FINITE ELEMENT METHOD FOR FLUID FLOW OF MODELLING OF THE DESCEMET MEMBRANE DETACHMENT AND RHEMATOGENEOUS RETINAL DETACHMENT

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To my beloved

father, mother,

brother, sisters

&

my supervisors,

Assoc. Prof. Dr. Sharidan Shafie,

and

Dr. Zuhaila Ismail.

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ABSTRACT

A numerical computation of the ocular fluid mechanics incorporate with Descemet membrane detachment (DMD) and Rhemathogeneous retinal detachment (RRD) were presented. DMD and RRD are serious diseases of the human eye that might cause vision impairment and blindness.The developing mechanisms of both diseases are related to the fluid flow in the anterior chamber (AC) and vitreous cavity (VC) of the human eye. However, the measurement of the fluid data in the human eye is difficult due to the small size of the eye and extremely low velocity of the fluid flow. Therefore, mathematical models were proposed to investigate the fluid flow in the human eye with DMD and RRD respectively. The aqueous humour (AH) in the AC and the vitreous humour (VH) in the VC have similar properties to water. Thus the AH and VH were assumed as an incompressible Newtonian fluid. Navier-Stokes equations were applied to govern the fluid flow in AC and VC. In AC, the AH flow was driven by the buoyancy forces. The AH adjacent to the iris was heated and eventually rises as buoyant flow. While, the AH adjacent to the cornea becomes colder and heavier that it flows down in the direction of the gravitational force. In VC, the VH flow was induced by the movement of the eye. Finite element method was used to solve this problem. A source code (MATLAB) was constructed to run the iterative numerical procedure, to determine the approximate solutions and to display the results graphically. The results show that the circulation of the AH in the AC was affected by the types and location of the DMD. Additionally, change in the orientations (the direction of the gravitational force) and the shape (the area) of the AC do change the characteristics of the AH flow. On the other hand, the characteristic of the VH flows in the VC were dependent on the pattern and seriousness of the RRD. The VH flow in the VC were dominantly driven by the saccadic eye movement. It is observed that, the scale and location of the sclera buckling were the important factors that affect the outflow of the subretinal fluid flow into the VC.

ABSTRAK

Pengiraan berangka bagi mekanik bendalir okular yang melibatkan penanggalan membran Descemet (DMD) dan penanggalan retina jenis Rhegmatogenous (RRD) dipersembahkan. DMD dan RRD merupakan penyakit mata manusia yang serius berkemungkinan boleh menyebabkan gangguan penglihatan dan kebutaan. Mekanisme melaratnya kedua-dua penyakit ini berkait rapat dengan aliran bendalir di dalam *aqueous chamber* (AC) dan *vitreous chamber* (VC) mata manusia. Namun, pengukuran data bendalir di dalam mata manusia adalah sukar kerana saiz mata yang kecil dan halaju aliran bendalir yang amat rendah. Oleh itu, model matematik telah dicadangkan untuk menyelidik aliran bendalir di dalam mata manusia, masing-masing dengan DMD dan RRD. *Aqueous humour* (AH) dalam AC dan *vitreous humour* (VH) dalam VC mempunyai ciri-ciri yang serupa dengan air. Oleh itu AH dan VH diandaikan sebagai bendalir Newtonan tidak termampat. Persamaan Navier-Stokes telah digunakan untuk mentadbir aliran bendalir dalam AC dan VC. Dalam AC, aliran AH didorong oleh daya keapungan. AH bersebelahan dengan iris dipanaskan dan akhirnya akan meningkat sebagai aliran apung. Manakala, AH bersebelahan dengan kornea menjadi lebih sejuk dan lebih berat sehingga kemudiannya akan mengalir mengikuti arah daya graviti. Dalam VC, aliran VH diaruh oleh pergerakan mata. Kaedah unsur terhingga digunakan untuk menyelesaikan masalah ini. Satu kod sumber (MATLAB) telah dibina untuk melaksanakan prosedur berangka secara lelaran, bagi menentukan penyelesaian penghampiran dan memaparkan keputusan secara grafik. Keputusan menunjukkan peredaran AH di dalam AC dipengaruhi oleh jenis dan lokasi DMD. Di samping itu, perubahan orientasi (arah daya graviti) dan perubahan bentuk AC (keluasan) didapati mengubah ciri-ciri aliran AH. Sebaliknya, ciri-ciri aliran VH di dalam VC adalah bergantung kepada corak dan keseriusan RRD. Aliran VH di dalam VC didominasi oleh pergerakan mata sakadik. Diperhatikan bahawa, skala dan lokasi sklera melengkok adalah faktor penting yang mempengaruhi pengaliran keluar bendalir *subretinal* masuk ke dalam VC.

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LIST OF SYMBOLS

A_{e_i} - surface area of a finite element

AC - Anterior Chamber

AH - Aqueous Humour

CS - control surface

CV - control volume

DOF - degree of freedom

DM - Descemet’s Membrane

DMD - Descemet’s Membrane Detachment

e - internal energy

e_i - element i

E - energy per unit mass

$\vec{F}_{body}$ - body force

$\vec{F}_{surface}$ - surface force

$F\left(\vec{x},t\right)$ - extensive property of a system

$f\left(\vec{x},t\right)$ - extensive property of a system per unit volume

$\vec{g}$ - gravitational force

h - specific enthalpy

k - thermal conductivity

m - mass

$\vec{n}$ - normal unit vector

$N_j^{e_i}$ - shape function of 1D line element

P - pressure

P_a - normal atmospheric pressure

P_h - hydrostatic pressure

P_m - mean/dynamic pressure

Pr - Prandtl number

$\vec{q}$ - heat flux vector

Q - internal heat source

RTT - Reynold Transport Theorem

RD - Retinal Detachment

RRD - Rhematogeneous Retinal Detachment

SB - Scleral Buckling

t - time

T - temperature of the fluid

T_p - temperature at the iris

T_c - temperature of the cornea

T_∞ - temperature of the ambient medium

U - a typical velocity in human eye

U_0 - a constant velocity

U_w - a velocity at the wall

u - velocity component of fluid phase in x-direction

v - velocity component of fluid phase in y-direction

$V(t)$ - material volume

VH - Vitreous Humour

VC - Vitreous Cavity

w - velocity component of fluid phase in z-direction

$w_m(x)$ - weight function

w_i - weight of Gauss Quadrature

μ - dynamic viscosity coefficient

υ - kinematic viscosity coefficient

$\sigma_{i\,j}$ - stress tensor

$\tau_{i\,j}$ - viscous stress tensor

τ - tolerance value

$\varepsilon_{i\,j}$ - strain rate tensor

β - thermal expansion

ρ - density of the fluid

$\psi_k^{e_i}$ - shape function for 2D linear element

$\phi_j^{e_i}$ - shape function for 2D quadratic element

$\delta_{i\,j}$ - Kronecker Delta

Γ_{e_i} - boundary curve of a finite element

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CHAPTER 1

INTRODUCTION

1.1 Introduction

The eye is an organ that enables a human being to see. The eye helps the human being to understand and interpret the world around them. Descemet’s membrane detachment (DMD) and Rhematogeneous retinal detachment (RRD) are diseases of the eye that might cause blindness. The formation of both diseases involves the fluid flow inside the eye. However, the small dimension of the human eye and the extremely low velocities of the ocular fluid flow make it difficulties for in-vivo study on the human eye. Therefore, many aspects of fluid flow within the human eye have not yet been fully examined or quantitatively explained. Alternative, computational simulation of the ocular fluid flow is applied to understand the flow mechanisms in the human eye, especially when the eye has DMD and RRD. In this chapter, the background of the problem is explained. Then, the statement of the problem and the objectives of this research are highlighted. Some limitations and scopes of the research are also stated here. Finally, the significance of the study and the outline of the thesis are presented.

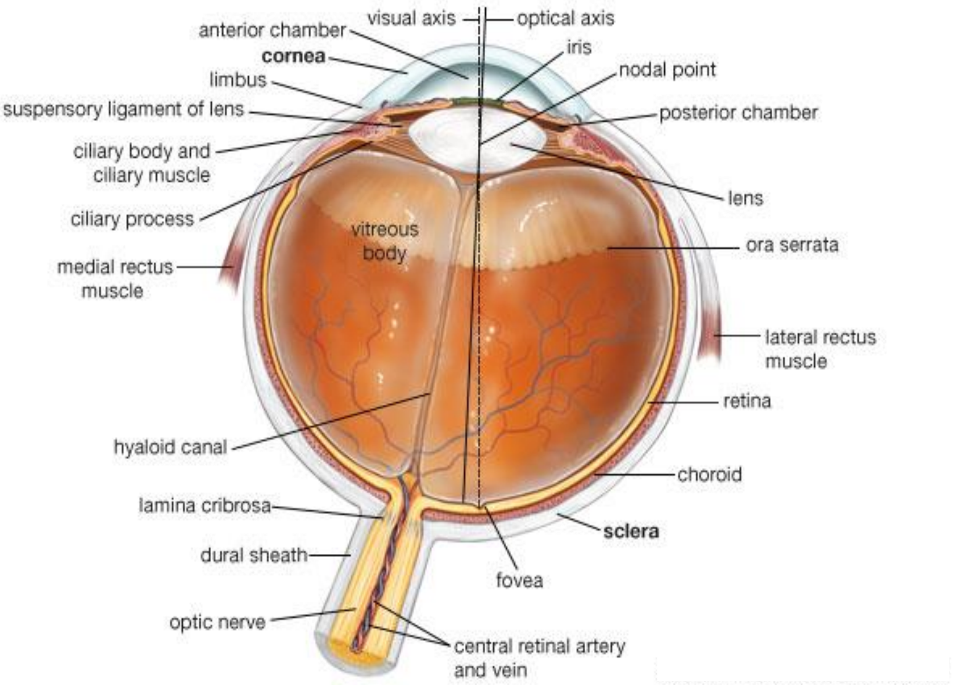

Figure 1.1 Structure of Human Eye (Kara, 2011).

1.2 Research Background

The clear dome like surface that exists at the front of a human eye is called as the cornea. The cornea covers the iris and the pupil, as shown in Figures 1.1. The region bounded by the cornea, the iris and the pupil is known as the anterior chamber (AC). There are three main layers and two auxiliary layers contained in the cornea, those are the epithelium, the stroma, the endothelium, the Browman layer and the Descemet membrane (DM). DM is a layer lies between the stroma and the endothelium layer of the cornea. DMD happens when DM is separated away from the stroma by the aqueous humour (AH) that flows into the space between the DM and the stroma through a tear or break on the DM. The detachment of DM may have serious adverse effects to the vision function of the human eye. Planar or non-planar, scrolled or non-scrolled and peripheral or non-peripheral with central corneal involvement are the types of DMD (Mulhern *et al.*, 1996; Menezo *et al.*, 2002; Potter and Zalatimo, 2005).

DMD due to cataract surgery, iridectomy, trabeculectomy, corneal transplantation, deep lamellar keratoplasty, holmium laser sclerostomy, alkali burn and viscocanalostomy have been reported by Mulhern *et al.* (1996), Potter and Zalatimo (2005), Hirano *et al.* (2002) and Ünlü and Aksünger (2000). Sevillano *et al.* (2008) reported the technique of curing of DMD caused by cataract surgery with sulphur hexafluoride injection. Potter and Zalatimo (2005) presented the case of treating scrolled DMD by injecting fourteen percent of intracameral perfluoropropane (C_3F_8) into AC.

Recently, Couch and Baratz (2009), investigated two cases of delayed bilateral DM and in one eye it was fixed surgically and the other eye improved spontaneously. They estimated that the spontaneous reattachment happens because of the buoyancy effects which cause the AH flow in AC. The spontaneous reattachment of DM has been supported by some observational and anecdotal evidences (Marcon *et al.*, 2002; Nouri *et al.*, 2002; Couch and Baratz, 2009; Ismail *et al.*, 2012). Fitt and Gonzalez (2006) had shown that under normal conditions the buoyancy effects due to the temperature gradient in AC enhance the AH to flow. Ismail *et al.* (2012) intended to explain the phenomena of the spontaneous reattachment and thus, developed a mathematical model to describe the AH flow in AC with DMD. They concluded that the temperature difference across the eye and the orientation of the patient may control the clinical outcomes for the DMD. Nonetheless, the model developed by Ismail *et al.* (2012) was based on the lubrication theory limit, which include a lot of simplification. Consequently, the model may only partially illuminate the behaviour of the fluid flow in AC. Therefore, in this thesis, fluid mechanical theory is applied to model the fluid flow in AC, in order to study the effect of the detached DM on the behaviour of the AH flow in AC.

DMD is categorized as planar or non-planar, scrolled or non-scrolled and peripheral or non-peripheral. Mackool and Holtz (1977) reported that non-planar detachments were difficult to reattach spontaneously compared to planar DMD. Assia *et al.* (1995) concluded that non-scrolled and non-planar DMD might be able to reattach spontaneously. And, Marcon *et al.* (2002) claimed that non-planar and non-scrolled DMD often reattach spontaneously if given enough time. These imply that the type of DMD may affect the AH flow, thus influencing the occurrence of the spontaneous reattachment of DMD in AC. Therefore, a model is developed is used to analyse the effect of different types of DMD to the fluid flow in AC. As, Amini and Barocas (2009) had shown that cornea indentation changed the contour of the iris and altered the AC angle, which is the angle between the iris and the cornea. Hence, it is possible for the indentation of cornea to affect the AH flow, which is driven by the temperature difference in the AC of human eye. Thus, the effects of the cornea indentation of the AH flow dynamics are investigated as well. In addition, the DMD problem during the indentation is also discussed in this study.

Retinal detachment happens in human eye when retina peels away from the cellular layer, which is called choroid and located between the retina and the sclera (Cook *et al.*, 1995; Dyson *et al.*, 2004; Daniel and Wilkinson, 2009; Ismail, 2013). Depending on the mechanism of subretinal fluid accumulation, retinal detachment can be categorized into Rhegmatogenous, Exudative and Tractional (Dyson *et al.*, 2004; Daniel and Wilkinson, 2009; Ismail, 2013). Rhegmatogenous Retinal Detachment (RRD) may occur when the liquefied vitreous humour (VH) flow through a tear or break into the space between the retina and the retinal pigment epithelium. Consequently, the patient will become permanently blind in the affected eye because the detached retina lost the ability to function. Exudative retinal detachment and Tractional retinal detachment happen without a retina break. Exudative retinal detachment occurs when retina is pulled away by subretinal fluid which produced by a tumour or an inflammatory disorder. The retinal detachments caused by proliferative diabetic retinopathy, cicatricial retinopathy of prematurity, proliferative sickle retinopathy and penetrating trauma are called as Tractional retinal detachment.

Retina, sclera and choroid are the main layers of human eyes as shown in Figure 1.1. Retina is the innermost layer of these three layers. Sclera is the outermost layer of the human eye and choroid is the intermediate layer. Retina is one of the most important parts of the human eye and may be considered to be a part of the brain. It is a complex nervous structure that is responsible for the visual process. Retina has three layers of nerve cells and two layers of synaptic connections (Davson, 1980; Kolb, 2003; Ismail, 2013). The nerve cell layers are photoreceptors, inner nuclear and ganglion cells. The photoreceptor layer is the outermost layer in the retina against the pigment epithelium and the choroid, and consists of the sensitive layer of rods and cones. The nuclear layer, which consists of 1 to 4 types of horizontal cells, 11 types of bipolar cells and 22 to 30 types of amacrine cells, lies between the photoreceptors and the ganglion cells. Then, in the innermost layer of the retina contains 20 types of the ganglion cells (Pirenne, 1967; Davson, 1980; Kolb, 2003).

Retina is also an important part in the visualization process because in the centre of the retina is a region with the highest visual acuity and it is named as fovea (Pirenne, 1967; Davson, 1980). When light passes through the cornea and the pupil to the lens, then it will be focused onto the fovea at the retina. To reach the photoreceptors, the light has to travel along the thickness of the retina. When the photoreceptors of the human eye receive the light, the light energy is transduced into electrical signals. Then, the electrical signals transmit back to the amacrine and the ganglion cells and the signals are propagated to the optic nerve by the axons of the ganglion cells. The optic nerve takes the ganglion cell axons to the brain for further visual processing (Pirenne, 1967; Davson, 1980; Kolb, 2003; Ismail, 2013).

RRD happens when the liquefied VH flows through a tear or break on the surface of the retina into the space between the retina and the retinal pigment epithelium (Daniel and Wilkinson, 2009; Ismail, 2013). The movement of the fluid in the subretinal space will pull the retina away from the choroid. To realize the mechanisms of the formation of the RRD, a number of researchers investigated the RRD problem clinically. Fatt and Shantinath (1971) reported that the RRD happen when there exist a hole in the retina. The mechanisms of the subretinal fluid in the pigment epithelium were discussed by Quintyn and Brasseur (2004). The recent trends in the managements of RRD can be found in the report prepared by Sodhi *et al.* (2008). However, few researchers studied the RRD problem mathematically. Gonzalez (2004) modelled the retinal detachment problem mathematically by assuming the detached retina as a free surface and the surface tension effect was taken into account. The effect of shear stress was neglected. In the study, the author found that the surface tension affect the progress of the retinal detachment.

More recently, Ismail (2013) considered the effect of pressure gradient to the fluid flow in the detached retina. A paradigm mathematical model was developed and solved by asymptotic method. The behaviour and the deformation of the detached retina were analysed. So far, some pioneering experimental and analytical researches had been performed, but the findings are short of data for the movements of liquefied vitreous in the RRD. Besides that, the results obtained in the previous researches were probably incomplete. In those researches, assumptions were made to simplify the governing equations, so that the analytical solutions can be determined. Thus, the obtained results cannot fully represent the actual fluid flow in the human eye with RRD. In this research, full two dimensional Navier-Stokes equations are used to represent the fluid flow in VC with RRD. Finite element method is applied to obtain the numerical solutions and used to analyse the complex flow field in the eye with a detached retina.

There are various treatments for RRD and the most common treatment for uncomplicated RRD is scleral buckling (SB) (Sodhi *et al.*, 2008; Ismail, 2013). A SB is a piece of fascia lata, palmaris trendon, semi-hard plastic, silicone sponge, rubber, or donor sclera that ophthalmologist places on the outside of the sclera or the white of the eye to treat the RRD. Silicone is the most popular material used to make the sclera buckle because silicon is nontoxic and nonallergenic (Daniel and Wilkinson, 2009). To relieve the traction on the retina, the SB will be sewn within the sclera. Therefore, the retinal tear will settle against the wall of the eye, so that the chorioretinal adhesions can be formed to seal the retinal break (Daniel and Wilkinson, 2009; Ismail, 2013).

SB has gained attention of many researchers due to its popularities. The anatomic and visual results in an asymptomatic clinical RRD repaired by SB were reported by Greven *et al.* (1999). Foster *et al.* (2010) found that rapid eye movements facilitate more rapid retinal reattachment. They have used a commercial software, the COMSOL Multiphysics to simulate the influence of the SB on the flow of subretinal fluid in a physical model of retinal detachment. The physical principles behind the SB surgery were analysed. Motivated by the above study, in the present work, the effect of the SB on the movement of liquefied VH in the subspace between the detached retina and choroid is investigated.

1.3 Statement of the Problem

As explained earlier, the flow data inside AC and VC are difficult to obtain due to the small scale of both chambers. In addition, the complexities of flow measurements inside a living eye also are obstacles for obtaining the data. As an alternative method, computational simulation of the flow can be very useful in producing the needed understanding of the flow mechanisms inside the AC and VC. In this way, the behaviour of the fluid flow when the human eye has DMD and RRD diseases can be comprehended. Further, most of the studies in the literature were conducted by using perturbation or asymptotic methods and were incapable to solve the complex fluid flow model used to describe the fluid flow in the AC with DMD and the VC with RRD respectively. Numerical method is needed. Therefore, finite element method is proposed in this study to obtain the numerical results of the flow. Although the DMD is a rare ocular problem, it may induce the loss of vision. The influence of the type of DMD, the cornea indentation and the position of the patient, to the flow of AH in AC may improve the occurrence of the spontaneous reattachment phenomena. Apart from that, as the SB treatment is the most common way to treat RRD, the response of the liquefied VH to SB is valuable in order to increase the success rate of the treatment.

Some questions need to be answered in order to accomplish this study. How to develop the governing equations that describe the behaviour of the fluid flow in the AC and the VC respectively? How to model the fluid flow in the AC with DMD and the VC with RRD respectively? Is the finite element method able to solve the corresponding governing equations? How does AH flow behave in the AC with DMD? In what way do the type of the DMD, the cornea indentation and the position of the patient influence the AH flow in the AC? How does the liquefied VH flow behave in the VC with RRD? And what are the effects of the SB of the fluid field in VC with RRD? This research aims to answer these questions successfully.

1.4 Objectives of the Study

The aim of this research is to investigate the characteristics of fluid flow in the human eye with DMD and RRD respectively. This includes the construction of suitable mathematical models of the flow by considering an appropriate governing equation and the boundary conditions and then solve the resulting equations using finite element method. Specifically, the objectives of this study are:

1. To develop the appropriate governing equations that describe the fluid flow in the AC with DMD and the VC with RRD respectively.
2. To develop a source code in MATLAB in order to solve the corresponding governing equations by using finite element method.
3. To investigate the AH flow, driven by the temperature gradient, in the AC with DMD.
4. To investigate the influence of the types of DMD, the indentation of the cornea and the position of the patient on the fluid behaviour in the AC.
5. To investigate the fluid flow in the human eye with RRD, as a simple thin layer channel flow where a detachment is present in two cases:
   (i) Both upper and bottom walls are not moving
   (ii) With a moving upper wall
6. To investigate the influence of the SB, which is used to treat RRD, on the liquefied VH flow in the subspace between the detached retina and choroid.

1.5 Scope of the Study

The fluid flow in the human eye with DMD and RRD are analysed. The fluid in the AC and VC is considered as an incompressible Newtonian fluid. The governing equations are solved numerically by using finite element method. The MATLAB 2013 software is used for the computation of numerical results as well as for plotting the graph for visual display. The fluid flow in the AC which is driven by the temperature gradient, for the DMD problem is considered. The types of DMD to be considered in this thesis are planar, non-planar, scrolled and non-scrolled. Additionally, the effects of the indentation of the cornea and the position of the patient in the fluid flow in the AC are analysed. Normally, retinal detachment can be characterized as rhegmatogeneous, tractional and exudative. The present study only focus on the RRD problem. This study is restricted to the effects of SB treatment. The effect of the scleral buckle on the outflow of the subretinal fluid through a hole in the retina is discussed.

1.6 Significance of the Study

The exact mechanism of RRD is complex and the picture of its development remains incomplete. Previous researches show only parts of the full puzzle. It is important to comprehend the development of the retinal detachment so that a new treatment method can be developed. It is hard to study the formation of the RRD in the real human eye. As a result, information on the RRD's formation is lacking. Thus, results obtained in this research fill up the incompletion of the information. Wong *et al.* (1999) reported annual incidences of RRD in Singapore based on ethnicity. The results showed that 11.6 cases per 100,000 in the Chinese population, 7 cases among the Malays, and 3.9 cases for the Indians.

The annual incidence of RRD of 18.2 cases per 100,000 people, in the Netherlands was reported by Van de Put *et al.* (2013). The result showed that peak incidence occurred for 55-59 year olds with 52.5 cases per 100,000 people. The data show that population of RRD is huge. Therefore, a full understanding of the RRD's formation is essential for the prevention. Also as sclera buckling is the most common treatment for the RRD, it is important to increase the percentage of the successful treatment. The understanding of the dynamic effect of SB on retinal detachment is essential to reduce the percentage of treatment failure.

DMD is an uncommon disease in the eye, mostly occurs during cataract extraction. 50% of people between ages 65 and 74 and 70% of people over age 75 have cataract (Kara, 2011). Cloudy vision, glare, colour vision problems and double vision are some of the symptoms of cataract. To treat cataracts, surgery is normally performed to remove the affected lens and replace it by an artificial lens within the eye. The mechanism that causes the DM to tear from the stroma is unclear and the most popular hypothesis is that it is caused by the mechanical force applied to the cornea during surgery. This allows the AH to flow through the tear into the space between DM and the stroma and generate full DMD. An understanding of the role of the fluid flow in the devolving of full DMD is important. This will assist ophthalmologists to figure out better treatment to heal DMD.

The finite element analysis used in this research is originally applied in the structural engineering. The application of the method in this study explores the technique in the field of fluid mechanics. Furthermore, numerical model obtained in this research is expected to be also suitable for other fluid problems such as fluid flow in a sphere or fluid flow inside the Earth. In addition, the numerical results obtained in this study can be used to validate new results obtained.

1.7 Outline of the Thesis

This thesis contains seven chapters are included in this thesis. The statement of problems, objectives, scope and significance of the research make up the introduction as Chapter 1. The literature review of the interested problems is elaborated and established in Chapter 2. There are two sections in the chapter, the fluid flow in the AC and the fluid flow in the VC. The achievements of researchers on fluid flow in the human eye as well as the methods that used by them to accomplish the task are presented. The implementation of finite element method in solving the proposed problem is explained in Chapter 3 and 4. The fundamental procedures of using finite element method are given in Chapter 3. In additions, the Taylor-Hood element used to mesh the domain and the iteration manner applied to solve the nonlinear equation are also explained in the chapter. The computer implementation of finite element method has been explored in Chapter 4. The verification of the source code also has been done and presented in the chapter.

The effect of the DMD to the fluid flow in AC is studied in Chapter 5. The effect of the different types of DMD, the shape of the AC and the position of the patient to the fluid flow are discussed. In Chapter 6, the RRD problem is demonstrated. Two cases are considered: (1) both upper and bottom walls are not moving and (2) with a moving upper wall, in order to analyse the behaviour of the fluid flow in the human eye with RRD. The alteration of the fluid flow due to the type of the RRD, the change of the gravity direction and the indented VC by the SB is explored and discussed. In the final chapter of this thesis, the summary of the research is presented. Additionally, some suggestions for the future study based on the present research are highlighted.

1.8 Conclusions

In this present chapter, the background of the problem, the statement of the problem, the objectives of the research, the scope of the research, significance of the study and the outline of the thesis are presented. The difficulties for in-vivo study on human eye lead researchers to use computational simulation of the ocular fluid flow to understand the flowing mechanical of the fluid in the human eye, especially in respect to the diseases of DMD and RRD. Six objectives are proposed to achieve in this thesis.

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