LEAST SQUARES VARIANCE COMPONENT ESTIMATION FOR SURVEYING NETWORK ADJUSTMENT

NUR KHALILAH BINTI BIDI

A thesis submitted in fulfilment of the
requirements for the award of the degree of
Master of Philosophy

Faculty of Built Environment and Surveying
Universiti Teknologi Malaysia

MAY 2019

DEDICATION

Special dedication to…

To my beloved father and mother…

Bidi b Mat Nor

Normah bt Sulong

My dear families…

To all my siblings

My supportive lab mates…

My supportive friend…

Muhammad Khairil Adzhar b Radzali

For finding me the light

Only Allah S.W.T will return your kindness…I am really appreciated it

ACKNOWLEDGEMENT

Praise to Lord of Universe, Allah S.W.T for the guidance and His overflow, I can complete and finish my Master study. May peace and blessings to Prophet Muhammad S.A.W, His last messenger.

Firstly, I would like to express my deep and sincere gratitude to my supervisor, Dr. Ami Hassan because of his insightful guidance helped me in all the time of research and writing this thesis. Without his help and encouragement, the thesis would not have reached its completion. His constructive comments and advice have always played a significant role in improving the content of my thesis and publications.

Special thanks go to Mr Zainal Abidin as my co-supervisor for his efficient supervision and kindness to guide and helpful advice during my study. His understanding, encouragement and personal guidance make me struggle to finish this research. He is also very much acknowledged for the scientific discussions we have had afterward.

A much appreciate to Department of Surveying and Mapping Johor Bahru (DSMM) for providing me cadastral data in order to realise this research. Thanks also to Dr. Dudy Wijaya for helping me to understand LS-VCE concept, algorithm and software.

Besides my supervisors, special thanks to all my friends in GIG, UTM especially Anim Zanariah, Suraya, Astina, Fadilah, Najihah, Adilla and Jehan for helping me and useful discussion over the last four semesters. Above all, my heartfelt gratitude goes to my beloved parents and siblings for their unconditional love, spiritual support, and continuous encouragement all the way through these years since 2016.

ABSTRACT

The stochastic model of least squares adjustment plays an essential role in geodetic network data processing because the model describes the accuracy of the measurements and their correlation with each other. Knowledge of weights of the observables is necessary to provide a better understanding of the sources of errors and to model the error, hence the weights need to be determined correctly. For geodetic applications, it is crucial to have knowledge about covariance matrix of the observables since variance components are most commonly used to determine a realistic precision. This study focuses on the estimation of variance components from different types of data for geodetic applications, which include deformation survey and cadastral survey. Least Squares Variance Component Estimation (LS-VCE) method was used in this study because the method is simple, flexible and attractive due to the precision of variance estimators that can be directly obtained. For deformation monitoring network, simulations of geodetic and Kenyir dam networks data were performed. Meanwhile, for cadastral observation data from several types of instruments such as chain measurement, Electronic Distance Measurement and total station were utilized. The results revealed that the estimated variance components for distance scale error σ_p seem to become unrealistic for each data tested as the baseline of the networks was not long enough. In addition, it was found that the traverse network which included chain survey, showed insignificant result to the precision of station coordinates when the measurements were combined. The distance-dependent model was selected as the best model for Kenyir dam network since W-test values of Epoch 1 and 2 were 0.30 and 0.50, where the expectation and the variance of W-test values were 0 and 1, respectively. The findings showed that LS-VCE method was very reliable in various geodetic applications. In conclusion, the program developed is valuable for professional groups which include surveyors and engineers, as well as geophysics and geologists.

ABSTRAK

Model stokastik bagi pelarasan kuasa dua terkecil memainkan peranan penting dalam pemprosesan data jaringan geodesi, kerana ia menerangkan ketepatan pengukuran dan kolerasi antara satu sama lain. Pengetahuan tentang pemberat dalam cerapan adalah perlu untuk memberikan pemahaman yang lebih baik tentang sumber-sumber selisih dan pemodelan selisih tersebut, oleh itu ia perlu ditentukan dengan betul. Bagi aplikasi geodesi, adalah penting untuk mempunyai pengetahuan mengenai matriks kovarians yang paling biasa digunakan untuk menentukan ketepatan yang realistik. Kajian ini memberi tumpuan kepada penganggaran komponen varians dari pelbagai jenis data untuk aplikasi geodesi termasuk ukur deformasi dan ukur kadaster. Kaedah Penganggaran Komponen Kuasa Dua Varians (PKKV) digunakan dalam kajian ini kerana kaedah ini lebih mudah, fleksibel dan menarik disebabkan ketepatan varians anggaran boleh diperoleh secara langsung. Bagi jaringan deformasi, data jaringan simulasi dan jaringan empangan Kenyir telah dijalankan. Sementara itu, data cerapan kadaster dari beberapa jenis peralatan seperti pengukuran rantai, pengukuran jarak elektronik dan total station telah digunakan. Keputusan menunjukkan bahawa komponen-komponen varians yang dianggarkan untuk selisih skala jarak σ_p tidak realistik bagi setiap data yang diuji kerana garis dasar jaringan tidak cukup panjang. Di samping itu, didapati bahawa jaringan terabas yang melibatkan ukur rantai menunjukkan hasil yang tidak signifikan kepada kejituan koordinat stesen apabila pengukuran digabungkan. Model jarak bersandar dipilih sebagai model terbaik bagi jaringan empangan Kenyir kerana nilai ujian W bagi epok 1 dan 2 adalah 0.30 dan 0.50, di mana jangkaan dan varians nilai ujian W masing-masing adalah 0 dan 1. Keputusan menunjukkan bahawa kaedah PKKV sangat dipercayai dalam pelbagai aplikasi geodesi. Kesimpulannya, program yang dibangunkan adalah sangat berguna kepada kumpulan professional termasuk juru ukur dan jurutera, serta ahli geofizik dan geologi.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

1D	-	One-Dimensional
2D	-	Two-Dimensional
3D	-	Three-Dimensional
BIQUE	-	Best Invariant Quadratic Unbiased Estimator
BLUE	-	Best Linear Unbiased Estimation
CNC	-	Cadastral Network Computation
DNAC	-	Deformation Network Analysis Computation
EDM	-	Electronic Distance Measurement
GNSS	-	Global Navigation Satellite System
GPS	-	Global Positioning System
GUIDE	-	Graphical User Interface Development Environment
DSMM	-	Department of Survey and Mapping Johor Bahru
MATLAB	-	Matrix Laboratory
MLE	-	Maximum Likelihood Estimation
MINQUE	-	Minimum Norm Quadratic Unbiased Estimator
NDCDB		National Digital Cadastral Database
LSA	-	Least Squares Adjustment
LS-VCE	-	Least Squares Variance Component Estimation
REML	-	Restricted Maximum Likelihood Estimator
TLS	-	Terrestrial Laser Scannning
TS	-	Total Station
UAV	-	Unmanned Aerial Vehicle
UTM	-	Universiti Teknologi Malaysia
VCE	-	Variance Component Estimation

LIST OF SYMBOLS

b	-	Degrees of freedom in functional model
df	-	Degrees of freedom in stochastic model
d	-	Distance measurement
e	-	Residual vector in stochastic model
k	-	Number of distance measurement
l	-	Observation vector in functional model
$£$	-	Likelihood function
m	-	Number of observations
n	-	Number of unknown parameters
p	-	Number of variance component
s	-	Number of directions
t	-	Redundant observation
v	-	Residual vector in functional model
w	-	W-test in stochastic model
x	-	Unknown parameter vector
y	-	Observation vector in stochastic model
A	-	Design matrix
B	-	Null space of A^T
P	-	Weight matrix in functional model
N	-	Normal matrix
W	-	Weight matrix in stochastic model
χ^2	-	Chi-Squares test
ε	-	Total errors
σ_k	-	Variance components
σ_d		Distance-dependent component
σ_p		Distance scale component
σ_b		Horizontal angle/bearing azimuth component
σ_o^2	-	A priori variance factor
$\hat{\sigma}_o^2$		A posteriori variance factor

σ_k	-	Initial variance component
$\hat{\sigma}_k$	-	Estimated variance component
σ_d^2	-	Variance of distance-dependent
σ_p^2	-	Variance of distance scale
σ_b^2	-	Variance of bearing/horizontal angle
θ_m	-	Variance covariance component
Σx	-	Covariance matrix in functional model
C_y	-	An *m* x *m* symmetric matrix
$\hat{L}^a$	-	Adjusted observation in functional model
Q_o	-	Known part of covariance matrix
K, Q_y	-	Covariance matrix in stochastic model
Q_k	-	Cofactor matrix in stochastic model
$P\frac{1}{A}$	-	Orthogonal projector in stochastic model
T_m	-	Coefficient matrices

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Background of the Research

Least squares adjustment (LSA) of a geodetic application is always an important topic in surveying. In general, principle of least squares is to minimize the sum of squares of the weighted residuals (Caspary, 2000). Back in the early 18th century, Gauss and Legendre introduced the concept of least squares estimation mostly for the purpose of reducing physical and astronomical data (Olyazadeh *et al.*, 2011). In geodetic data processing, principle of least squares method deals with these two types of models, which are 'functional model' and 'stochastic model'. The functional model contains the geometrical relationship between measurements and parameters to be estimated, while the stochastic model gives information about the precision of measurements.

Data processing in geodetic network often count on the least squares method, thus an appropriate stochastic model of the observables is constantly required (Amiri-Simkooei, 2007). In addition, information of weight of the observables is important to estimate the correct least squares solution (Wentworlh, 1965). According to Ghilani and Wolf (2006):

> "The weight of an observation is a measure of its relative worth compared to other measurements. Weights are used to control the sizes of corrections applied to measurements in an adjustment. The more precise an observation, the higher its weight; in other words, the smaller the variance, the higher the weight."
>
> (Ghilani and Wolf, 2006, pp. 159)

Based on these sentences, it can be stated automatically that weights are inversely proportional to variances. Hence, it also follows that correction sizes should be inversely proportional to the weights. Inappropriate weight (variance component) can affect the LSA results, which is significant in geodetic network. The input data of the geodetic network representing the measurement accuracy is also referred to as the variance components of the measurement data. Therefore, the estimation of the unknown variance components is called variance component estimation (VCE).

For many applications, information available on the covariance matrix of the observation is crucial. This is because every measurement data or observation data of geodetic network are necessary to know their level of accuracy in the form of a standard deviation or variance. Thus, the standard deviation of the measurement has supposedly become one of the important elements in LSA. A realistic covariance matrix allows us to determine a correct precision description of the unknown parameters. According to Amiri-Simkooei (2007), the information that we can obtain through covariance matrix allows us, first, to study the different contributing errors factors in observation, second, to acquire minimum variance estimators of the parameters in a linear model and third, to perform hypothesis testing correctly and to evaluate other quality control measures such as reliability.

However, the covariance matrix is only partially known in most of the modern geodetic applications, as a result, the unknown part needs to be estimated from the redundant observation. For example, our knowledge of stochastic model is still at an elementary level in the field of Global Navigation Satellite System (GNSS). This is different with a functional model (i.e. observation equations), which is common in many GNSS textbooks available (Amiri-Simkooei, 2007). In recent years, the geodetic community has become increasingly interested in the method of estimating variance components. The interest in VCE in geodetic applications is more relevant within the context of the least squares adjustment. Variance component estimation brings geodetic perspective, which refers to the technique of estimating the variance or the variance factors for individual groups of observation from an adjustment.

Nowadays, variance component estimation method has become one of the familiar topics in statistical geodetic literatures. There are various available methods for computing variance components. However, these methods employ different principles of estimation. Moghtased-Azar *et al.* (2014), Eshagh & Sjöberg (2008) and Teunissen & Amiri-Simkooei (2008) have listed many pioneer studies on VCE work published in several statistic related journals. One of the most popular methods, presented by Rao (1970), is the Minimum Norm Quadratic Unbiased Estimator (MINQUE) to avoid any distributional assumption. Then in 1983, the MINQUE method was extended to the general Gauss-Helmert model by Sjöberg, namely condition adjustment with unknowns (Moghtased-Azar *et al.*, 2014).

Maximum Likelihood Estimation (MLE) method was first presented for geodetic applications by Kubik (1970) to estimate ratios of weight in a network direction. Another approach is Helmert's method presented by Kelm (1978), followed by Least Squares Variance Component Estimation (LS-VCE) method by Amiri-Simkooei (2007), which explored a new kind of least squares estimator to the VCE in GPS application. The VCE approach is being proposed to be implemented in the attempt to develop more realistic weighting factors for geodetic network adjustment. The principle of VCE method generally is based on the 'unconstrained' estimation (Amiri-Simkooei, 2016). Least squares variance component estimation (LS-VCE) method is used in this study, which was first proposed by Teunissen (1988). Even though the method is rarely used in VCE methods, Amri-Simkooei (2007) found that LS-VCE is a more simple, flexible and attractive method for the estimation of unknown variance and covariance components. Therefore, this study is aimed to investigate whether the least squares estimation method can be used to solve the problem of variance component estimation in geodetic applications which include deformation survey and cadastral survey.

1.2 Problem Statement

The weight matrix is computed by using variance of the observations that were initially provided as input data in the form of standard deviation for each observation data accordingly. Grodecki (1997) showed that the weights assigned to the observations can directly influence the LSA results. The solution of LSA computation is dependent on the weight of the observations. Furthermore, the weight matrix is then used again for the computation of LSA global test (or χ^2 test) to determine whether the LSA solution passed or failed. Failure in global test renders the LSA solution as questionable, and as the result, the weight matrix used in such computation has become the possible element that needs to be examined.

Currently, there is a limitation on the information of the covariance matrix of the observables in many modern geodetic applications. For geodetic applications, it is crucial to have knowledge about the covariance matrix of the observables since that variance component is the most commonly used to determine a realistic precision. In addition, it allows us to investigate the various contributing error factors in the observations (Amiri-Simkooei, 2007). Such an example in Figure 1.1, what if a set of measurement data is computed by using StarNet software have failed the Global test (Chi-Square test). However, by removing blunders in observations or use trial and error method by changing the standard deviation values seem not practical to be done in bundles of data. Thus, determination of variance component in more systematic ways are crucially needed. This shows the importance of variance component estimation methods.

```
                    Adjustment Statistical Summary
                    ==============================

                   Iterations              =      2

                   Number of Stations      =      6

                   Number of Observations  =     27
                   Number of Unknowns      =      8
                   Number of Redundant Obs =     19

       Observation    Count    Sum Squares         Error
                                 of StdRes         Factor
            Angles      15        63.833           2.459
         Distances      12       900.016          10.324

             Total      27       963.848           7.122

    Warning: The Chi-Square Test at 5.00% Level Exceeded Upper Bound
                    Lower/Upper Bounds (0.685/1.315)
```

Figure 1.1 Example of a set of measurement data have failed Chi-Square test in StarNet software.

In recent years, conventional approaches have been used to estimate the variance component. Such methods that used include employing a rigorous computation, adopting the accuracy of specification given by the manufacturer, based on analysis of previous works, using individual groups of the observations for separating adjustments of the network, and last but not least, trial and error method, in which different combinations of the suspected variances of the observations are entered into the adjustment (Dennler, 1980; Chen, 1983). Obviously, the conventional methods are too rigid and need rigorous computation. Also, they are not suitable for current practice and with current needs such as new technologies like GPS, UAV, datum unification, photogrammetry, etc.

Since the conventional methods have their own weakness, the geodetic community has been trying to explore a better alternative. As a result, several approaches based on statistical methods have shown the potential to be more convincing. At present, many different methods exist for estimating the variance component (VCE). One of the well-known methods is the Minimum Norm Quadratic Unbiased Estimator (MINQUE). The other methods are the Best Invariant Quadratic

Unbiased Estimator (BIQUE), the Restricted Maximum Likelihood Estimator (REML), the Bayesian method, and Least Squares Variance Component Estimator (LS-VCE) to the VCE.

Regarding the previously stated scenario, the conventional methods are not challenged enough and cannot be achieved by the following problem statement. The determination of standard deviation (or the estimation of variance component) is required for geodetic networks. The present practice of determining the variance component through conventional means is faced with difficulties. Thus, the geodetic community is actively seeking an alternative method for the determination of VCE.

To address this challenge, Amiri-Simkooei (2007) come out with a new method called LS-VCE, where using the principle of least square and weight matrix is defined by the user. The main idea of this method is to identify whether it is possible to apply the least squares estimation method and solve VCE problems. LS-VCE method also sustains all the basic properties of a least squares estimator. The advantages of these methods are that the existing knowledge of least squares theory can be applied for various aspects of VCE and the precision of variance components can directly be achieved (Teunissen and Amiri-Simkooei, 2008). Therefore, this research will highlight and focus on the estimation of variance components from different types of data by using LS-VCE method for the geodetic applications which includes deformation monitoring survey and cadastral survey. Finally, the LSA and VCE computational package will be developed to estimate the variance components of the geodetic observation data.

1.3 Research Question

The research question is expected to be the key and the important indicator that will highlight the main idea for this research and to solve the issue that is needed to be encountered. Thus, the following questions need to be properly addressed:

i) How to estimate the variance components for geodetic applications; deformation monitoring network and cadastral network?

ii) How to determine the optimum variance of geodetic observation network using W-test statistic?

1.4 Aim and Objectives of the Research

The aim of this research is to estimate variance components from the different types of data for geodetic applications which include deformation survey and cadastral survey. Therefore, the following specific objectives were set towards achieving this aim:

i) To perform least square adjustment and variance component estimation using LS-VCE method for geodetic surveying network.

ii) To evaluate the optimum variance components of the geodetic network for selecting the best stochastic model using W-test statistic.

1.5 Scope of the Research

i) Two-dimensional (2D) Geodetic Network Application

This study focuses mainly on two types of geodetic networks, which include applications on deformation monitoring network and cadastral network. This research is concentrated only on the geodetic analysis in two-dimensional (2D) network for simplicity and ease of understanding. The types of observation data involved consist of distances, horizontal angles, and bearing azimuth.

ii) Data Acquisition

Deformation Monitoring Network

For deformation monitoring network, there are three types of data tested. The first dataset based on the simulation deformation geodetic network was carried out at the field of Universiti Teknologi Malaysia (UTM). The second dataset was obtained from the existing data through FREDY04 software. Then, the last data was based on the real work at Kenyir dam acquired from the supervisor's past project. All sets of data are made in the two epochs measurement.

Cadastral Network

Cadastral network utilized observation data from several types of instrument used, such as chain measurement, electronic distance measurement (EDM), and total station (TS). Scope of the area is focused on the small area of Johor, Malaysia, where Mukim Tebrau region is selected (refer to Figure 1.2). These data are provided from the Department of Survey and Mapping Johor Bahru (JUPEM). Year of data is selected randomly from 1966 to 2014 to differentiate the advancement of instruments used by the surveyors.

Figure 1.2 The study area for cadastral network data

iii) Software Used

The geodetic observation data is processed using MATLAB programming. A package was developed by using MATLAB GUIDE to solve least square adjustment computation and to estimate the optimum variance component from different type geodetic network data. Then, the package is compared with commercial software MicroSurvey StarNet (a powerful adjustment commercial software) for validation process.

iv) VCE Method Used

As mentioned in Section 1.2, LS-VCE method is used for VCE computation after network adjustment is done since it is based on the least squares principle. The simple routine is developed by using MATLAB programming in order to solve LSA and VCE computational problem. Thus, it is hoped that it will become a user-friendly package and easy to understand for educational purpose only.

1.6 Significance of the Research

The significance of this study was triggered to estimate standard deviation (variance component) in geodetic network applications from covariance matrix elements particularly on deformation monitoring network and cadastral network. In Malaysia, surveyors and practitioners usually depend on commercial software, such as MicroSurvey StarNet and GeoLab. However, a limited number of this commercial software is available, especially for deformation monitoring. Therefore, in order to encounter this limitation, this study has developed a program which able to perform LSA and VCE computation directly.

An additional aim of this study is to develop a computational LSA and VCE package by using the simple, flexible and attractive method, which is LS-VCE method, in order to gain interest and draw attention to the important of variance components in the field of geodetic network applications. Implementing the VCE method after the least square adjustment or LSA can be used as comprehensive way of estimating a proper weighting component in different type of observables corresponding to different characteristics.

Lastly, this research hopes that development of computational LSA and VCE package can be useful for attracting practicing surveyors in Malaysia as well as Geomatics Engineering students in local universities to become interested in estimation of variance component in integrated manner.

1.7 General Research Methodology

The general research methodology in this study is shown in Figure 1.3. This thesis highlighted literature review, data acquisition, data processing and data analysis.

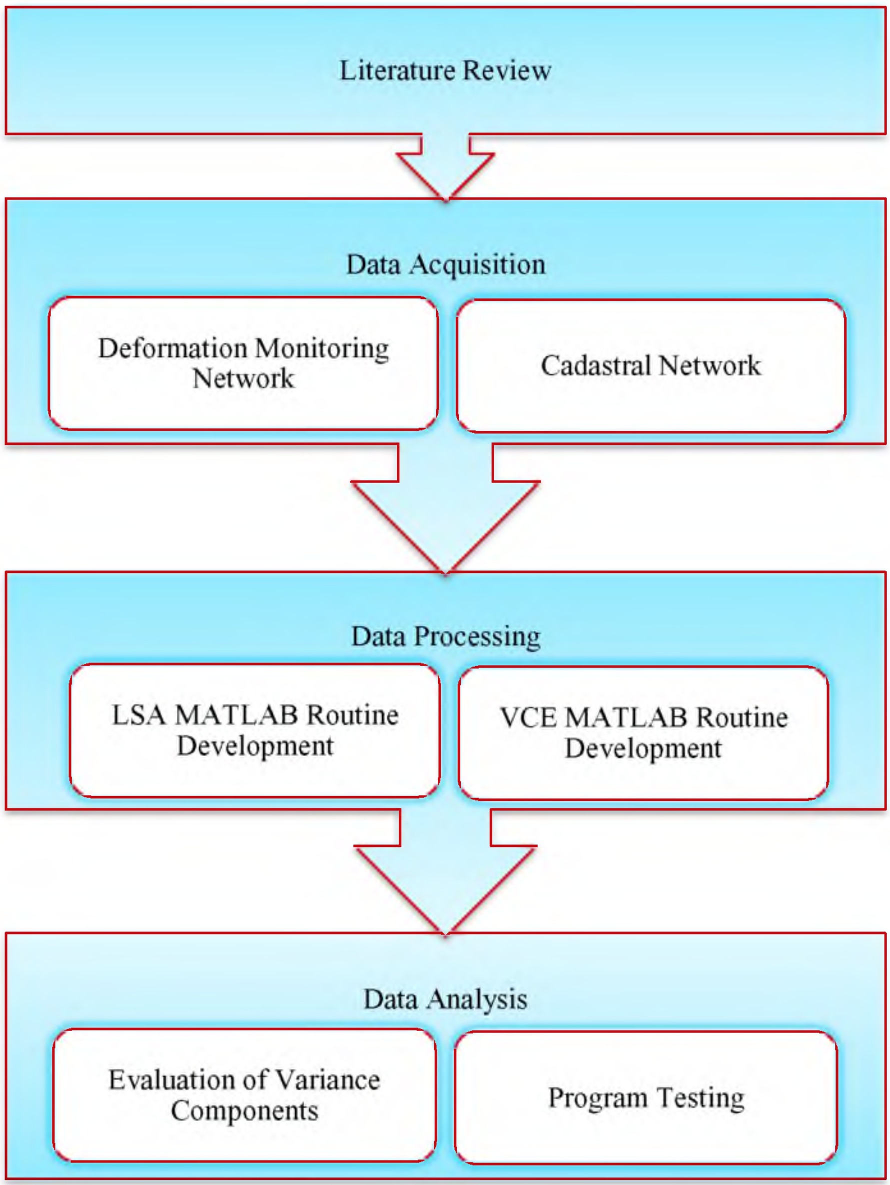

Figure 1.3 Overview of general research methodology

Literature Review

The literature review stage is required as a reference and guidance in executing this study. The topics discussed are focused on:

i) An overview of variance component estimation, errors in the measurement, and VCE applications.

ii) Types of variance component models

iii) Methods available for variance component estimation

iv) The principle of Least Squares Variance Component Estimation (LS-VCE) method which is applied in this study.

v) Applying the VCE to the geodetic applications, where this study focused on deformation monitoring network and cadastral network.

Data Acquisition

The data acquisition of this study is gathered as follows:

i) Deformation monitoring network

There are three sets of data, which consist of 2 sets of simulation data and real work data at Kenyir dam. All the data are measured in the 2D network which includes measurement of distances, horizontal angle and bearing azimuth. Every deformation network is made up of two epochs measurement.

ii) Cadastral Network

In this study, cadastral data are requested from the Department of Survey and Mapping Johor Bahru (DSMM) where the data come with various type of instruments used. These instruments include observation by chain measurement, electronic distance measurement (EDM), theodolite and total station (TS). The study area is in Johor state at Mukim Tebrau region. The data are chosen randomly from the year 1966 to 2014. This is because, with the advancement of technology, various modern instruments are developed.

Data Processing and Programming

The geodetic observation data is processed using MATLAB programming. This stage involved the development of the least square adjustment (LSA) routine and followed by variance component estimation (VCE) computation. The developed package is divided into two part, which is deformation monitoring survey and cadastral survey. Each geodetic application will create their own MATLAB computation routine. This is because the format of the input data for each application is different. The data are processed by the least squares adjustment of the network and proceeded with VCE computation in order to acquire the realistic precision of measurement.

Data Analysis

As for data analysis, the result of the stochastic model from VCE computation, which is the case here, variance components need to be evaluated. The W-test statistic is practiced in order to find misspecifications in the stochastic model, to enhance an existing covariance matrix, and to test the general validity of the stochastic model. Subsequently, the developed package is compared with MicroSurvey StarNet for validation of the program developed.

1.8 The Structure of The Thesis

This thesis is organised into six chapters. The introduction to the study and research explanation has been detailed in **Chapter 1**. It starts with the background of the study followed by the problem statement, the aims and objectives of this study, the scope of this study and lastly the significance of this study.

Chapter 2 reviewed literature on several topics and begins with an overview of the principle of variance component estimation. In addition, a simple review of the available methods for VCE that includes minimum norm quadratic unbiased estimator (MINQUE), the best invariant quadratic unbiased estimator (BIQUE), maximum likelihood estimation (MLE), the Helmert method to VCE, and lastly, a detailed on least squares variance component estimation (LS-VCE) is presented since this study employed the method. This was followed by a discussion on the geodetic application, which includes deformation monitoring network and cadastral network.

Chapter 3 presents the research methodology of this study. In this chapter, data acquisition of deformation monitoring and cadastral network is explained. This chapter also included the development of least square adjustment (LSA) routine followed by variance component estimation (VCE) computation. This is to complete Objective 1 of this study. Then, a methodology to evaluate the variance component estimation was presented and discussed. The W-test was generalized for hypothesis testing to the stochastic model. This is towards achieving Objective 2 of this research.

This chapter also explained the developed computation package using MATLAB programming. The package will be divided into two choices, either deformation network computation or cadastral computation. Each geodetic application will create their own MATLAB computation routine. This is because the format of the input data of each application is different. A detailed explanation about the function of the package and how to employ it for different types of geodetic observation data was also described in this chapter.

Chapter 4 and **Chapter 5** discuss the result and analysis of deformation monitoring network and cadastral network, respectively. The purpose of this chapter was to come up with a realistic optimum variance component based on various types of geodetic observation data. The analysis was made to select the best model of variance component for each geodetic network tested in this study. Next, the developed computational package was compared with a powerful adjustment commercial software for validation process.

Chapter 6 is the last chapter of this research, where the overall study and recommendations listed for further research study and enhancements for future work are summarized.

REFERENCES

Abdallah, A., Mohamed, A. and Elkhabeer, A. (2015) 'Adjustment and gross errors detection of free triangulation geodetic network using minimum-norm least-squares inverses and data snooping', *Global Journal of Earth Science and Engineering*, 2(2), pp. 31–40.

Alizadeh-Khameneh, M. A. (2017) *Optimal Design in Geodetic GNSS-based Networks*. Doctoral Thesis in Geodesy, KTH Royal Institute of Technology, Stockholm, Sweden.

Amiri-Simkooei, A. R. (2007) *Least-Squares Variance Component Estimation: Theory and GPS Applications*. PhD Thesis, Delft University of Technology, Delft, Netherlands.

Amiri-Simkooei, A. R., Teunissen, P. J. G. and Tiberius, C. C. J. M. (2007) 'Assessment of noise in GPS coordinate time series: Methodology and results', *Journal of Geophysical Research*, 112(7), pp. 1–19. doi: 10.1029/2006JB004913.

Amiri-Simkooei, A. R., Teunissen, P. J. G. and Tiberius, C. C. J. M. (2009) 'Application of least-squares variance component estimation to GPS observables', *Journal of Surveying Engineering*, 135(4), pp. 149–160.

Amiri-Simkooei, A. R., Asgari, J., Zangeneh-Nejad, F., Zaminpardaz, S. (2012) 'Basic concepts of optimization and design of geodetic networks', *Journal of Surveying Engineering*, 138(4), pp. 172–183.

Amiri-Simkooei, A. R. (2013) 'Application of least squares variance component estimation to errors-in-variables models', *Journal of Geodesy*, 87(10–12), pp. 935–944. doi: 10.1007/s00190-013-0658-8.

Amiri-Simkooei, A. R., Zangeneh-Nejad, F. and Asgari, J. (2013) 'Least-squares variance component estimation applied to GPS geometry-based observation model', *Journal of Surveying Engineering*, 139(4), pp. 176–187.

Amiri-Simkooei, A. R. (2016) 'Non-negative least-squares variance component estimation with application to GPS time series', *Journal of Geodesy*. Springer Berlin Heidelberg, 90(5), pp. 451–466.

Bähr, H., Altamimi, Z. and Heck, B. (2007) *Variance Component Estimation for Combination of Terreatrial Reference Frames*. Publication series of the program Geodesy and Geoinformatics, University of Karlsruhe, No. 6.

Baarda, W. (1968) 'A testing procedure for use in geodetic networks', *Publications on Geodesy*, 2(5), Netherlands Geodetic Commission, Delft.

Baryla, R., Paziewski, J., Wielgosz, P., Stepniak, K. and Krukowska, M. (2014) 'Accuracy assessment of the ground deformation monitoring with the use of GPS local network: Open pit mine Koźmin case study', *Acta Geodynamica et Geomaterialia,* 11(4), pp. 317–24.

Bhatia, V. K. (2011) *Variance Component Estimation and Best Linear Unbiased Prediction (BLUP)*. New Delhi: Indian Agricultural Statistics Research Institute (ICAR) Library Avenue. Retrieved August 15, from http://www.iasri.res.in/ebook/EBADAT/index.htm.

Bidi, N. K., Som, Z. A. M., Din, A. H. M., Omar, A. H. (2018) 'The development of computational routine for deformation modelling and analysis: A case for two-dimensional geodetic technique', In: Pradhan B. (eds) GCEC 2017. GCEC 2017. Lecture Notes in Civil Eng., vol. 9. *Springer*, pp. 723-740.

Bozic, B., Gospavic, Z. and Milosavljevic, Z. (2011) 'Estimation of the variance components in various covariance matrix structures', *Survey Review*, 43(323), pp. 653–662. doi: 10.1179/003962611X13117748892434.

Cai, C., Pan, L. and Gao, Y. (2014) 'A precise weighting approach with application to combined L1/B1 GPS/BeiDou positioning', *Journal of Navigation*, 67(5), pp. 911–925.

Caspary, W. F. (1987) *Concepts of Network and Deformation Analysis*. Technical report, School of Surveying, The University of New South Wales, Kensington.

Caspary, W. F. (2000) *Concepts of Network and Deformation Analysis*. 3rd corrected, Edited by J. M. Rueger, Monograph 11. Sydney: School of Geomatic Engineering, University of New South Wales.

Chen, Y. Q. (1983) *Analysis of Deformation Surveys – A Generalized Method*. Technical Report No. 94. Department of Geodesy and Geomatics Engineering, University of New Brunswick, Fredericton, N.B.

Chen, Y. Q., Chrzanowski, A. and Secord, J. M. (1990) 'A strategy for the analysis of the stability of reference points in deformation surveys', *CISM Journal ACSGC*, 44(2), pp. 141–149.

Chrzanowski, A., Chen, Y. Q. and Secord, J. (1986) 'Geometrical Analysis of Deformation Surveys', *Proceedings of the Deformation Measurements Workshop*, MIT, Cambridge, Mass., pp. 170–206.

Crocetto, N., Gatti, M. and Russo, P. (2000) 'Simplified formulae for the BIQUE estimation of variance components in disjunctive observation groups', *Journal of Geodesy*, 74, pp. 447–457.

Civil Seek (2018) *Chain Surveying: Its Procedure, Instruments, and Principles*. Retrieved November 9, from https://civilseek.com/chain-surveying/.

Eshagh, M. and Sjöberg, L. E. (2008) 'The modified best quadratic unbiased non-negative estimator (MBQUNE) of variance components', *Studia Geophysica et Geodaetica*, 52(3), pp. 305–320.

Dennler, S. M. (1980) *Evaluation of Micro-Geodetic Networks for Monitoring Tectonic Movements in Peru*. M.Eng. Report, Dept. of Surveying Engineering, University of New Brunswick, Fredericton, N. B., Canada.

Fan, H. (2010) 'Theory of errors and least squares', *Nature*, 98(2464), pp. 385–386. doi: 10.1038/098385b0.

Förstner, W. (1979) 'A Method for Estimating Variance and Covariance Components', *General Surveying News*, pp. 446–453.

Fotopoulos, G. (2003) *An Analysis on the Optimal Combination of Geoid, Orthometric and Ellipsoidal Height Data*. Report No. 20185, Department of Geomatics Engineering, University of Calgary, Canada.

Fotopoulos, G. (2005) 'Calibration of geoid error models via a combined adjustment of ellipsoidal, orthometric and gravimetrical geoid height data', *Journal of Geodesy*, 79, pp. 111−123.

Ghilani, C. D. and Wolf, P. R. (2006) *Adjustment Computations: Spatial Data Analysis*, International Journal of Geographical Information Science. Edited by Fourth Edition. John Wiley & Sons, Inc.

Gopaul, N. S., Wang, J. and Guo, J. (2010) 'On posteriori variance-covariance component estimation in gps relative positioning', *CPGPS 2010 Technical Forum*, pp. 1–10.

Grafarend, E., Kleusberg, A. and Schaffrin, B. (1980) 'An Introduction to the Variance-Covariance Component Estimation of Helmert Type', *ZFV*, 105(4), pp. 161–179.

Grafarend, E. W. (1984) 'Variance-covariance component estimation of Helmert type in the Gauss-helmert model', *ZFV*, 109, pp. 34–44.

Grafarend, E. W. (1985) 'Variance-covariance component estimation: theoretical results and geodetic applications', *Statist Decis Supplement*, 2, pp. 407–441.

Grodecki, J. (1997) *Estimation of Variance-Covariance Components for Geodetic Observations and Implications on Deformation Trend Analysis*. Technical Report No. 186. Department of Geodesy and Geomatics Engineering, University of New Brunswick, Fredericton, N.B.

Guo, D. M. and Xu, H. Z. (2015) 'Application of variance components estimation to calibrate geoid error models', SpringerPlus. *Springer International Publishing*, 4(1), pp. 1–12.

Helmert, F. R. (1907) *The Least Squares Adjustment Calculation*. 2nd Edition, publisher of B.G Teubner, Leipzig.

Helmert, F. R. (1924) *The Least Squares Adjustment Calculation*. 3rd Edition, publisher of B.G Teubner, Leipzig.

Junhuan, P., Yun, S., Shuhui, L. and Honglei, Y. (2011) 'MINQUE of variance-covariance components in linear gauss-markov models', *Journal of Surveying Engineering*, 138(3), pp. 129–139.

Kalkan, Y. (2014) 'Geodetic deformation monitoring of Ataturk Dam in Turkey', *Arabian J. Geosci.*, 7(1), pp. 397−405. doi: 10.1007/s12517-012-0765-5.

Kavitha, M., Viswanath, R., Kavibharathi, P., Aakash, K. and Balajimanikandan, M. (2018) 'A Comparative Study of Conventional Surveying Techniques With Total Station and GPS', *Intern. Journal of Civil Eng. and Tech.*, 9(1), pp. 440–446.

Kelm, R. (1978) 'Is the variance estimate according to Helmert MINQUE?', *General Survey News*, 85(2), pp. 49−54.

Kiamehr, R. and Eshagh, M. (2008) 'Estimating variance components of ellipsoidal, orthometric and geoidal heights through the GPS/levelling network in Iran', *Journal of the Earth & Space Physics*, 34(3), pp. 1–13.

Koch, K. R. (1978) 'Estimation of variance components', *General Surveying News*, 85, pp. 264–269.

Koch, K. R. (1986) 'Maximum likelihood estimate of variance components', *Bull. Geod.*, 60, pp. 329–338.

Koch K. R. (1997) *Parameter Estimation and Hypothesis Testing in Linear Models*. 3rd Edition, Dümmler, Bonn.

Koch, K. R. (1999) 'Parameter estimation and hypothesis testing in linear models', *Springer Verlag*, Berlin.

Kubik, K. (1970) 'The estimation of the weights of measured quantities within the method of least squares', *Bulletin Géodésique,* 95(1), pp. 21–40.

Kusche, J. (2003) 'A Monte-Carlo technique for weight estimation in satellite geodesy', *Journal of Geodesy*, 76, pp. 641–652.

Li, B. F., Shen, Y. Z., Lou L. Z. (2011) 'Efficient estimation of variance and covariance components: A case study for GPS stochastic model evaluation', *IEEE Transactions on Geoscience and Remote Sensing*, 49(1), pp. 203–209.

Mahboub, V. (2014) 'Variance component estimation in errors-in-variables models and a rigorous total least-squares approach', *Studia Geophysica et Geodaetica*, 58(1), pp. 17–40.

Moghtased-Azar, K., Tehranchi, R. and Amiri-Simkooei, A. R. (2014) 'An alternative method for non-negative estimation of variance components', *Journal of Geodesy*, 88(5), pp. 427–439.

Mohamed, A. A. M. (2013) *Development of Geodetic Deformation Analysis Software Based on Iterative Weighted Similarity Transformation Technique*. Msc. Thesis, Universiti Teknologi Malaysia, Johor, Malaysia.

Olyazadeh, R., Setan, H. and Fouladinejad, N. (2011) 'Network adjustment program using MATLAB', *Geospatial World Forum*, (January).

Ou, Z. (1989) 'Estimation of variance and covariance components', *Bull. G´eod.*, 63, pp. 139–148.

Pace, R. K. (2014) *Maximum Likelihood Estimation*. In: Fischer, M., Nijkamp, P. (eds) Handbook of Regional Science. Springer, Berlin, Heidelberg.

Pope, A. J. (1976) *The Statistics of Residuals and The Detection of Outliers*. NOAA Tech. Rep. NOS 65 NGS 1, US Department of Commerce, Rockville.

Rasch, D. and Mašata, O. (2006) 'Methods of variance component estimation', *Czech Journal of Animal Science*, 51(6), pp. 227–235.

Rao, C. R. (1970) 'Estimation of heteroscedatic variances in linear models', *Journal of the American Statistical Association*, 65, pp. 161-172.

Rao, C. R. (1971a) 'Estimation of variance and covariance components—MINQUE theory', *Journal of Multivariate Analysis*, 1(3), pp. 257-275.

Rao, C. R. (1971b) 'Minimum variance quadratic unbiased estimation of variance components', *Journal of Multivariate Analysis*, 1, pp. 445–456.

Rao, C. R. (1972) 'Estimation of variance and covariance components in linear models', *Journal of the American Statistical Association*, 67, pp. 112–115.

Rao, C. R. (1973) *Linear Statistical Inference and Its Applications.* John Wiley & Sons, Inc. New York, London, Sydney, Toronto.

Rao, C. R. (1979) 'MINQE theory and its relation to ML and MML estimation of variance components', *Sankhya: The Indian Journal of Statistics*, 41(B), pp. 138–153.

Rao, C. R. and Kleffe, J. (1988) *Estimation of variance components and Applications*. Vol 3, North-Holland, Amsterdam.

Scaioni, M., Barazzetti, L., Giussani, A. et al. (2014) 'Photogrammetric techniques for monitoring tunnel deformation', *Earth Sci. Inform*, 7(2), pp. 83–95. doi: 10.1007/s12145-014-0152-8.

Schaffrin, B. (1981) 'Best invariant covariance component estimators and its application to the generalized multivariate adjustment of heterogeneous deformation observations', *Bull. G'eod.*, 55, pp. 73–85.

Schaffrin, B. (1983) *Variance-Covariance Components Estimation in the Adjustment of Heterogeneous Repeat Measurements*. Dissertation, Universität Bonn. Deutsche Geodätische Kommission Reihe C, Heft 282, München.

Scholz, F. W. (2006) 'Maximum Likelihood Estimation', *Encyclopedia of Statistical Sciences*, pp. 1–13. doi: 10.1002/0471667196.ess1571.pub2.

Searle, S. R., Casella, G. and McCulloch, C. E. (1992) *Variance Components*. John Wiley & Sons, New York.

Setan, H. and Singh, R. (2001) 'Deformation analysis of a geodetic monitoring network', *Geomatica*, 55(3), pp. 1–9.

Setan, H. (2014) *Lecture notes for least squares adjustment computation*, Slide 1–34, Universiti Teknologi Malaysia (Unpublished).

Sjöberg, L. E. (1983) 'Unbiased estimation of variance-covariance components in condition adjustment with unknowns - A MINQUE approach', *Journal of Surveying*, 108(9), pp. 382–387.

Starplus Software (2000) *STARNET V6 Least Squares Survey Adjustment Program*. Reference Manual.

Teunissen, P. J. G. (1988) *Towards a least-squares framework for adjusting and testing of both functional and stochastic model*. Tech. Rep. 26, Mathematical Geodesy and Positioning Series. Delft University of Technology, Faculty of Civil Engineering and Geosciences, Department of MGP.

Teunissen, P. J. G. and Amiri-Simkooei, A. R. (2008) 'Least-squares variance component estimation', *Journal of Geodesy*, 82(2), pp. 65–82. doi: 10.1007/s00190-007-0157-x.

The Constructor (2017) *Electronic Distance Measurement Instrument - Types, Functions & Operations*. Retrieved November 9, from https://theconstructor.org/surveying/electronic-distance-measurement-instrument/6576/.

Tomás, R., Romero, R., Mulas, J. et al. (2014) 'Radar interferometry techniques for the study of ground subsidence phenomena: A review of practical issues through cases in Spain', *Environmental Earth Science*, 71, pp. 163–181.

Wang, J., Gopaul, N. and Scherzinger, B. (2009) 'Simplified algorithms of variance component estimation for static and kinematic gps single point positioning', *Journal of GPS*, 8(3), pp. 43–52.

Wentworlh, W. E. (1965) 'Rigorous least squares adjustment', *Journal of Chemical Education*, 42(2), pp. 96–103.

Xu, P., Shen, Y., Fukuda, Y. and Liu, Y. (2006) 'Variance component estimation in linear inverse ill-posed models', *Journal of Geodesy*, 80(2), pp. 69–81. doi: 10.1007/s00190-006-0032-1.

Xu, P. L., Liu, Y. M., Shen, Y. Z., & Fukuda, Y. (2007) 'Estimability analysis of variance and covariance components', *Journal of Geodesy*, 81: pp. 593–602. doi: 10.1007/s00190-006-0122-0.

Xu, P. and Liu, J. (2014) 'Variance components in errors-in-variables models: estimability, stability and bias analysis', *Journal of Geodesy*, 88(8), pp. 719–734. doi: 10.1007/s00190-014-0717-9.

Yavuz, E., Baykal, O. and Ersoy, N. (2011) 'Comparison of variance component estimation methods for horizontal control networks', *International Journal of the Physical Sciences*, 6(6), pp. 1317–1324.

Yi, T. H., Li, H. N. and Gu, M. (2013) 'Experimental assessment of high-rate GPS receivers for deformation monitoring of bridge', *Journal of the Intern. Meas. Confederation*, 46(1), pp. 420–432. doi: 10.1016/j.measurement.2012.07.018.

Yu, Z. C. (1996) 'A universal formula of maximum likelihood estimation of variance-covariance components', *Journal of Geodesy*, 70(4), pp. 233–240.

Yusoff, M. Y., Jamil, H., Halim, N. Z., Yusof, N. A. and Zain, M. A., (2013) 'Ekadaster: A Learning Experience for Malaysia', *FIG Pacific Small Island Developing States Symposium*, 18-20 September 2013, Suva, Fiji.

Zainal Abidin Md Som (2000) 'The implementation of variance components estimation (VCE) using MINQE approach into an adjustment of height networks ', *ISG 2005*, Batu Feringgi, Penang Malaysia.