

RENDERING OF LIGHT SHAFT AND SHADOW FOR INDOOR  
ENVIRONMENTS ENHANCING TECHNIQUE

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*I would like to dedicate this work for my beloved wife and lovely kids 'Nour, Hasan,  
and Haneen' for being patience, supportive, and understanding*

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## ABSTRACT

The ray marching methods have become the most attractive method to provide realism in rendering the effects of light scattering in the participating media of numerous applications. This has attracted significant attention from the scientific community. Up-sampling of ray marching methods is suitable to evaluate light scattering effects such as volumetric shadows and light shafts for rendering realistic scenes, but suffers of cost a lot for rendering. Therefore, some encouraging outcomes have been achieved by using down-sampling of ray marching approach to accelerate rendered scenes. However, these methods are inherently prone to artifacts, aliasing and incorrect boundaries due to the reduced number of sample points along view rays. This study proposed a new enhancing technique to render light shafts and shadows taking into consideration the integration light shafts, volumetric shadows, and shadows for indoor environments. This research has three major phases that cover species of the effects addressed in this thesis. The first phase includes the soft volumetric shadows creation technique called Soft Bilateral Filtering Volumetric Shadows (SoftBiF-VS). The soft shadow was created using a new algorithm called Soft Bilateral Filtering Shadow (SBFS). This technique was started by developing an algorithm called Imperfect Multi-View Soft Shadows (IMVSSs) based on down-sampling multiple point lights (DMPLs) and multiple depth maps, which are processed by using bilateral filtering to obtain soft shadows. Then, down-sampling light scattering model was used with (SBFS) to create volumetric shadows, which was improved using cross-bilateral filter to get soft volumetric shadows. In the second phase, soft light shaft was generated using a new technique called Realistic Real-Time Soft Bilateral Filtering Light Shafts (realTiSoftLS). This technique computed the light shaft depending on down-sampling volumetric light model and depth test, and was interpolated by bilateral filtering to gain soft light shafts. Finally, an enhancing technique for integrating all of these effects that represent the third phase of this research was achieved. The performance of the new enhanced technique was evaluated quantitatively and qualitatively a measured using standard dataset. Results from the experiment showed that 63% of the participants gave strong positive responses to this technique of improving realism. From the quantitative evaluation, the results revealed that the technique has dramatically outpaced the state-of-the-art techniques with a speed of 74 fps in improving the performance for indoor environments.

## ABSTRAK

Kaedah *ray marching* telah menjadi kaedah yang paling menarik untuk memberi kesan penyebaran cahaya yang realisme dalam media yang mengambil bahagian dalam pelbagai aplikasi. Ini secara signifikan telah menarik perhatian masyarakat saintifik. Peningkatan persampelan bagi kaedah *ray marching* amat sesuai untuk menilai kesan penyerakan cahaya seperti bayangan berisipadu dan alur cahaya sebagai pendekatan penjanaan realistik, tetapi mengalami banyak kos untuk penyerakan. Oleh itu, sebahagian dapatan sebelum ini telah menghasilkan pencapaian yang mengalakkan melalui pengurangan persampelan pendekatan *ray marching* untuk kaedah mempercepatkan pemandangan yang diberi. Walau bagaimanapun, kaedah ini terlalu mudah mengundang kecacatan, kesamaran dan kesilapan sempadanan disebabkan pengurangan bilangan titik sampel di sepanjang sinar pandangan. Kajian ini mencadangkan teknik baru untuk penyebaran cahaya dan pengambilan bayangan bagi mendapatkan penjanaan masa nyata, bayangan berisipadu dan bayangan untuk persekitaran dalaman. Kajian ini mengandungi tiga proses utama yang terdiri daripada pelbagai jenis kesan yang diketengahkan dalam tesis. Dalam fasa pertama, bayangan halus dihasilkan menggunakan teknik baru dipanggil Penapisan Bayangan Halus Dua Hala (SBFS). Bayangan halus ini dihasilkan dengan menggunakan satu algoritma yang dipanggil Bayangan Halus Pandangan Gandaan Tidak Sempurna (IMVSSs) berasaskan pengurangan sampel titik cahaya secara berganda (DMPL) dan peta kedalaman secara berganda yang kemudiannya diproses oleh penapis dua hala untuk menghasilkan bayangan halus. Seterusnya, model pengurangan penyerakan cahaya digunakan dengan SBFS untuk menghasilkan bayangan berisipadu dengan menggunakan penapis gandaan bersilang. Pada fasa kedua, alur cahaya halus dihasilkan menggunakan teknik baru yang dipanggil Penapisan Alur Cahaya Halus Realistik Dua Arah Dalam Maya Nyata (realTSoftLS). Teknik ini mengira alur cahaya berdasarkan pengurangan sampel model cahaya berisipadu dan ujian kedalaman dan kemudian ia adalah sisipan oleh penapis dua arah untuk mendapatkan alur cahaya halus. Akhir sekali, teknik terhasil dengan menggabungkan semua kesan ini menunjukkan tujuan penyelidikan ini tercapai. Prestasi teknik peningkatan baru dinilai secara kuantitatif dan kualitatif yang diukur menggunakan set data berpiawai. Hasil uji kaji menunjukkan 63% peserta memberi respon positif yang tinggi terhadap peningkatan teknik tersebut dalam meningkatkan realisme. Berkenaan dengan penilaian kuantitatif keputusan telah menunjukkan bahawa teknik yang dicadangkan menyediakan 74 fps dalam meningkatkan prestasi untuk persekitaran dalaman.

## TABLE OF CONTENTS

CHAPTER	TITLE	PAGE
	<b>DECLARATION</b>	ii
	<b>DEDICATION</b>	iii
	<b>ACKNOWLEDGEMENT</b>	iv
	<b>ABSTRACT</b>	v
	<b>ABSTRAK</b>	vi
	<b>TABLE OF CONTENTS</b>	vii
	<b>LIST OF TABLES</b>	xiii
	<b>LIST OF FIGURES</b>	xiv
	<b>LIST OF ABBREVIATIONS</b>	xix
	<b>LIST OF SYMBOLS</b>	xx
	<b>LIST OF APPENDICES</b>	xxi
<b>1</b>	<b>INTRODUCTION</b>	1
	1.1 Introduction	1
	1.2 Problem Background	3
	1.3 Problem Statement	8
	1.4 Research Goal	8
	1.5 Research Objectives	9
	1.6 Justification	9
	1.7 Research Contributions	10
	1.8 Research Scope and Limitations	10
	1.9 Thesis Organization	11
<b>2</b>	<b>LITERATURE REVIEW</b>	13
	2.1 Introduction	13
	2.2 Research Background	14
	2.2.1 Fundamental Techniques of Real-Time Shadows	19
	2.2.1.1 Shadow Volume Algorithm	19

	2.2.1.2	Shadow Mapping Algorithm	22
	2.2.2	Computing Soft Shadows	25
	2.2.2.1	Soft Shadows of Multiple Light Sources	26
	2.2.2.2	Soft Shadows of Multiple Objects	27
	2.2.3	Fake Shadows	28
	2.2.3.1	Shadows from an Area Light Source	28
	2.2.3.2	Penumbra Region	30
	2.2.3.3	Umbra Region	32
2.3		Reviews	32
	2.3.1	Soft Shadows	32
	2.3.1.1	Combining Multi Hard Shadows	33
	2.3.1.2	Soft Shadows Using Plateaus	33
	2.3.1.3	Smoothies	34
	2.3.1.4	Soft Shadow Volumes	35
	2.3.2	Soft Shadow Maps	37
	2.3.2.1	Combination of Multiple Point Image-Based Shadow	37
	2.3.2.2	Layered Attenuation Maps	38
	2.3.2.3	Quantitative Information in Shadow Map	38
	2.3.2.4	One Sample Soft Shadows	39
	2.3.2.5	Convolution Image-Based Shadows	40
2.4		Light Transport	41
	2.4.1	Light Scattering	41
	2.4.2	Participating Media	43
	2.4.2.1	No Scattering	43
	2.4.2.2	Single Scattering	44
	2.4.2.3	Multiple Scattering	47
	2.4.3	Phase Function	49
	2.4.4	Visibility Function	50
2.5		Illumination in Participating Media	51
	2.5.1	Non-Real-Time Approaches	51
	2.5.2	Real-Time Approaches	52
	2.5.3	High Performance Approaches	54

2.6	Deferred Rendering	59
	2.6.1 Geometry Stage	61
	2.6.2 Lighting Stage	62
	2.6.3 Post Processing Stage	63
	2.6.4 Final Stage	63
2.7	Volumetric Shadows	63
2.8	Volumetric Lighting	66
	2.8.1 Light shafts	68
	2.8.2 Soft Light shafts	69
2.9	Discussion	70
<b>3</b>	<b>RESEARCH METHODOLOGY</b>	<b>74</b>
3.1	Introduction	74
3.2	Research Framework	75
3.3	Phase 1: Problem Formulation	79
3.4	Phase 2: Technique Design and implementation	79
	3.4.1 Soft bilateral filtering Volumetric Shadow (SoftBiF-VS) technique	79
	3.4.1.1 Downsampling Multiple Point Lights (DMPLs)	80
	3.4.1.2 Multiple Depth Maps	80
	3.4.1.3 Imperfect Multi-View Soft Shadow (IMVSSs) algorithm	81
	3.4.1.4 Bilateral Filtering	81
	3.4.1.5 Soft Bilateral Filtering Shadows (SBFSs) Algorithm	83
	3.4.1.6 Downsampling Light Scattering Model	83
	3.4.1.7 Volumetric Shadows	84
	3.4.1.8 Cross-Bilateral Filter	84
	3.4.1.9 Soft Volumetric Shadow	84
	3.4.2 Soft Light Shafts	85
	3.4.2.1 Downsampling Volumetric Light Model	85
	3.4.2.2 Depth Test	87
	3.4.2.3 Light Shafts	87
	3.4.2.4 Bilateral Interpolation	87
	3.4.2.5 Soft Light Shafts	88
	3.4.3 Enhancing Technique	88



3.5	Phase4: Testing and Evaluation	88
	3.5.1 Quantitative Evaluation	89
	3.5.2 Qualitative Evaluation	89
3.6	Dataset	90
	3.6.1 Bunny Stanford	90
	3.6.2 Happy Buddha	91
	3.6.3 Dragon	92
	3.6.4 Lucy	92
	3.6.5 Armadillo	93
3.7	Summary	94
<b>4</b>	<b>LIGHT SHAFTS AND SOFT SHADOW ENHANCING TECHNIQUE</b>	<b>95</b>
4.1	Introduction	95
4.2	Soft Volumetric Shadows	97
	4.2.1 Imperfect Multi-View Soft Shadows Al- gorithm	98
	4.2.1.1 Multiple Position and Direction of Light Point	98
	4.2.1.2 Multiple Depth Maps	100
	4.2.2 Bilateral Filtering	102
	4.2.3 Soft Bilateral Filtering Shadows Algo- rithm	104
	4.2.4 Downsampling Light Scattering Model	114
	4.2.5 Volumetric Shadows	116
	4.2.6 Cross-Bilateral filter	117
	4.2.7 SoftBiF-VS Technique	118
4.3	Soft Light Shafts	129
	4.3.1 Downsampling Volumetric Light Model	130
	4.3.2 Depth Test	130
	4.3.3 Light Shafts	130
	4.3.4 Bilateral Interpolation	131
	4.3.5 realTiSoftLS Technique	132
4.4	Integrating Effects for Indoor Environments	134
	4.4.1 Enhancing Technique	134
	4.4.1.1 Rendering Off-screen	134
	4.4.1.2 Pipeline	135
4.5	Summary	140

<b>5</b>	<b>QUALITATIVE AND QUANTITATIVE EXPERIMENTAL RESULT</b>	<b>142</b>
5.1	Introduction	142
5.2	SoftBiF-VS Technique and SBFS Algorithm	143
5.2.1	Qualitative Evaluation of SoftBiF-VS Technique and SBFS algorithm	143
5.2.1.1	Visual Human Inspection of SBFS Algorithm	143
5.2.1.2	User Study of SBFS algorithm	146
5.2.1.3	Visual Human Inspection of SoftBiF-VS Technique	147
5.2.1.4	User Study of SoftBiF-VS technique	148
5.2.2	Quantitative Evaluation of SBFS algorithm	149
5.2.3	Quantitative Evaluation of SoftBiF-VS Technique	151
5.3	realTiSoftLS Technique	152
5.3.1	Qualitative Evaluation of realTiSoftLS Technique	152
5.3.1.1	Visual Human Inspection of realTiSoftLS Technique	153
5.3.1.2	User Study of realTiSoftLS technique	154
5.3.2	Quantitative Evaluation of realTiSoftLS Technique	155
5.4	Enhancing Technique	156
5.4.1	Qualitative Evaluation of Enhancing Technique	156
5.4.1.1	Visual Human Inspection of Enhancing Technique	157
5.4.1.2	User Study of Enhancing Technique	159
5.4.1.3	Quantitative Evaluation of Enhancing Technique	159
5.5	Summary	160
<b>6</b>	<b>CONCLUSION</b>	<b>162</b>
6.1	Overall Conclusion	162

6.2	Methods Proposed	163
6.3	Research Achievements	164
6.4	Future Works	166
<b>REFERENCES</b>		167
Appendices A – C		183 – 190

## LIST OF TABLES

<b>TABLE NO.</b>	<b>TITLE</b>	<b>PAGE</b>
2.1	Summary of the most representative rendering approaches in participating media	58
2.2	Summary of the most representative rendering approaches according to types of techniques	59
2.3	Shows summary of most important methods of light shafts and volumetric shadows	70
3.1	The roadmap of this research	78
5.1	Performance comparison between SBFS, Bidirectional Penumbra Map (Lili <i>et al.</i> , 2010), GEARS (Wang <i>et al.</i> , 2014b), and HSM (Hybrid Shadow Maps) (Kolivand and Sunar, 2012) with two different models	151
5.2	Performance comparison between SoftBiF-VS, Biri <i>et al.</i> (2006)(ms), and Wyman and Dai (2013) with different models	152

## LIST OF FIGURES

FIGURE NO.	TITLE	PAGE
1.1	Left: outdoor, Right: indoor shows shadows, volumetric shadows, and light shafts of the Masjid Sultan Ismail UTM.	2
1.2	Research focus area (ACM)	3
2.1	Left: Analysis of shadows by Leonardo da Vinci (1490), Right: Shadow generation by Lambert (1774)	15
2.2	The understand of relative object position in a scene	16
2.3	Shadows reveal information about the shape folded of the receiver	17
2.4	Shadows give information about the shape of the occluder	17
2.5	Radiance at a point $x$ represents incoming radiance (left), radiance outgoing at $x$ in direction (center). BRDF model (right)	18
2.6	Model to interact a light in participating media	19
2.7	Shadow volume theory	21
2.8	Shadow map. Left: scene from the camera. Right: depth value, view from light source	23
2.9	Theory of shadow maps	24
2.10	(Left) geometry of hard shadows, (Right) illustration of hard shadows	25
2.11	(Left) geometry of soft shadows, (Right) illustration of soft shadows	26
2.12	Shadow due to multiple light sources	27
2.13	The shadow of couple of the blockers is not easy combination of the couple of individual shadows	28
2.14	The changing of size of the light source, (Left) teapot cast faint shadows from large light source, (Middle) teapot cast more clear shadows from medium light source. (Right) teapot cast clearly shadows from small light source	29
2.15	The umbra region outwards	30
2.16	Overlapping between shadows	31

2.17	(Left) The bidirectional penumbra map, (Right) theory of both outer and inner penumbra	31
2.18	Expanding the geometric-based shadow of a blocker with cones and planes	34
2.19	The penumbra wedge of a silhouette boundary: Left circle light source, Right area light source	35
2.20	Many connected boundaries. The part of the light source that is blocked is equal to sum of the parts of the light source blocked by different boundaries	36
2.21	Visible ratio of a linear light source	39
2.22	Umbra and penumbra from extended light source	40
2.23	Left: the convolution image-based shadows. Middle: uncover conventional image-based shadows where point $x_1$ is lit, point $x_2$ is in shadow. Right: illustrates result of convolution image-based shadows ignore any light bleeding	41
2.24	Single scattering in the participating media	44
2.25	The illuminated face of Saturn's Rings (left).The unilluminated face of Saturn's Rings (right)	45
2.26	The multiple light scattering	48
2.27	A phase function suitable for a hazy atmosphere	50
2.28	The phase function describes, (left) light scattering in the backward, (middle) light scattering equally in all directions, (right) light scattering in forward	50
2.29	Diagram of deferred rendering. Image courtesy of	60
2.30	Visualization of four goals of the G-Buffer. (a) normal, (b) diffuse, (c) specular and (d) depth	61
2.31	Output image created from texture	62
2.32	Volumetric shadows	64
2.33	History of light scattering a brief timeline	67
3.1	Framework of research	77
3.2	The bunny model Stanford	91
3.3	The Happy Buddha model Stanford	91
3.4	The Dragon model Stanford	92
3.5	The Lucy model Stanford	93
3.6	The Armadillo model Stanford	93
4.1	Downsampling of multiple point lights in black dash line	100

4.2	(a) single shadow map with umbra, (b) the multiple shadow maps using multi-frustum view of light sample points. The scene from the light source and from the camera where the red rectangle indicates boundaries of the penumbra	101
4.3	Boundaries of the multiple shadow maps using multi-frustum view of sample point of the light in color blue, red, and green	102
4.4	(a) hard shadow shows the umbra, and (b) soft shadow explains appearance of the penumbra	104
4.5	Interpolate the color for pixels on the screen using a bilateral filter of the edges of penumbra region, yellow dashed lines indicate the distances	105
4.6	(a) the input from multiple image-based shadow with artifacts, (b) the domain filtering of the input image and the smoothing effects, (c) the range filtering with sharp boundaries, (d) weights multiplication of range by domain, (e) the bilateral filter effects	107
4.7	the soft shadow, (a) the penumbra from rendering multiple image-based shadows with artifacts and aliasing, (b) a smooth penumbra region that maintains the shapes of the edges using bilateral filtering	110
4.8	The two different scenes of Elephant, the results of SBFS method using three light sample points and $hd = 3$ and $hr = 0.5$	111
4.9	The two different scenes of Dragon and Buddha models, the results of SBFS technique using five light sample points and $hd = 5$ and $hr = 0.5$	112
4.10	generating soft shadows using SBFS. Employing parameters $hd$ and $hr$ to control on effective and efficient of the soft shadows according to sample points of light source	113
4.11	The parameters of light scattering model to volumetric shadows	115
4.12	The downsampling of the points along view ray	117
4.13	Soft volumetric shadows	118
4.14	Bilateral filtering process in two 1D	119
4.15	Process of Soft Bilateral Filtering Volumetric Shadows (SoftBiF-VS)	120
4.16	. Procedure of vertex shader and fragment shader for input of main program (SoftBiF-VS)	122

4.17	Visual framework of SoftBiF-VS, (A) Original input image without lighting, (B) with lighting, (C) HDR linear, (D) HDR volume, (E) compositing between (C) and (D), (F) render cube map shadow, and (G) render bilateral filtering	125
4.18	The two different scenes of the Elephant model	126
4.19	Buddha model using bilateral filtering, (a) the scene with 4 taps, (b) the scene with 8 taps, (c) the scene with 16 taps, and (d) the scene with 32 taps	127
4.20	Bunny and Fence models on shadows: Right side: the effect of Fence model; Left side: the effect of Bunny and Fence together, bottom the effect of bunny on shading the scene	128
4.21	The effect soft volumetric shadows of our method using models in different sizes	129
4.22	Single scattering along view ray from object to eye	130
4.23	Light scattering model	132
4.24	(left) the downsampling of ray marching for evaluating the light shafts the blue arrow refer to ray view and red points to samples, (right) the results of our technique using bilateral interpolation	133
4.25	The results of prototype development	139
4.26	The downsampling of ray marching to appear effect of light shafts and shadow	140
5.1	Rendering the Dragon and Buddha models: Left is the results of Bidirectional Penumbra Map (Lili <i>et al.</i> , 2010), Middle: our technique with background similar to in (left). Right: our method texture background	144
5.2	rendering soft shadows of the Bunny and Elephant models: left: the result of the Bidirectional Penumbra Map (Lili <i>et al.</i> , 2010), Middle: result of the SBFS with gray scale background, Right: result of the SBFS method in texture background	145
5.3	Overlapping shadows of multiple objects, (left) the theory of our method, while middle of is the result of (Lauritzen and McCool, 2008) with light bleeding, (right) the result of our method	146
5.4	Histogram of questionnaire of evaluation realistic SBSF algorithm with previous works	147



5.5	Rendering different models using interpolation: (a) and (c) the imperfect results of Wyman and Dais (2013) VSV method, (b) and (d) the results of SoftBiF-VS method	148
5.6	Histogram of questionnaire of evaluation realistic SoftBiF-VS technique with previous works	149
5.7	Frame rate of SBFS technique compare with the Bidirectional Penumbra Map Method	150
5.8	Rows from top to bottom: R1) Brute force (ray marching method) with 150 sample; R2) ray marching with 32 sample; R3) downsampling with 32 blur by Gaussian filter; R4) our technique with 32 sample.	154
5.9	The graph for performance of our technique comparison with other techniques	155
5.10	The graph for performance of our technique comparison with other techniques	156
5.11	The comparison between our results (bottom), and results of naive method (top)	158
5.12	The graph of questionnaire of evaluation our integrated framework with previous techniques	159
5.13	The comparison in frame per second between our method and previous methods	160

**LIST OF ABBREVIATIONS**

CPU	-	Central Processing Unit
GPU	-	Graphics Processing Unit
FPS	-	Frame Per Second
ms	-	millisecond
PCF	-	Percentage-Closer Filtering
PCSS	-	Percentage Closer Soft Shadows
VSSM	-	Variance Soft Shadow Mapping
LVSM	-	layered Variance Shadow Maps
VSM	-	Variance Shadow Mapping
MVSS	-	Multi-View Soft Shadow
IMVSS	-	Imperfect Multi-View Soft Shadow
SBFS	-	Soft Bilateral Filtering Shadow
SoftBiF-VS	-	Soft Bilateral Filtering Volumetric Shadow
realTiSoftLS	-	Realistic Real-Time Soft Light Shaft
BRDF	-	Bidirectional Reflectance Distribution Function
P-N	-	Spherical Harmonic
G-Buffer	-	Geometry Buffer
FBO	-	Framebuffer Object
HDR	-	High Dynamic Range
DMPL	-	Downsampling Multiple Point Lights

**LIST OF SYMBOLS**

$g(p)$	-	filtered image
$G_{hd}$	-	domain filtering
$G_{hr}$	-	range filtering
$tE$	-	extinction coefficient
$tS$	-	scattering coefficient
$U_{sh}$	-	visibility function
$L_s$	-	light scattering
$F$	-	phase function

**LIST OF APPENDICES**

<b>APPENDIX</b>	<b>TITLE</b>	<b>PAGE</b>
A	User Study	183
B	List of Publications	188
C	List of Participates	190

## CHAPTER 1

### INTRODUCTION

#### 1.1 Introduction

Light scattering in the participating media has become a popular rendering approach similar to a light rendering in virtual environments. Since light scattering adds visualization (pun intended) to the scenes, many researchers have been motivated to solve this topic (Blinn, 1982; Dobashi *et al.*, 2002; Kaneda *et al.*, 1991; Kilpatrick and Ramshaw, 2015; Klassen, 1987; Li *et al.*, 2007; Max, 1986; Nishita *et al.*, 1996; Stumpfel, 2004; Sunar *et al.*, 2003; Tadamura *et al.*, 1993). In contrast, while the effects of light represent an interaction that only occurs at the surface of the object, the effects of light scattering is an interaction with suspended particles in air gives rise to effects such as light shafts and volumetric shadows.

Light shaft is light passing through participating media that is scattered towards the viewer, making the participating media visible (Wyman and Ramsey, 2008). Volumetric shadow term is part of the great subject of scattered light in participating media that arises due to object casting a shadow in the participating media (Eisemann *et al.*, 2011). Therefore, the volumetric shadows significantly enhance realism in rendering scenes based on the physical properties and density of the surrounding particles and due to exist the occlude (Engelhardt and Dachsbacher, 2010).

The participating medium is a media that contributes to interaction of light such as dusty air, fog, or smoke. In other words, the light does not travel over the media without affecting it (Wyman and Ramsey, 2008). The medium may scatter and absorb of the radiance rays due to refractions and reflections of suspended particles in air. Calculating the correct scattering effects, with multiple refraction or reflections as the light travels through the media, is still computationally expensive in real-time applications. However, restricting the scattering to a single reflection makes

calculations less complex. Thus, single scattering achieves requirements in real-time applications while providing eye-pleasing results.

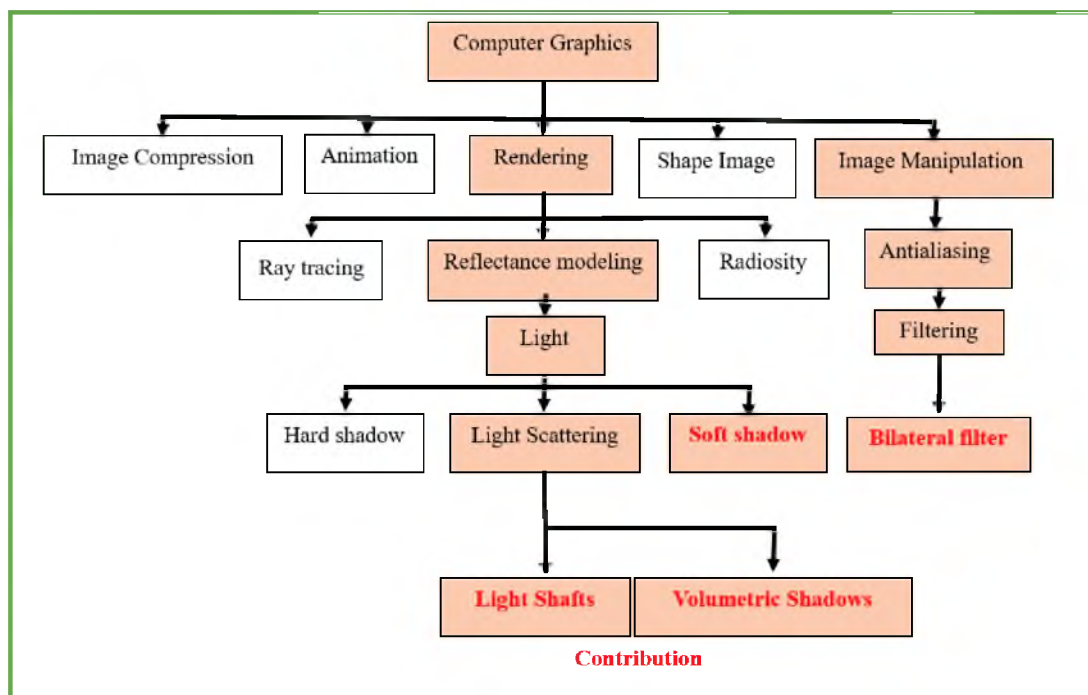
The light transport equation is used to govern energy emission process in participating media (Segal *et al.*, 1992). Therefore, the methods of rendering light transport equation in the presence of participating media can be classified into three categories, analytic, deterministic and statistical (Blinn, 1982; Cerezo *et al.*, 2005; Rogers *et al.*, 2014).

In interactive applications, shadows, volumetric shadows, and light shafts makes users feel a sense of interacting with a scene as it would appear in the real world. Then, applications without these effects are not satisfactory for users. Currently, the user interacts with simulator devices and his desire for more and more realism. To generate realistic scenes, shadows play an important role in providing information about the distances between objects in the whole scene. In addition, it is a major factor to determine occluded areas from those visible areas for the light source (Kolivand and Sunar, 2013; Li *et al.*, 2007; McGuire and Enderton, 2011; Wyman and Ramsey, 2008). Figure 1.1 (left) and (right) shows the shadows, light shafts, and volumetric shadows of outdoor and indoor of the Masjid Sultan Ismail at Universiti Teknologi Malaysia. (Rahim, 2016; Zuhaka, 2007).



**Figure 1.1:** left: outdoor, Right: indoor shows shadows, volumetric shadows, and light shafts of the Masjid Sultan Ismail UTM. (Rahim, 2016; Zuhaka, 2007)

This thesis focuses on rendering in computer gaming and interactive simulation that related with light and light scattering as shown in Figure 1.2 in orange rectangles. Specifically, light shafts, volumetric shadows, and soft shadow are generated using bilateral filtering as shown Figure 1.2 in red words according to ACM diagram 2013.



**Figure 1.2:** Research focus area(ACM 2013)

## 1.2 Problem Background

Rendering scenes of the natural phenomena is a topic well-sought for virtual environments in various contexts: flight simulators, stage lighting designs, video games, computer games, entertainment, and safety analysis. Blinn (1982) was the first to use light effects to visualize Saturn's rings. Moreover, noisy means were used to display natural phenomena for rendering in the presence participating media which are extremely difficult (Gardner, 1985; Perlin, 1985; Yaeger *et al.*, 1986). In spite of acceleration in real-time applications, the realism is also a very important factor (Boulanger, 2008; Feng, 2008; Xing *et al.*, 2012, 2013).

Recently, computer-generated imagery of the natural phenomena has attracted the interest of the research community due to its numerous applications. In fact, the

aim is to generate scenes similar to what the viewers see in real life, but bottleneck in attaining the goal is computational complexity. Nature scenes of light scattering depend on the number of phenomena for interaction such as absorption, in-scattering, out-scattering, scattering, emission and shadows which are hard to integrate (Bavoil, 2011; Chen *et al.*, 2016; Engelhardt and Dachsbacher, 2010; Jansen and Chalmers, 1993; Siegel and Howell, 1992), consume a lot of time to render and require a large amount of memory. The intricacy mainly due to equation of radiance transfer. This equation intricacy arises from taking high number of sample points along view rays to compute amount of radiance that up to a viewer.

In computer graphics, a trade-off between quality and performance is a highly interconnected issue. In order to obtain a desirable outcome, an increment in Frame Per Second (FPS) will be at the expense of sacrificing quality. Therefore, increment in frame rate is an undesirable for low quality and vice versa. Introducing a compromise approach to improve the frame rate with preserving quality could be a desirable accomplishment for the current study.

Numerous techniques are proposed for shadow creation. Shadow maps and shadow volumes are conventional real-time algorithms. Shadow volumes are effective enough, however they are object-space and need complex computations. Although shadow volumes are applied in many applications, they have couple of costly steps; silhouette detection, and several passes to update volume rendering (Assarsson and Akenine-Möller, 2003; Billeter *et al.*, 2010; Jung *et al.*, 2004; Raskar and Cohen, 1999). Moreover, shadow volumes are ineffective in generating soft shadows.

Unlike object-space shadows, shadow mapping is image-based and simple to be carried out. Williams (1978) was the first researcher who proposed shadow maps which are a milestone in shadow creation. Although image-based shadows are more popular than object-space shadows due to cheap rendering, but they are prone to appear aliasings. This problem is a rigorous issue of shadow maps (Billeter *et al.*, 2010; Eisemann *et al.*, 2009; Lauritzen and McCool, 2008; Lokovic and Veach, 2000; Reeves *et al.*, 1987; Scherzer *et al.*, 2011; Williams, 1978; Woo *et al.*, 1990). Enhancement of this category of shadow creation technique is another contribution of the current study.

Donnelly and Lauritzen (2006) suggested a technique entitled Variance Shadow Maps (VSMs) that adopted classical shadow maps to eliminate aliasing. In this technique, squared values and mean for depths distribution were stored instead of storing z-value. Afterward, (Dimitrov, 2007) presented an approach to generate



soft shadows depending on shadow maps that they called Cascaded Shadow Maps (CSMs). CSMs attempt to remove aliasing using z-partitioning. Layered Variance Shadow Maps (LVSMs) technique was proposed by (Lauritzen and McCool, 2008). This technique works by dividing depth map into layers but the textural accuracy is decreased.

Atty *et al.* (2006); Guennebaud *et al.* (2006) were the first who used the term soft shadow mapping. They partitioned the scene and made each sample as a blocker, and then applied the less complex scene to calculate the radiance. Projection overlap and light leak are the shortcomings of this method. Lawlor (2006) presented a technique using linear interpolation between the outer penumbra and inner penumbra without silhouette detection. This technique cannot maintain the shapes of edges; therefore, it does not produce any improvement other than smoothing the shadowing between texels.

Yang *et al.* (2010) extended variance shadow maps technique by proposing Variance Soft Shadow Mapping (VSSM) technique. This technique exploits VSMs also is based on PCF. VSSM has ability to render soft shadow faster than percentage closer soft shadow technique. Furthermore, this method is depended on the framework of PCSS technique to calculate of occluder distances. Lili *et al.* (2010) proposed a method to render soft shadow. This method works on edge detection of frustum that the light launched in 3D space and extends the edge shape outwards to get penumbra map. Wang *et al.* (2014b) work on determining the potential blocking triangles uses bitmask process and rectified shadow map to render soft shadow.

Volume rendering is a part of the large topic of scattered light in the participating medium. The main issue of light scattering and shadows is the visual realism (Tóth and Umenhoffer, 2009). The effect of volumetric shadows that gives rise to phenomenon such as shadows in the participating media very important for rendered scenes. Indeed, shadows are often soft due to multiple point lights, which lead to overlap regions that becoming softer at boundaries of shadows. These regions are extremely important whether at surface or space to give realistic scenes. However, Most rendering approaches have concentrated on rendering of simple lighting in static scenes due to intricacy of the dynamic scenes in rendering light scattering that contains on multiple point lights and shadows in virtual environments (Biri *et al.*, 2006).

Max (1986) introduced scan-line-based method to determine the regions of lit and unlit atmosphere by contributing to the shadow volumes. (Dobashi *et al.*, 2002)

proposed a solution to create volumetric shadow effects using sub-sampling. Although the results of volumetric shadows were acceptable for global illumination, it leads to the appearance of artifacts for local illumination due to use sub-planes.

Biri *et al.* (2006) presented mathematical formulations for radiance transfer equation to simulate volumetric shadows based on shadow volumes and fog. However, these methods exploited capability and capacities of graphics hardware. Although efficient, the effects of volumetric shadows did not appear accurately. This was because the variation in participating media density did not coincide with variation in scattered light.

Wyman and Ramsey (2008) in their approach for rendering the effect of volumetric shadows induced scattering of light. This method combined sub-sampling and geometry-based shadows to accelerate the brute force algorithm. The ray view that contributed in scattering was computed using sub-sampling method. Then, geometry-based shadows were applied to determine lit and unlit space. However, the geometry-based shadows were impractical and slow for complex scenes.

Billeter *et al.* (2010) proposed a method to generate a volumetric shadows grid using the shadow map. This grid is created with one vertex per shadow map texel, then displaces by changing the depth values in the shadow map. This method has high efficient of the coarse shadow maps, but is significantly cumbersome with high resolution of shadow map. Baran *et al.* (2010) presented incremental integration method using epipolar recitation to get high performance. The shortcoming of this method was the appearance of artifacts. Chen *et al.* (2011) extended the algorithm incremental integration method by relying on 1D min-max mipmap. This method not only computed epipolar lines of the light, but also viewed rays to obtain a high performance. This method suffered from numerical stability issues when the light source was inside the viewing frustum of camera.

Wyman (2011) proposed voxelized shadow volumes method for visibility queries, which used epipolar space for downsample of voxelize. This method applied a parallel scan along view rays using bitwise OR, rather than adding operator to generate voxelized shadow volumes. The advantages of this method were: 1) separated geometric complexity, 2) reduce visibility costs, and 3) a few cache coherent lookups. These advantages drop into existing participating media techniques to improve visibility. Artifact and aliasing being limited to point light source were the

shortcomings of this method. Later (Wyman and Dai, 2013) improved the above method in order to avoid the problems when using wide light sources.

Lin *et al.* (2013) proposed a method to reduce the number of sample points for generating volumetric shadows in dynamic scenes. This method is based on the decision of whether a pixel should be re-computed or should the information from previous frame be used. However, acceleration was reduced when moving objects and their geometry-based shadows contained many screen pixels or the camera moved continuously.

Most research study found was based on Klehm *et al.* (2014), which proposed a method called rectify shadow maps to reduce samples instead of using brute force method. This method is performed well and produced high quality outcomes. Nevertheless, this method was not successful when view and light rays were not parallel to each other, which meant the light source was directly invisible. Many approaches have exploited a graphics hardware to obtain appropriate textures. Since the first work of (Dobashi *et al.*, 2000), many techniques (Chen *et al.*, 2006; Dobashi *et al.*, 2002; Li *et al.*, 2007; Tóth and Umenhoffer, 2009) have been proposed. Most of these techniques were focused on making the interactive rendering of typical light shafts caused by global illumination in atmosphere. Imagire *et al.* (2007) proposed a technique to render of volumetric effect using sampling hulls. However, it is a valid of small regions within view ray.

On the other hand, numerous epipolar-based methods have been proposed for rendering volumetric lighting. Engelhardt and Dachsbacher (2010) used geometry epipolar sampling to reduce the points of rays. Wyman (2011) presented an algorithm to determine points of sample inside volumetric that was viewed from light source using binary voxel grid in epipolar coordinates. Baran *et al.* (2010) presented incremental integration method using epipolar rectification to get good performance, (Chen *et al.*, 2011) in their method to extend the algorithm of above method, relied on 1D min-max mipmap. All these techniques above seek to reduce a number of samples to improve the performance, which is called downsampling.

In summary, downsampling is one of the main issues in efficiently rendering the effects of light scattering and shadows for interactive application. The important components in synthesis of realistic scenes are the shadows, volumetric shadows, and light shafts and integration among them. A few research have sought to improve the realism of light scattering but most research have concentrated on traditional lighting

at the surfaces. A new techniques in generating shadows, volumetric shadows, and light shafts free of artifact is required to improve the realism of virtual environments.

### 1.3 Problem Statement

Realistic rendering of the light shafts, volumetric shadows, and shadows in virtual environments crucially depends on the smooth boundaries (Chen *et al.*, 2016; Engelhardt and Dachsbacher, 2010; Klehm *et al.*, 2015) which are considered important issues in the appearance of these effects. Typically, Ray Marching Methods (RMMs) contribute in calculating these effects by adopting the sampling terms of a sample points that are commonly used before conducting the calculations. Downsampling term is widely used in calculating such as effects based on reducing the number of sample points along view rays which is called coarser sampling (Baran *et al.*, 2010; Chen *et al.*, 2011). This approach is commonly used to improve the run-time rendering performance. However, it leads to the appearance of some problems such as artifacts and aliasing in light shafts, volumetric shadows, and shadows. Moreover, the light shaft and shadow still suffer harshness and crispness at boundaries (Klehm *et al.*, 2015). Therefore, to resolve this issue, an enhancing technique is required to render these effects that are indistinguishable from the real-world.

Several approaches were proposed to solve this issue of real-time rendering in photo-realistic virtual environments for static and simple scenes (Shin and Olano, 2013; Tóth and Umenhoffer, 2009). These approaches aimed to integrate these effects in real-time rendering which is the most salient features of the current study to solve the problem of sharpness at their boundaries (Chen *et al.*, 2011), Therefore, enhancement these boundaries in rendering light shafts and volumetric shadows with respect to shadows that may be increasing the realism of rendered scenes. This thesis will primarily focus on how to render correctly boundaries of these effects in real-time performance.

### 1.4 Research Goal

The primary goal of this research is to propose a technique to achieve realistic real-time rendering taking into account integration between light shaft and shadow with respect to volumetric shadow for indoor environments.

## 1.5 Research Objectives

To accomplish this goal, the following objectives must be followed:

1. To propose a new soft volumetric shadow technique for enhancing the realism at boundaries of volumetric shadows based on a new soft shadow algorithm
2. To introduce a new technique for rendering soft light shaft at interactive rates and increasing realism at boundaries of light shaft
3. To enhance indoor environments by proposing a new technique for integrating light shaft and shadow with respect to volumetric shadow

## 1.6 Justification

This study has proposed three new techniques to generate soft volumetric shadow, soft light shaft, and integrate light shaft and shadow with respect to volumetric shadow for indoor environments. First technique depends on a new soft shadow algorithm to create soft volumetric shadow. This algorithm computes multiple point lights and multiple depth maps to generate imperfect multi-view soft shadows. Therefore, The new algorithm to create soft shadow plays an important role in generating soft volumetric shadow technique. The soft volumetric shadow generats can be used in interactive applications with high efficient and high frame rate. The new technique to create soft light shafts can be used in interactive applications. The enhancing technique for integrating these effects, the outcomes obtained from this study are prepared to become grounds for the much required industrial standard in computer graphics. This technique solves the problem of image and image-based shadow, also it is thought to have an attractive effect and influence on the realism and real-time performance.

For the final enhancing technique, three of the groups are from audiences. The first group is interested in safety analysis. The second group are animators (e.g. Zootopia and Moana), game developers (e.g. Overwatch and Forza Horizon 3) and movies (e.g. Star Trek Beyond and Vampire Hunter). The third group are drive simulators and space/flight simulators.

## 1.7 Research Contributions

- The soft shadow creation technique is one of the contributions to keep maintain the tradeoff between quality and performance of outcomes. An algorithm for imperfect multi-view soft shadow based on multiple point lights and multiple depth maps, then used bilateral filter to obtain soft shadows are the first contributions of this study. In the case downsampling of a number of points of light this improvement on multiple shadow maps increases the performance. The bilateral filtering improve the quality of soft shadows also increases the performance.
- The soft volumetric shadows technique is one of contributions to increase the performance and quality of outcomes. The model scattering introduced for generating volumetric shadow based on soft shadows and by applying downsampling with used cross bilateral filter to obtain soft volumetric shadows.
- The third contribution in this study is to present a new technique soft light shafts based on volumetric lighting model to generate light shafts by using downsampling, then bilateral interpolation uses to produce soft light shafts. The main contribution of this work is achieving the aim in rendering the shadows, volumetric shadows, and soft light shafts in integrating framework in virtual environments.

## 1.8 Research Scope and Limitations

The soft shadows creation algorithm can be used for opaque objects in virtual environments, the transparent and translucent objects are excluded. The soft bilateral filtering shadow technique could improved both the efficient and accurate of soft shadows.

The soft volumetric shadows technique is based on particle density in the air and opaque objects that casting shadow in the participating media of virtual environments. In this research the multiple light scattering is excluded.

The soft light shaft technique is generated in isotropic media and does not involve atmosphere phenomena such as aerial perspective, rainbows, halos, glories, and underwater scenes.

Implementation of the algorithms is programmed in OpenGL language and GLSL as well as implemented with OBJ and PLY data. As well as is based on points of light source, the wide light sources is excluded such as area light source and directional light source.

In contrast of outdoor environments, the indoor rendering includes accurate details such as the boundaries of light shafts, volumetric shadows, and shadows which are important factors to consider for effective realism (Kolivand *et al.*, 2015). The focus of this research is on producing a high quality Light shaft and volumetric shadow rendering system with respect to shadows in the real-time performance environment. It is applicable to computer games and rendering animation systems industry.

## 1.9 Thesis Organization

This thesis includes 6 main chapters: chapter 1 for introduction, chapter 2 a review of some of the relevant literatures to current research. Chapter 3 research methodology. Chapter 4 for implementation of soft Shadow and light scattering rendering techniques. Chapter 5 qualitative and quantitative experimental results. Finally Chapter 6 is for conclusion and future works. The six chapters is organized as outlined in the following:

**Chapter 1**, begins with the introduction. Then problem background and problem statement. Then comes goal and objectives, and the scope and limitation. At end of the chapter is presented the research outline.

**Chapter 2**, provides an in-depth review of relevant literatures in soft shadows creation in virtual environments, volumetric shadows generation, and light shafts generation. It is emphasized on the different limitations and contributions for the proposed techniques and algorithms. Therefore, a comprehensive discussion is provided on diverse methods used so far in generating effects mentioned.

**Chapter 3**, presents the research methodology in three phases. Phase 1 problem formulation which involves investigating in problems of the shadow and light scattering. Phase 2 technique design and implementation which includes three stages, generation soft volumetric shadow, and creation soft light shafts, and enhancing

technique which comprises integrating soft shadow, soft volumetric shadows, and soft light shafts mentioned in Phase 2. Phase 3 is testing and evaluating of the results.

**Chapter 4**, discusses the proposed methods of the three objectives of this study. It begins with implement the soft volumetric shadows which consists. Firstly build multiple point light based on multi-view matrices of light. Secondly, multiple depth maps are rendered based on multiple point light. Thirdly, generating imperfect multi-view soft shadows based on multiple depth maps. Fourthly, bilateral filtering is used to generate soft shadows. fourth step is a mathematical formulation of the light scattering model. The fifth step generate volumetric shadows based on light scattering model. The sixth step uses cross-bilateral filter to obtain soft volumetric shadows. The next objective to implement the soft light shafts which involves three steps. The first is mathematical formulation of the volumetric lighting model. The second step generate light model. the third step applied bilateral interpolation to product. The second step generate light shafts based on volumetric light model. The third step applied bilateral interpolation to product soft light shafts. Third objective for enhancing technique to integrate soft light shafts, soft volumetric shadows and soft shadows to smooth these effects and preserving on boundaries.

**Chapter 5**, provides the experimental results emanating from the applied of the three techniques. It explains the qualitative and quantitative measurements that are conducted for the performance assessments of the technique for every phase separately. The soft volumetric shadow is part which assesses the outcomes of the generated soft volumetric shadow which involves also The soft shadow is part which evaluates the results of the created soft volumetric shadow in virtual environments. The soft light shafts is section which evaluates the outcomes of the created soft light shafts in virtual environments. The last section of this chapter is testing and assessment of the enhancing technique, the qualitative measurements are based on visual human inspections and user study, while the quantitative measurements are performed using frame per second. In addition, each process is benchmarked against the comparable techniques.

**Chapter 6**, concludes by emphasizing the main contributions, objectives and proposed techniques, and ends with highlighting the accomplishments and suggestions for future research.



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