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Mathematical Model for Steady State Subsea Cable Laying Problem

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Abstract Subsea cable laying process is a difficult task for an engineer due to many uncertain situations which occur during the operation. It is very often that the cable being laid out is not perfectly fit on the route being planned, which results in the formation of slack. In order to control wastages during installation, the slack needs to be minimized and the movement of a ship/vessel needs to be synchronized with the cable being laid out. The current problem was addressed using a mathematical model by considering a number of defining parameters such as the external forces, the cable properties and geometry. Due to the complexity, the model is developed for a steady-state problem assuming velocity of the vessel is constant, seabed is flat and the effect of wind and wave is insignificant. Non-dimensional system is used to scale the engineering parameters and grouped them into only two main parameters which are the hydrodynamic drag of the fluid and the bending stiffness of the cable. There are two solutions generated in this article; numerical and asymptotic solutions. The result of these solutions suggests that the percentage of slack can be reduced by the increase of the prescribed cable tension, and also the increase in either the drag coefficient of the sea water or the bending stiffness of the cable, similarly will result in lower slack percentage

Keywords Subsea cable laying; minimization of slack; steady-state problem

Mathematics Subject Classification 35A99, 00A71, 00A99

1 Introduction

Subsea cable laying is one of the important tasks for an engineer to install cables and pipelines under the sea [1]. This operation is done slowly by using ships that carry the submarine cable on board and based on the plans given by the cable operator.

Vessel of opportunity is used in the operation of submarine cable laying. The modular and easy transportation by commercial logistics for the tools used in this operation facilitate the working of vessel of opportunity. The mechanical cable engine is one of these modular tools. This engine is used to maintain the correct amount of speed and to residual tension for subsea laying cable

Depending on the sea conditions, the seabed in the area of cable laying, the equipment used in the operation and the typeof plough, the slack can be occurred when deploying the cable. The slack formation is one of the significant challenges that can be faced by the subsea cable laying where it will increase the possibility of laying too long cable along the route. Consequently, the detection of cable break location will be inaccurate and the cable break instances will increase as well (cable is laid not exactly on the route planned). The problem of uncontrolled slack leads also to the wastage of budget where the length of the cable laid is more than what is planned

To control the wastages, it is required to minimize the slacks and synchronize the movement of vessel with the cable being laid out. To achieve this, a mathematical model is developed in this article to determine the optimal speed of cable roller engine during the cable deployment and to control the residual tension of the cable. The model is solved numerically by using MATLAB software and analytically in a special case.

The model is developed for a steady-state case so as to get the final configuration of a cable during the laying down of the cable on the sea bed. This configuration of the cable can be used to guide the engineers to control the prescribed tension during the deployment of the cable from the vessel

The speed of cable roller engine is calculated by considering the speed of vessel lay, the vessel position and the depth of water. Since, for any sudden swell detected, a false slack might occurred. Therefore the pitch and roll of the vessel position is included in the model formulation to eliminate the false slack

2 The Mathematical Modelling

By wide increase of using the cables in deep-ocean around the world [2-6], determining the configuration of cables become an important parameters in the submarine cable laying process. In this article, a mathematical model is developed for the operation of subsea cable laying from a vessel under a steady-state condition as shown in Figure 1. Howison [7] discussed this problem by using the Euler–Bernoulli model for the displacement of a slender nearly straight beam.

The model is determined by the cable shape from the point at the vessel where the cable passes through a 'tensioner' to the touching point for the cables with the seabed. The initial tension T_0 of the cable is applied at the vessel. The arc-length (s) is assumed 0 and $-L$ at the vessel and seabed respectively (where L is the actual cable length).

The depth of the sea is denoted by h and $\theta(s)$ represents the angle between x-axis and the cable curve so that

$$\frac{dX}{ds} = \cos\theta, \quad \frac{dY}{ds} = \sin\theta, \quad \frac{d\theta}{ds} = \kappa, \tag{1}$$

where κ is the curvature. Assuming small elements of beam with length δs for each element as illustrated in Figure 2. The forces acting on the ends of the element are simply the internal elastic forces. The body force has components f_x and f_y per unit length in x and y direction

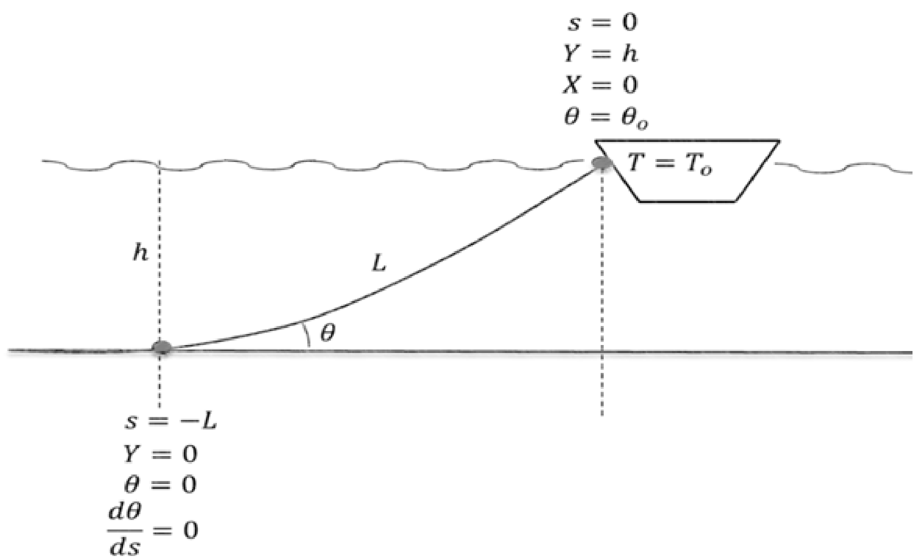

Figure 1: Layout of the Cable

respectively. The elastic forces at the ends of our elements are written as F_x and F_y in x and y direction respectively. Based on the equilibrium principal, the difference between the forces must be zero, and therefore, the equilibrium equations are

$$\begin{aligned} &\frac{dF_x}{ds} = f_x, \quad \frac{dF_y}{ds} = f_y \\ &\frac{dM}{ds} - F_x \sin\theta + F_y \cos\theta = 0, \end{aligned} \tag{2}$$

where M is the internal bending moment which is generated to balance the moment of internal forces.

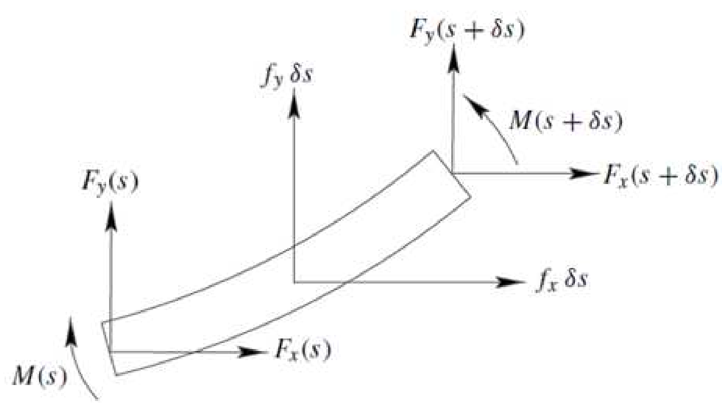

Figure 2: Forces and Moments on an Element of a Beam [1]

The external forces on the cable are the drag force, f_d and a buoyancy force, f_b, which are written as follows [7]:

$$\begin{aligned} &f_d = C_d A \rho_w U^2 \sin(\theta) \\ &f_b = gA\Delta\rho, \end{aligned} \tag{3}$$

where ρ_w is the sea water density, $\Delta\rho$ is the difference of the water density and cable, U is the object speed relative to the fluid, A is the cross sectional cable area, C_d is the drag coefficient, and g is the gravitational acceleration. In this model, the two forces f_d and f_b is considered to be equal f_x and f_y respectively.

The constitutive equation for a beam that is started moving from straight and bent into a curve is given as

$$\frac{dM}{ds} = EI\frac{d^2\theta}{ds^2}, \tag{4}$$

with EI is the cable bending stiffness. Based on Equations (2),(3) and (4), the governing equation for this particular problem can be written as:

$$EI\frac{d^2\theta}{ds^2} - F_x \sin\theta + F_y \cos\theta = 0 \tag{5a}$$

$$\frac{dF_x}{ds} = C_d A \rho_w U^2 \sin\theta \tag{5b}$$

$$\frac{dF_y}{ds} = gA\Delta\rho \tag{5c}$$

$$\frac{dX}{ds} = \cos\theta \tag{5d}$$

$$\frac{dY}{ds} = \sin\theta. \tag{5e}$$

The unknown parameters in Equations 4 and 5(a-e) are θ, F_x, F_y, X and Y with the following boundary conditions

$$\begin{aligned} &\text{at } s = 0;\ \theta = \theta_0,\ Y = 1,\ X = 0\ T = T_0 \\ &\text{at } s = -1;\ \theta = 0,\ \frac{d\theta}{ds} = 0,\ Y = 0. \end{aligned} \tag{6}$$

2.1 Non-dimensional Governing Equations

To simplify the governing equation, it is convertedinto a scaled or non-dimensionalised system. For example, consider Equation (5c), having a scale of

$$F_y \sim f_0\hat{F}_y, \quad F_x \sim f_1\hat{F}_x s \sim s_0\hat{s},\ f_0 \quad = f_1 = gAh\Delta\rho \text{ and } s_0 = Lh \tag{7}$$

where $\hat{F}_y$ and $\hat{s}$ are the non-dimensional variables, while f_0, f_1 and s_0 are the scaling factor so that

$$\frac{1}{L}\frac{d\hat{F}_y}{ds} = 1 \text{ and } \frac{1}{L}\frac{d\hat{F}_x}{ds} = \alpha\sin\theta. \tag{8}$$

Therefore, all Equations (5a–e), can be converted into the following dimensionless equations where the symbol ($\hat{}$) is removed to simplify the equation:

$$\begin{aligned}
&\frac{1}{L^2}\beta\theta_{,ss} - F_x\sin\theta + F_y\cos\theta = 0\\
&\frac{1}{L}F_{x,s} = \alpha\sin\theta\\
&\frac{1}{L}F_{y,s} = 1\\
&\frac{1}{L}F_{,s} = \cos\theta\\
&\frac{1}{L}X_{,s} = \cos\theta
\end{aligned}\tag{9}$$

in which

$$\begin{aligned}
\alpha &= \frac{C_d A\rho_w U^2}{\Delta\rho g A} \text{ (hydrodynamic drag) and}\\
\beta &= \frac{EI}{\Delta\rho g A h^3} \text{ (bending stiffness)}
\end{aligned}\tag{10}$$

(a subscript variable with a comma at the behined ($,s$) means a derivative with respect to the variable). Now, the dimensionless boundary conditions become

$$\begin{aligned}
&\text{at } s = 0\ \theta = \theta_0,\ Y = 1,\ X = 0\ T = T_0\\
&\text{at } s = -1;\ \theta = 0,\ \frac{d\theta}{ds} = 0,\ Y = 0
\end{aligned}\tag{11}$$

2.2 Numerical solution

The governing Equation (9)-(11) can be solved via a numerical approach except that the second order term makes the solution rather complicated. Consider an extra equation

$$\gamma = \frac{1}{L}\frac{d\theta}{ds} \Longrightarrow \theta_{,s} = L\gamma. \tag{12}$$

So, the governing equation becomes:

$$\begin{aligned}
&\gamma_{,s} = \frac{L^2}{\beta}\left(-F_x\sin\theta + F_y\cos\theta\right)\\
&F_{x,s} = L\alpha\sin\theta\\
&F_{y,s} = L\\
&X_{,s} = L\cos\theta\\
&Y_{,s} = L\sin\theta
\end{aligned}\tag{13}$$

This set of equations was solved numerically using MATLAB software as shown in appendix A

2.3 Asymptotic solution

The governing Equations in (9)-(11) can be further simplified by considering an asymptotic solution, for example in the case where the bending stiffness coefficient is very small so that the effect can be neglected. Consider the first three Equations in (9)

$$\beta\theta_{,ss} - F_x \sin\theta + F_y \cos\theta = 0, \tag{14a}$$
$$F_{x,s} = \alpha \sin\theta, \tag{14b}$$
$$F_{y,s} = 1 \quad \Longrightarrow \quad F_y = F_0 + s. \tag{14c}$$

If $\beta \ll 1$, then Equation (14a) becomes

$$F_x \tan\theta = F_y. \tag{15}$$

Substituting Equation (14c) into Equation (15) yields

$$F_x \tan\theta = F_0 + s \tag{16}$$

Differentiating both sides and rearranging them, leads to

$$F_{x,s} = -(F_0 + s)\, csc^2(\theta)\, \theta_{,s} + \cot\theta \tag{17}$$

By substituting Equation (14b) into Equation (17) gives

$$\begin{aligned} &\alpha \sin\theta = -(F_0 + s)\, csc^2(\theta)\, \theta_s + \cot\theta \\ &\Longrightarrow (F_0 + s)\, \theta_{,s} = -\alpha \sin^3\theta + \cos\theta \sin\theta. \end{aligned} \tag{18}$$

Let $s = L\zeta$ and suppose $F_0 = L$. Then Equation (18) can be transformed into

$$(1 + \zeta)\, \theta_{,\zeta} = -\alpha \sin^3\theta + \cos\theta \sin\theta. \tag{19}$$

Equation (19) is considered as the equation of slack with $\zeta = 1, \theta = 0$, and $\int_{-1}^{0} \sin\theta d\zeta = \frac{1}{L}$. Equation (19) is a separable equation where:

$$\begin{aligned} &(1 + \zeta)\frac{d\theta}{d\zeta} = -\alpha \sin^3\theta + \cos\theta \sin\theta \\ &\Longrightarrow \frac{1}{-\alpha \sin^3\theta + \cos\theta \sin\theta} d\theta = \frac{1}{1 + \zeta} d\zeta \end{aligned} \tag{20}$$

Integrate both sides of Equation (20) gives

$$\int_0^{\theta} \frac{1}{-\alpha \sin^3\theta + \cos\theta \sin\theta} d\theta = \int_{-1}^{\zeta} \frac{1}{1 + \zeta} d\zeta. \tag{21}$$

The right integral can be calculated directly by

$$\int_{-1}^{\zeta} \frac{1}{1 + \zeta} d\zeta = \log(1 + \zeta)_{-1}^{\zeta}. \tag{22}$$

To solve the left integral, a substitution of $u{=}\cos\theta$ is used

$$\begin{aligned}&\int_0^\theta \frac{1}{-\alpha\sin^3\theta{+}\cos\theta\sin\theta}d\theta{=}\int_0^\theta \frac{1}{\sin\theta(-\alpha\sin^2\theta{+}\cos\theta)}d\theta\\ &=\int_0^\theta \frac{1}{\sin\theta(\alpha\cos^2\theta{+}\cos\theta{-}\alpha)}d\theta=\int_0^\theta \frac{\sin\theta}{(1-\cos^2\theta)(\alpha\cos^2\theta+\cos\theta-\alpha)}\,d\theta \end{aligned}\tag{23}$$

Let $u=\cos\theta$, then $du=-\sin\theta\,d\theta$. Therefore

$$\begin{aligned}&\int_0^\theta \frac{\sin\theta}{(1-\cos^2\theta)(\alpha\cos^2\theta+\cos\theta-\alpha)}\,d\theta=\int_1^{\cos\theta}\frac{-1}{(1-u^2)(\alpha u^2+u-\alpha)}du\\ &=\int_1^{\cos\theta}\frac{-1}{(1-u)(1+u)(u+r_1)(u+r_2)}du\end{aligned}\tag{24}$$

where r_1 and r_2 are the roots of $\alpha u^2+u-\alpha=0$.

By using integration by partial fractions approach to solve the previous integral leads to

$$\frac{-1}{(1-u)(1+u)(u+r_1)(u+r_2)}=\frac{A}{1-u}+\frac{B}{1+u}+\frac{C}{u+r_1}+\frac{C}{u+r_2}\tag{25}$$

where A,B,C and D are constants, which can be solved analytically to obtain

$$A=\frac{-1}{2},\ B=\frac{1}{2},\ C=-\alpha-\frac{\alpha r_2-1}{r_1-r_2},\ D=\frac{\alpha r_2-1}{r_1-r_2}.\tag{26}$$

Then Equation (24) becomes:

$$\begin{aligned}&\int_1^{\cos\theta}\frac{-1}{(1-u)(1+u)(u+r_1)(u+r_2)}du=\int_1^{\cos\theta}\frac{A}{1-u}+\frac{B}{1+u}+\frac{C}{u+r_1}+\frac{C}{u+r_2}du\\ &=-\frac{1}{2}\log(1-u)+\frac{1}{2}\log(1+u)-\left(\alpha+\frac{\alpha r_2-1}{r_1-r_2}\right)\log(u+r_1)+\frac{\alpha r_2-1}{r_1-r_2}\log(u+r_2)\\ &=-\frac{1}{2}\log(1-\cos\theta)+\frac{1}{2}\log(1+\cos\theta)+C\log(\cos\theta+r_1)+D\log(\cos\theta+r_2)\\ &=\log\left[\frac{\sqrt{(1+\cos\theta)}\cos\theta+r_1)^C(\cos\theta+r_2)^D}{\sqrt{(1-\cos\theta}}\right]_0^\theta.\end{aligned}\tag{27}$$

Finally, by substituting back the values of integral from Equations (22) and (27) into Equation (21) gives

$$\log\left[\frac{\sqrt{(1+\cos\theta)}(\cos\theta+r_1)^C(\cos\theta+r_2)^D}{\sqrt{(1-\cos\theta}}\right]_0^\theta=\log[1+\zeta]_{-1}^{\zeta}.\tag{28}$$

Equation (28) relates the angle θ with arc cable length ζ . This equation has two singularities at 0 and -1. Subsequently, the space variable (X and Y) can be determined using Equations (5d) and (5e). Nonetheless, it is important to note that this asymptotic solution is only accurate when $\beta\ll 1$ as mentioned earlier.

3 Results and Discussion

The numerical solution developed by using MATLAB program for Equations (12) and (13) give the values of θ, F_x, F_y, X, Y and also L. For calculation purposes, and to let the analysis becomes easier, a presumed value is set to the two parameters α and β (without comparing to any engineering parameters). We only need to have a general idea about the cable behaviour as the changes of defining parameters.

Figure 1 shows the cable shape in 2D domain from the point (0,1) at the vessel to the point (-X,0) at the sea bed. The values of α and β are assumed to be equal 1, and the initial angle θ_0 is assumed 1 radian. The prescribed tension T_0 value is taken to be varied from 0 to 20 so we can observe the effect on the cable shape. It is found that higher tension makes the cable being suspended are longer. Although the high tension of the cable gives more control of the cable laying, the extremely high tension may lead to the cable damage

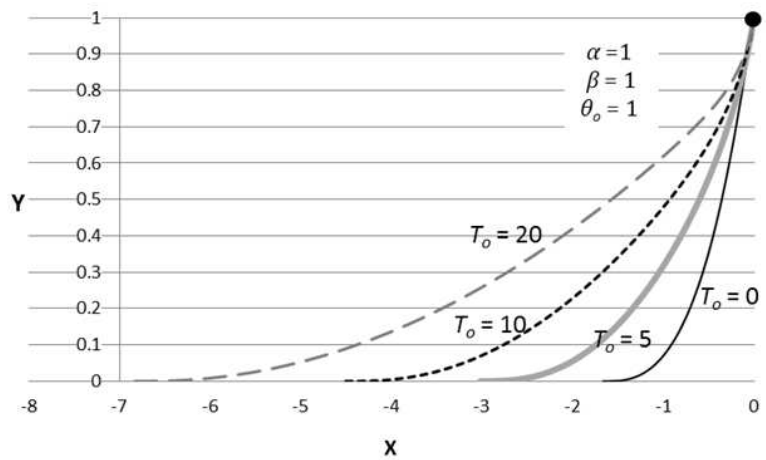

Figure 3: Shape of Cables in Different Prescribed Tension

In this particular study, the slack S is defined as the difference between the straight distance from (0,1) to $(-X, 0)$ and the actual cable length (L), as follows

$$S = \left(\frac{L}{\sqrt{X^2 - Y^2}} - 1 \right) \times 100\% \tag{29}$$

in which X is the maximum horizontal distance from the origin and Y is the sea depth which is 1. Figures 3–6 show the influence of different parameters on the slack percentage. It is noticed that increasing the controlled tension from the vessel reduce the percentage of the slack as discussed earlier. Also, the high tension makes the suspended cable becomes longer and this gives nearly straight shape, and consequently reduces slack as well. The other two figures show a similar result where increasing the coefficients gives lower slack percentages. For α, increasing the 'positive' drag force (in the normal direction to the curve) serves as an 'uplift' force to the cable. Thus we can avoid the cable to be further sagging. For β, it may explained due to the cable stiffness *EI*. By considering that the cable already has the curve shape, it is difficult at the higher rigidity of the cable to change its shape to a straight line unless higher tensile force is applied

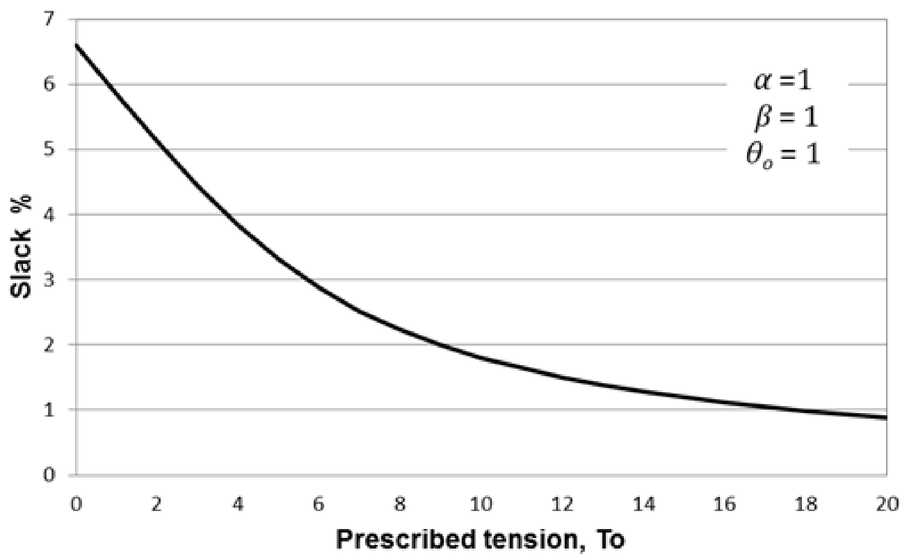

Figure 4: The Effect of Prescribed Tension on the Slack

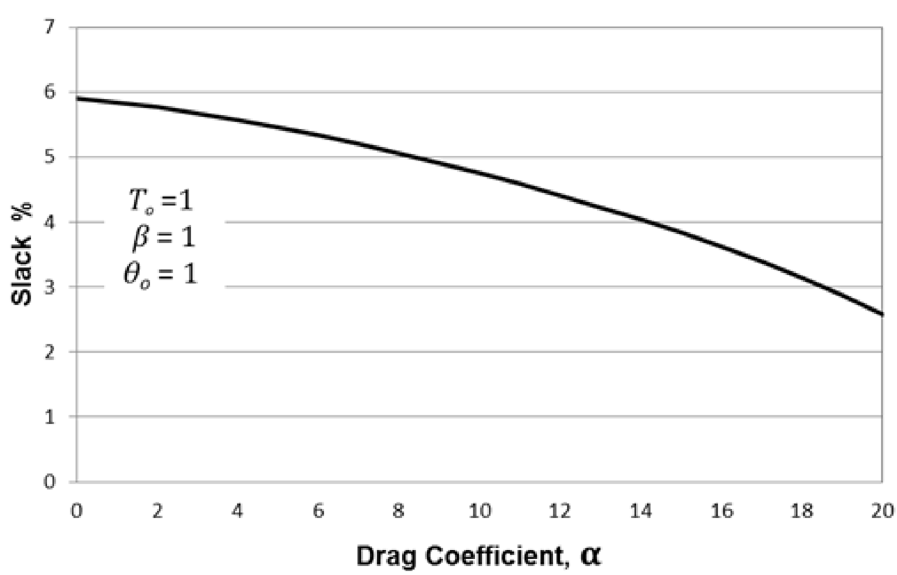

Figure 5: The Effect of Hydrodynamic Drag on the Slack

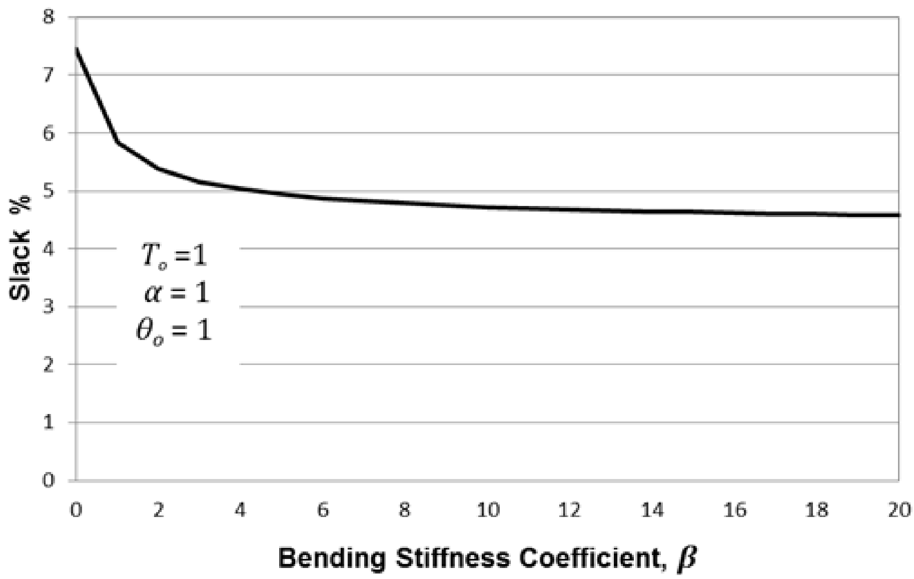

Figure 6: The effect of Cable Bending Stiffness on the Slack

4 Conclusion

This article provides a mathematical model for a steady-state subsea cable laying problem based on Euler–Bernoulli equation derived by Howison [7]. This equation can predict the layout of the laying cable at a particular time by knowing the fluid well as the fluid behavior. The model assumes that the vessel velocity is constant and as the seabed is flat. The model neglect the effect of waves and wind. The non-dimensional set of equations that are developed for the model leads to only two defining parameters, which are basically related to the bending stiffness of the cable and hydrodynamic drag of the fluid

The governing equations are solved by numerical solution conducted in MATLAB program, and also by asymptotic solution obtained via analytical calculation. The results show the cable shape in 2D domain. It is also found that increasing the prescribed cable tension can reduce the slack percentage. Moreover, increasing the drag coefficient or bending stiffness parameter can also reduce the slack percentage as well. The model results are good enough to extend it to consider a time-dependent problem, in which the effect of waves and seabed surface shape are now significant

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Appendix A
MATLAB Coding for Subsea Cable Laying Solution

```
function [l,Xn] = cable2a()
% Full cable solution

fn = 'cableIFactor';

% PARAMETERS
alf = 1;        % hydrodynamic drag
bet = 0.01;       % bending stiffness
T0 = 1;           % tension
kap = -0;         % curvature at boat
thetastar = 1;    % angle at boat (alternative bc)
h = 1;            % water depth


% INITIAL GUESS FOR SOLUTION
x_guess = linspace(0,1,100);
y_guess = [0; 0; 0; 0; 0; 0];
l_guess = 1;
solinit = bvpinit(x_guess,y_guess,l_guess);
solinit.y(1,:) = atan(solinit.x+1); % approximate solution
solinit.y(2,:) = (1+(solinit.x+1).^2).^(-1);
sol = solinit;


% SOLVE
options=bvpset('Vectorized','on','FJacobian',@fjac2,'NMax',10000);
sol = bvp4c(@geq2,@gbc2,sol,options);

l = sol.parameters;
s = sol.parameters*sol.x;
theta = sol.y(1,:);
Fx = sol.y(3,:);
Fy = sol.y(4,:);
X = sol.y(5,:);
Y = sol.y(6,:);


% PLOT
figure(1); clf;
    plot(s,theta,s,Fx,s,Fy);
    plot(s,theta);
    xlabel('x'); ylabel('y');
    l0 = (2*T0-1).^(1/2);
    hold on; plot(s,atan((s-s(1))./(T0.^2-l0.^2).^(1/2)),'k--');
```

```
    l0 = (2*T0-1).^(1/2);
    hold on; plot(s,atan((s-s(1))./(T0.^2-l0.^2).^(1/2)),'k--');
% asymptotic solution for alpha = 0
figure(2); clf;
    plot(X,Y);

save(fn);

% PLOT
figure(2); clf;
set(gcf,'Paperpositionmode','auto','units','centimeters','position',[2  2
20 10]);
    plot(sol.y(5,:),sol.y(6,:),'linewidth',2);
    xlabel('x'); ylabel('y');
    ylim([0 1]);
    shg;

% print(gcf,'-depsc2',fn,'-loose');



%% SUBFUNCTIONS %%

% BOUNDARY CONDITIONS
function res = gbc2(ya,yb,l)
    res(1,:) = ya(1,:);
    res(2,:) = ya(2,:);
%     res(3,:) = yb(2,:)-kap;
    res(3,:) = yb(1,:)-thetastar;  % ( alternative bc )
%     res(4,:) = yb(3,:).*cos(yb(1,:))+yb(4,:).*sin(yb(1,:))-T0;
    res(4,:) = yb(3,:)-T0;  % (alternative bc )
    res(5,:) = ya(6,:);
    res(6,:) = yb(6,:)-h;
    res(7,:) = yb(5,:);
end



% JACOBIAN OF ODE SYSTEM
function [dfdy,dfdp] = fjac2(x,y,l)
    theta = y(1,:);
    psi = y(2,:);
    Fx = y(3,:);
    Fy = y(4,:);
    X = y(5,:);
    Y = y(6,:);

    dfdy = [0 1 0 0 0 0;
            l./bet.*(  Fx.*cos(theta)  + Fy.*sin(theta)  ) 0 l./bet.*
              ( sin(theta)  ) l./bet.*(  -cos(theta)  ) 0 0;
            l.*alf.*cos(theta)  0  0  0  0  0 ;
            0  0  0  0  0  0 ;
```

```
    dfdp = [psi;
            1/bet.*( Fx.*sin(theta) - Fy.*cos(theta) );
            alf*sin(theta);
            1;
            cos(theta);
            sin(theta)];
end


% ODE SYSTEM
function dydx = geq2(x,y,l)
    theta = y(1,:);
    psi = y(2,:);
    Fx = y(3,:);
    Fy = y(4,:);
    X = y(5,:);
    Y = y(6,:);

    dydx(1,:) = l.*psi;
    dydx(2,:) = l./bet.*( Fx.*sin(theta) - Fy.*cos(theta) );
    dydx(3,:) = l.*alf*sin(theta);
    dydx(4,:) = l.*1;
    dydx(5,:) = l.*cos(theta);
    dydx(6,:) = l.*sin(theta);
end

end
```