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RESEARCH ARTICLE

Intuitionistic L-fuzzy set and intuitionistic N-fuzzy set

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Abstract

In this paper we prove that the concept of intuitionistic N-fuzzy sets (briefly INFS) proposed by Akram et al in [5] is equivalent to ILFS and it's not a generalization of IFS. We concluded that IFS, ILFS, INFS and L-fuzzy sets are all equivalent.

***Keywords*:** Fuzzy Set; L-Fuzzy Set; Intuitionistic fuzzy sets; Intuitionistic L-fuzzy sets; Intuitionistic N-fuzzy sets

INTRODUCTION

In 1965, Zadeh introduced the concept of fuzzy sets in his classical paper and two years later Goguen introduced the idea of L-fuzzy sets as the generalization of Zadeh's fuzzy sets in 1967. In another direction, Atassanov and Stoeva introduced another fuzzy object called Intuitionistic Fuzzy Sets (IFS) and Intuitionistic L-Fuzzy Sets ILFS) in 1983 and 1986, as the generalization of both Fuzzy set and L-fuzzy sets.

In 2000, G.J Wang and Y.Y. He shows that intuitionistic fuzzy sets, intuitionistic L-fuzzy sets and L-fuzzy sets are equivalent. In this paper we prove that the concept of intuitionistic N-fuzzy sets (briefly INFS) proposed by Akram et al in [5] is indeed equivalent to ILFS and it's not a generalization of IFS and we concluded that IFS, ILFS, INFS and L-fuzzy set are all equivalent

The rest of this paper is organized as follows. Prelimenaries briefly review some related literature. Next we state and proves two theorems and a corollary which show that Intuitionistic N-fuzzy set (INFS) can be transform to Intuitionistic L-fuzzy set (ILFS) and IFS, ILFS, INFS and L-fuzzy set are all equivalent. Finally,we gives some conclusion

PRELIMENARIES

Here we present some fundamental definitions that are needed.

Definition 1. [1] Let L be a non-empty partially ordered set

I. If $x \vee y$ and $x \wedge y$ exist for all $x, y \in L$, then L is called a **lattice.**
II. If $\wedge S$ and $\vee S$ exist for all $S \subseteq L$, then L is called a **complete lattice.**

Definition 2. [2] Let X be a collection of objects, with a generic element of X denoted by x. A Fuzzy set F in X is characterized by a membership function $\mu_F : X \to [0,1]$, with the value of $\mu_F(x)$ representing the grade of membership of x in F.

Definition 3. [3] Let X be a non-empty crisp set and let L be a complete lattice. An L-Fuzzy set, on X, is a mapping:

$$A: X \longrightarrow L$$

If the poset L correspond with the interval $[0,1]$, we obtain the definition of fuzzy set.

Definition 4. [4] Let E be a non-empty crisp set. An IFS A^* in E is an object having the form

$$A^* = \{\langle x, \mu_A(x), v_A(x)\rangle : x \in E\},$$

where $\mu_A: E \longrightarrow [0,1]$ and $\nu_A: E \longrightarrow [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in E$ to $A \subset E$, respectively, and

$$(\forall x \in E)(0 \leq \mu_A(x), +\nu_A(x) \leq 1),$$

Definition 5. [4] Let E be a non-empty crisp set. An ILFS A^* in E is an object having the form

$$A^* = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in E\},$$

where $\mu_A: E \longrightarrow L$ and $\nu_A: E \longrightarrow L$ satisfying the condition

$$(\forall x \in E)(\mu_A(x) \leq N(\nu_A(x)),$$

where N is the order-reversing involution on L.

The authors of [5] used the idea of negative fuzzy set (briefly N-fuzzy set) to proposed the concept of intuitionistic N-fuzzy sets

Definition 6. [5] An intuitionistic N-fuzzy set (INFS) A in a non-empty set X is an object of the form

$A^* = \{\langle x, \bar{\mu}_A, \gamma_A\rangle : x \in X\}$, where $\bar{\mu}_A: X \longrightarrow [-1, 0]$ and $\gamma_A: X \longrightarrow [-1, 0]$ such that $-1 \leq \bar{\mu}_A(x), +\gamma_A(x) \leq 0$ for all $x \in X$.

THE EQUIVALENCE

Here we state and proves two theorems and a corollary which show that Intuitionistic N-fuzzy set (INFS) can be transform to Intuitionistic L-fuzzy set (ILFS) and IFS, ILFS, INFS and L-fuzzy set are all equivalent

Theorem 1. The concept of Intuitionistic N-fuzzy set (INFS) and Intuitionistic L-fuzzy set (ILFS) are equivalent.

Proof. Recall that a fuzzy set A^* in E is an object having the form

$$A^* = \{\langle x, \mu_A(x)\rangle : x \in E\}, \quad (1)$$

In other word (1) means a corresponding

$$(\forall x \in E)(x \longmapsto \mu_A(x))$$

In general case, it can be written as $f: E \longrightarrow [0, 1] \ni \; f = \{\langle x, \mu_f(x)\rangle : x \in E\}$. The function $f': E \longrightarrow [0, 1]^2$ can be written as

$$f' = \{\langle x, \mu_f(x), \nu_f(x)\rangle : x \in E\}$$

whereby $\mu_f(x)$ and $\nu_f(x)$ are the first and second coordinates of $f^*(x)$ in $[0, 1]^2$, respectively

Now when f^* is restricted with the condition $(\forall x \in E)(0 \leq \mu_A(x), +\nu_A(x) \leq 1)$, then f^* will be intuitionistic fuzzy set (IFS)

The set, $[0, 1]^2$ can be replaced by $[-1, 0]^2$ or simply $[0, 1]$ can be replaced by $[-1, 0]$, by using a function $t: [0, 1] \longrightarrow [-1, 0] \ni t(x) = x - 1$ which is one to one and onto. Hence $[0, 1] \cong [-1, 0]$.

Furthermore, $[-1, 0]$ is a lattice since we can show that it is partially ordered set (poset) in the form of $([-1, 0], \leq)$ and every $x, y \in [-1, 0]$ has a least upper bound and a greatest lower bound. Therefore, we can introduce $f^{**}: E \longrightarrow [0, 1]^2$ and can be written as

$$f^{**} = \{\langle x, \mu_f(x), \nu_f(x)\rangle : x \in E\}$$

When f^* is restricted with the condition$(\forall x \in E)(-1 \leq \mu_A(x), +\nu_A(x) \leq 0)$, then f^{**} will be intuitionistic N-fuzzy set (INFS).

In other words, INFS can be transformed in to IFS. Without loss of generality, INFS ≅ IFS and by [6] INFS ≅ ILFS ■

Corollary. Intuitionistic N-fuzzy set is equivalent to L-fuzzy set.

Proof. Observe that by the theorem above INFS ≅ ILFS, and by the theorem 2 in [6] we have ILFS ≅ ILFS. Therefore, INFS ≅ L-fuzzy set.

Theorem 2. The concept of IFS, ILFS, INFS and L-fuzzy set are all equivalent.

Proof. By the theorem 1, corollary and theorem 2 of [6], IFS, ILFS, INFS and L-fuzzy set are equivalent.■

By studying the properties of IFS, ILFS and INFS, we will obtain properties similar to that of L-fuzzy set, which is not surprising since there are all equivalent. Therefore that INFS it is not a generalization of IFS as suggested by [5]

CONCLUSION

In this paper, we state and proved a theorem which shows that INFS is equivalent to ILFS, further, using the theorem and theorem 2 in [6], we proved that IFS, ILFS, INFS and L-fuzzy set are all equivalent

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