCTION

Multilateration is a passive wireless positioning system used by the air traffic monitoring (ATM) center for surveillance purposes within its flight information region (FIR) [1]. The position estimation (PE) process of the system is in two stages [2]. The first stage involves the time difference of arrival (TDOA) estimation of the emitter's emission detected at pair of ground receiving station (GRS)s [1], [3–6], while the second stage, which is the scope of this work, involves using the TDOA estimated from the first stage to determine the position of the emitter with a lateration algorithm.

A hyperbolic equation relates the path difference (PD) measurements (TDOA measurement in distance) from the first stage with the emitter position which forms the basis for the lateration algorithm [7]. It presents a non-linear relationship between the input variable (PD measurement) and the output variable (emitter position). Several approaches have been developed to linearize this relationship which resulted into the different lateration algorithms and can be grouped as: linear and non-linear lateration algorithm [2, 7]. The non-linear lateration algorithm involves the used of linearization techniques and iteration process to obtain a linear relationship [2], [8, 9]. It suffers from convergence issue due to the iteration process and is most suitable for an active positioning system in which a rough estimate of the emitter position is known [9].

The use of algebraic manipulation to obtain the linear relationship is utilized in the linear lateration algorithm [10–15]. This approach suffers no convergence issue and is most suitable for a passive system but has high PE error due to bias introduced in the algebraic manipulation [16, 17].

The more the GRS deployed for a multilateration system, the higher its PE accuracy. Thus, it was suggested that for 3-D PE, a minimum of five GRSs should be deployed even though it is possible with four GRSs [1]. In [10], a condition number based multiple GRSs reference selection (GREPS) technique was proposed to improve the 3-D PE accuracy of a multilateration system with four deployed GRSs. Its performance in emitter PE was compared with the fixed GRS reference pair approach used in [11, 12] which is also based on four GRSs. As an extension of the work performed in [10], this paper compares the PE accuracy of the linear lateration algorithm combined with the GREPS technique with other techniques that are based on five GRSs. This is to validate if the used of a reference selection technique can make the PE accuracy of a four GRS based lateration algorithm comparable to the five GRS based lateration algorithms.

The remainder of the paper is organised as follows. Section 2 and Section 3 respectively gives a summary of the GRS reference pair linear lateration algorithm and the GREPS technique. The simulation results and discussion are presented in Section 4 followed by the conclusion in Section 5.

2. GRS REFERENCE PAIR LINEAR LATERATION ALGORITHM

In this section of the paper, a summary of the GRS reference pair lateration algorithm for a minimum configuration 3-D multilateration is presented.

Let $\mathbf{x}_e = [x, y, z]^T$ be the position of an emitter in 3-D Euclidean space and the coordinate of the i -th, j -th, k -th and m -th GRSs be $\mathbf{S}_i = [x_i, z_i, z_i]$, $\mathbf{S}_j = [x_j, z_j, z_j]$, $\mathbf{S}_k = [x_k, z_k, z_k]$ and $\mathbf{S}_m = [x_m, z_m, z_m]$ respectively. Since GRS pair is used as reference for the TDOA estimation and lateration algorithm, let the i -th and j -th GRSs to be chosen as reference pair while the non-reference GRSs are be labelled m -th and k -th. The PD measurements obtained with the i -th and j -th GRS as reference pair as presented in [10] are as follows:

$$d_{i,k} = c \times \tau_{i,k} = d_i - d_k \quad (1)$$

$$d_{i,m} = c \times \tau_{i,m} = d_i - d_m \quad (2)$$

$$d_{j,k} = c \times \tau_{j,k} = d_j - d_k \quad (3)$$

$$d_{j,m} = c \times \tau_{j,m} = d_j - d_m \quad (4)$$

where $c = 3 \times 10^8 \text{ m/s}$, $\tau_{i,k}$ and $\tau_{i,m}$ are the TDOA measurements obtained using the i -th reference GRS with the k -th and m -th as non-reference respectively; $\tau_{j,k}$ and $\tau_{j,m}$ are the TDOA measurements obtained using the j -th reference GRS with the k -th and m -th as non-reference respectively.

In practical application, signals are corrupted by noise which will result in PD measurement estimation error. By modelling the PD estimation (PDE) error as a zero mean Gaussian random variable with probability density function as $N(0, \sigma)$ [8], the PD measurements in Equation (1) to Equation (4) are estimated as:

$$\hat{d}_{i,k} = d_{i,k} + N(0, \sigma_{i,k}) \quad (6)$$

$$\hat{d}_{i,m} = d_{i,m} + N(0, \sigma_{i,m}) \quad (7)$$

$$\hat{d}_{j,k} = d_{j,k} + N(0, \sigma_{j,k}) \quad (8)$$

$$\hat{d}_{j,m} = d_{j,m} + N(0, \sigma_{j,m}) \quad (9)$$

where $\sigma_{i,k}$ and $\sigma_{i,m}$ are the PDE error standard deviations (STD) between the i -th reference GRS and the k -th and m -th non-reference GRSs respectively while $\sigma_{j,k}$ and $\sigma_{j,m}$ are the PDE error standard deviations between the j -th reference GRS and the k -th and m -th non-reference GRSs respectively. The PDE error STD depends on the received effective SNR between the GRS pair.

Algebraically manipulating Equation (6), Equation (7), Equation (8) and Equation (9) will result in a pair of 3-D plane equation in the form [10] :

$$A_{i,k,m} = xB_{i,k,m} + yC_{i,k,m} + zD_{i,k,m} \tag{10}$$

$$A_{j,k,m} = xB_{j,k,m} + yC_{j,k,m} + zD_{j,k,m} \tag{11}$$

where the coefficients of Equation (10) and Equation (11) are functions of the PD measurements and GRS coordinate which can be found in [10].

The pair of plane equations that is Equation (10) and Equation (11) can be presented in matrix form as follows:

$$\begin{bmatrix} B_{i,k,m} & C_{i,k,m} & D_{i,k,m} \\ B_{j,k,m} & C_{j,k,m} & D_{j,k,m} \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} A_{i,k,m} \\ A_{j,k,m} \end{bmatrix} \tag{12a}$$

$$\mathbf{Q}_{ij} \times \mathbf{x}_e = \mathbf{a}_{ij} \tag{12b}$$

The underdetermined LS equation in Equation (12) is known as the multilateration 3-D PE mathematical model with minimum GRS configuration. The location of the emitter is obtained by finding the inverse matrix solution of Equation (12) with TDOA or PD measurements and GRSs coordinates as inputs. Detail derivation of this approach can be found in [12].

3. GRS REFERENCE PAIR SELECTION METHODOLOGY

In [10], a condition number based reference technique called GREPS for a minimum configuration 3-D multilateration system was proposed. A matrix was derived that has as its entries only the PD measurements. The PD measurements obtained for each of the possible GRS pair combinations were substituted into the matrix, and the condition number was calculated. The mathematical expression for the condition number of the matrix based on only the PD measurements obtained using the i -th and j -th GRSs as reference is presented as follows [10]:

$$K(\mathbf{M}_{ij}) = \left(\frac{\hat{d}_{j,m} \times \hat{d}_{j,k}}{\hat{d}_{i,m} \times \hat{d}_{i,k}} \right) + \left(\frac{\hat{d}_{i,m} \times \hat{d}_{i,k}}{\hat{d}_{j,m} \times \hat{d}_{j,k}} \right) \tag{13}$$

where $\hat{d}_{i,k}$, $\hat{d}_{i,m}$, $\hat{d}_{j,k}$ and $\hat{d}_{j,m}$ are the estimated PD measurements in Equation (6), Equation (7), Equation (8) and Equation (9) respectively.

The GRS pair whose PD measurements resulted in the least condition number value using Equation (13) is chosen as the reference GRSs for the linear lateration algorithm. Summary of the approach for selecting the suitable GRS pair as reference for the lateration algorithm in Section 2 is describe as follows;

1. At a given emitter position, obtain the PD measurement set in the form of Equation (14) for each of the possible GRS pair (i, j) as references.

$$\mathbf{d}_{i,j,m,n} = [\hat{d}_{i,k}, \hat{d}_{i,m}, \hat{d}_{j,k}, \hat{d}_{j,m}] \tag{14}$$

2. Using the PD measurement set from (i) for each GRS pair, substitute into Equation (13) and solve for $K(\mathbf{M}_{ij})$.

Choose the GRS pair with the least $K(\mathbf{M}_{ij})$ value from step (ii) as the reference pair for the PE process with the lateration algorithm.

4. RESULTS AND ANALYSIS

In this section of the paper, the 3-D PE using the lateration algorithm describe in Section 2 with the GREPS technique in Section 3 is compared with other techniques that are based on five GRSs. The techniques considered are the TLS approach (SF-TLS) presented in [14] and the fixed GRS reference pair LS approach (MF-LS) presented in [13]. Position root mean square error (RMSE) is used as the performance measure for comparison. Mathematically, the horizontal coordinate and altitude RMSE are respectively obtained as follows:

$$H_{mse} = \frac{1}{N} \sqrt{\sum_{n=1}^N [(\hat{x}_n - x)^2 + (\hat{y}_n - y)^2]} \tag{15}$$

$$Alt_{mse} = \frac{1}{N} \sqrt{\sum_{n=1}^N (\hat{z}_n - z)^2} \tag{16}$$

where (x, y, z) is the known emitter position and $(\hat{x}_i, \hat{y}_i, \hat{z}_i)$ is the estimated emitter position at the n -th Monte Carlo simulation realization. Position RMSE are obtained after $N = 500$ Monte Carlo realization and it is assumed due to proximity of the GRSs that the PDE error STD in Equation (6) to Equation (9) are equal that is $\sigma_{i,k} = \sigma_{i,m} = \sigma_{j,k} = \sigma_{j,m} = \sigma$.

The PE accuracy of the multilateration system depends on the GRS configuration. According to Chan *et al* [18], for a total of four GRSs, square configuration with the a GRS at each vertex results in better PE accuracy. Thus, for this reason, the square GRS configuration is adopted for the analysis and the distribution is shown in Figure 1.

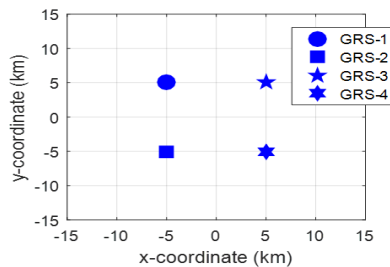


Figure 1. 10 km square GRS configuration

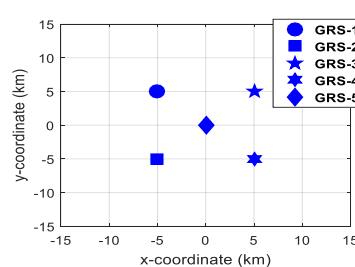


Figure 2. 10 km five-square GRS configuration

As for the SF-TLS and MF-LS approaches that are based on five GRSs, a five-square GRS configuration is adopted as shown in Figure 2.

For the analysis, six different emitter positions are considered with the coordinates shown in Table 1.

Table 1. Selected Emitter positions for analysis

Coordinates	Emitter position					
	A	B	C	D	E	F
x (km)	13	22	43	9	-87	-50
y (km)	22	13	-25	-50	-50	87
z (km)	4		7		10	

By varying the PDE error STD (σ) from 0 to 2 m, the horizontal coordinate and altitude RMSE of the lateration algorithm in Section 2 with the GREPS technique in Section 3 are obtained and compared with that obtained using the SF-TLS and MF-LS approaches. Figures 3, 4, 5, 6, 7 and 8 shows the horizontal coordinate and altitude RMSE comparison for PDE error STD range of 0 to 2 m at emitter positions A, B, C, D, E and F respectively. Irrespective of the PE algorithm used, the horizontal coordinate and altitude RMSE increased with increase in the PDE error STD from 0 to 2 m and it varies with the emitter position. Table 2 shows the position RMSE comparison at PDE error STD of 1 m. Comparison shows that the use of the GREPS technique with the lateration algorithm for the minimum configuration in Section 2 had improved on PE accuracy. It can be seen to outperform the SF-TLS and MF-LS which are based on five GRSSs at the selected emitter positions. For instance, at emitter position A, the horizontal coordinate RMSE with the GREPS technique is 17.43 m while that of the SF-TLS and MF-LS which are higher are 51.91 m and 36.04 m respectively. At emitter position C, the horizontal coordinate RMSE of the lateration algorithm with the GREPS technique and that of SF-TLS and MF-LS are 61.96 m, 91.64 m and 63.38 m respectively. On the average, based on the selected emitter positions, the use of the GREPS technique with the lateration algorithm for PE process outperformed the SF-TLS and MF-LS in horizontal coordinate estimated with a reduction in their horizontal coordinate RMSE by about 50% and 30% respectively. As for the altitude RMSE, the use of the GREPS technique with the lateration algorithm outperformed the SF-TLS and MF-LS with a reduction in their altitude RMSE by about 90%.

Table 2. Position RMSE comparison with other techniques

Emitter position	Horizontal coordinate RMSE (m)			Altitude RMSE (m)		
	SF-TLS	MF-LS	With GREPS	SF-TLS	MF-LS	With GREPS
A	51.91	36.04	17.43	296.80	256.2	11.39
B	54.10	35.61	17.23	304.20	249.20	11.20
C	91.65	63.38	61.96	271.50	372.10	28.13
D	112.90	78.16	90.15	309.90	387.3	30.80
E	737.70	481.40	246.00	1076.00	927.2	70.83
F	358.70	261.10	241.90	535.60	849.90	76.99

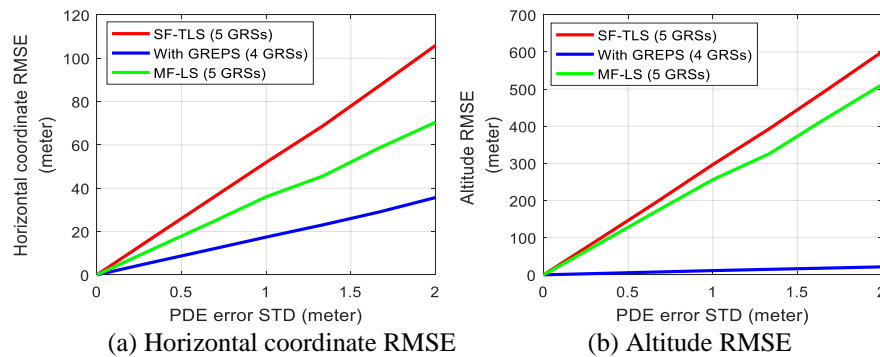


Figure 3. Position RMSE comparison at emitter position A

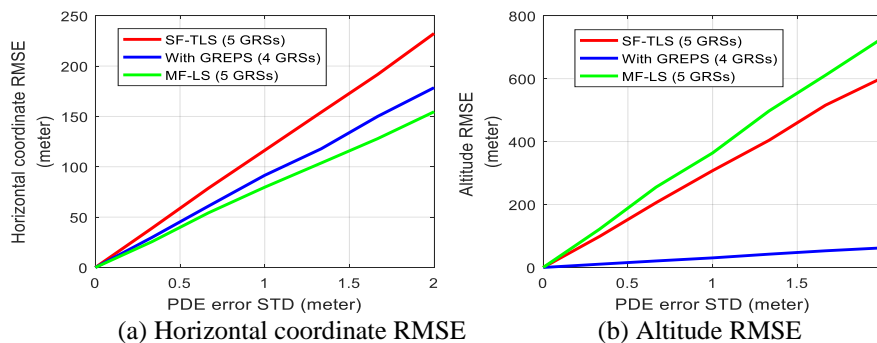


Figure 6. Position RMSE comparison at emitter position D

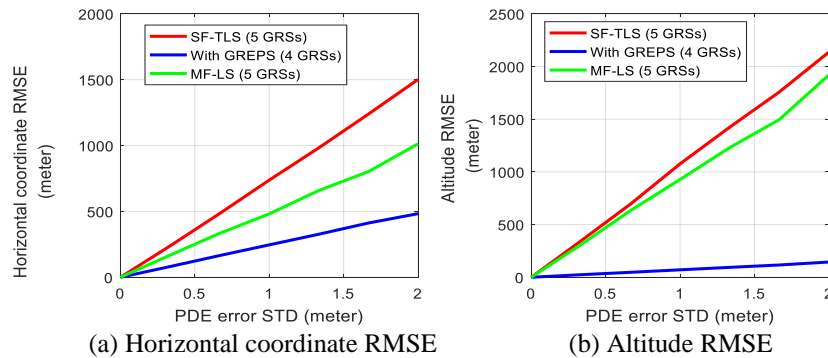


Figure 7. Position RMSE comparison at emitter position E

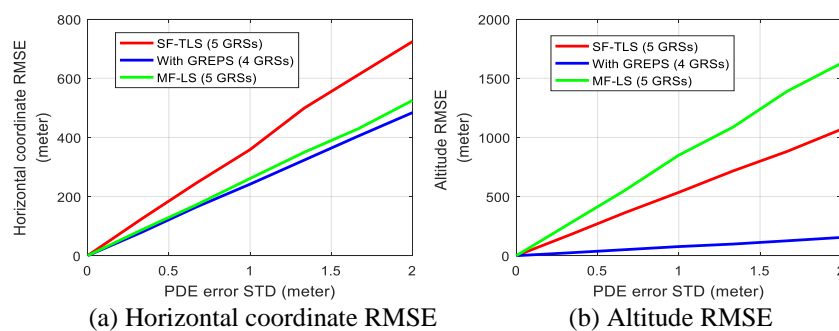


Figure 8. Position RMSE comparison at emitter position F

5. CONCLUSION

In this paper, the PE performance analysis of a minimum configuration 3-D lateration algorithm combined with a GREPS technique is presented. The linear lateration algorithm is considered as it is most suitable for a passive positioning system. The PE comparison was done with two five-GRS linear lateration algorithms which are SF-TLS and MF-LS. Monte Carlo simulation results was carried out at selected emitter positions with the GRS in square configuration. The PE RMSE results shows that the minimum configuration 3-D lateration algorithm when combined with the GREPS technique outperformed the SF-TLS and MF-LS approaches. This is through a reduction in the horizontal coordinate RMSE of about 50% and 30% respectively compared to the SF-TLS and MF-LS approaches. As for the altitude RMSE, there was a reduction of about 90%. In this research, it is assumed that the PD measurements have already been estimated but with error which is modelled as a zero mean Gaussian random variable.

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