

## **OPTIMISATION STUDIES ON INCLINED WEB WHEEL USING FINITE ELEMENTS**

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### **ABSTRACT**

*The optimisation studies on wheels have generated much interest among the researchers in order to upgrade the wheel performance. Wheels find application as wheel in automobiles, trains, as pulleys, helical gears etc. Optimum shape of wheel is the fundamental objective of designers. Finite Element Method (FEM) is an appropriate technique suitable for computer implementation. This study is mainly concerned about the analysis of stress and deflection of the web of wheel under static loading conditions. Modification of geometric parameters such as web inclination, fillet radius and rim-volume reduce the stress distribution and deflection of wheel axial loading condition. The investigation reveals information on shape optimisation and the resulting stress reduction.*

## 1.0 INTRODUCTION

Wheels are generally designed on empirical relations based on experience. This results in over designed wheels. A wheel consists of three main parts, the hub, web or disk and rim as shown below:

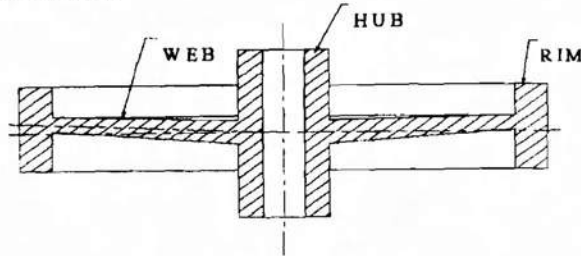


Fig. 1 Parts Of Wheel

In mechanical stress analysis, the geometric parameters of wheel play a very important role because the deflection and stress of wheel depends on these parameters. For example, the inclined web wheel is designed in order to support large axial loads. These parameters are indicated in the following figures:

- |                     |                     |
|---------------------|---------------------|
| a - Wheel diameter  | b- Hub diameter     |
| c- Thickness of hub | d- Depth of rim     |
| e- Thickness of web | f- Disk taper       |
| g- Disk Inclination | h- Thickness of rim |

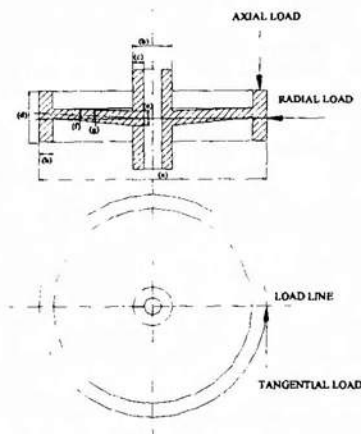


Fig. 2 Geometric Parameters Of Disc Wheel Model

Hub is generally press fitted on the shaft. It is appropriate to regard the bore of the hub as rigidly constrained. This will determine the kinematic boundary conditions of wheel where the inner edge of the hub will be fixed and supported as shown below:

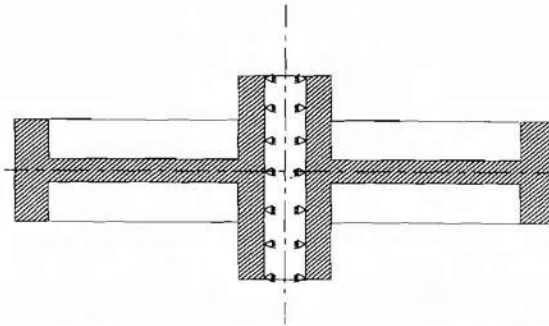


Fig. 3 Boundary Condition  $P_m$  for a Fitted Disk with Central Hole

The loading conditions on the wheel depend on its application. A transportation vehicle wheel experiences a radial force. When the wheel is operating, different types of loads, radial, tangential and axial loads will act on the structures of wheel. Apart from these, many researches and investigations of wheel design attempt to develop optimum shape of wheel in order to use the material economically. The wheel is designed to perform well with lower volume material but also to support the same or even higher loads due to their application.

H. G. Gibbs and T. H. Richards have done some research and investigation to determine the stress distribution in various designs of disc-type wheels. The typical rimmed disc wheel idealisation is shown in Figure 4. The design specifications of three types of loads: (a) pure radial load, (b) an offset radial load and (c) a combined radial and axial loads. The wheels of interest were made of low modulus material and the hub contained a force fit sleeve of metal. The elastic modulus of the wheel and sleeve were of different orders of magnitude, so that it was considered adequate to regard the hub as rigidly constrained.

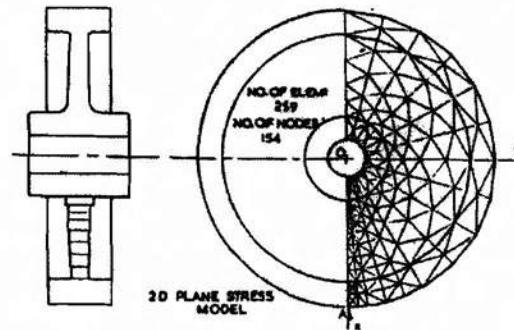


Fig. 4 Structural Idealisation of a Rimmed Disc

The effective stress (Von Mises Stress) is maximum at the hub-web junction. A set of calculations was aimed at finding the influence of rim size on the maximum Von Mises Stress. Figure 5 shows how increasing the rim volume reduces the maximum effectiveness stress in the disc. For the purpose of these calculations, rim was treated as the extra material added to an originally plane disc. Features of the results are:

1. The effectiveness of adding material to rim decreases as the rim size increases
2. Within the range of the proportions examined here (which were in the neighbour of those currently used ), the shape of rim does not affect matters.
3. The wheels of interest ranged from 4 in to 6 in o. d. (outer diameter)

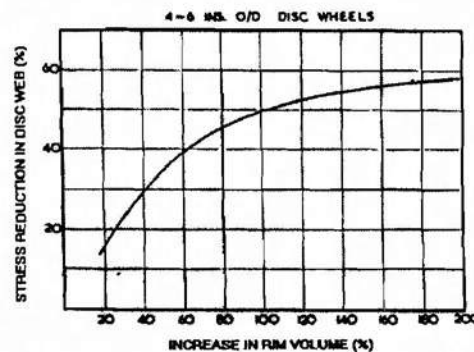


Fig. 5 Reduction of Stresses Due to Increase of Rim Volume

## 2.0 VALIDATION AND CONVERGENCE TEST

For our purpose of investigation, a profile of wheel and dimensions for hub, web and rim is shown in figure 6. A nodal force 5000 N will be applied at the distance of 230 mm from the axis on the surface of the rim. The depth for hub and rim are the same which is 60 mm.

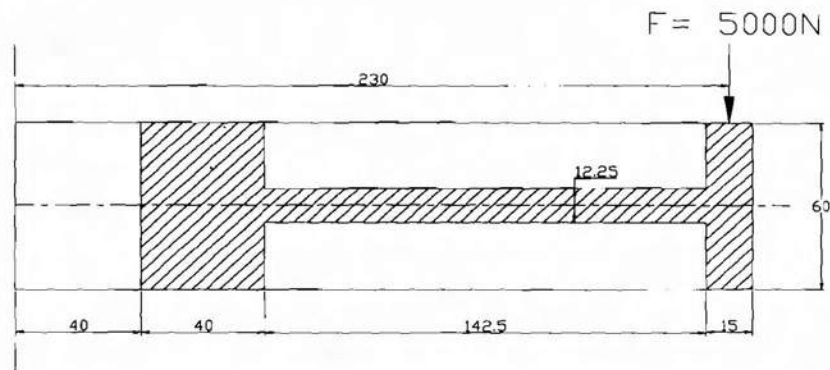


Fig. 6 Profile Of The Wheel With Point Loading 5000 N

The finite element software ANSYS 5.4 V is used to analyse the wheel structure. The wheel model is generated using the available options in the software and the solution is obtained through the postprocessor. However, validation and convergence tests are required before using finite element method to carry out the current investigation. Actually the FEM results depend much on type and size of elements chosen to mesh the model; refinement of the mesh plays important role to give reliable results. First and foremost step before further analysis is the convergence test on wheel as the direct mathematical solution is not available to this problem.

The wheel must be generated with sufficient elements and element increment along load line is required to indicate more reliable stress distribution results. Convergence is ensured before making comparison with the results available in literature. The results obtained by the current analysis are plotted in Figure 7. The dashed lines in the figure indicate the results obtained by Ramamohana Rao A. in

*Bending Of Web In A Wheel Subject To A Single Concentrated Lateral Load On The Rim.*

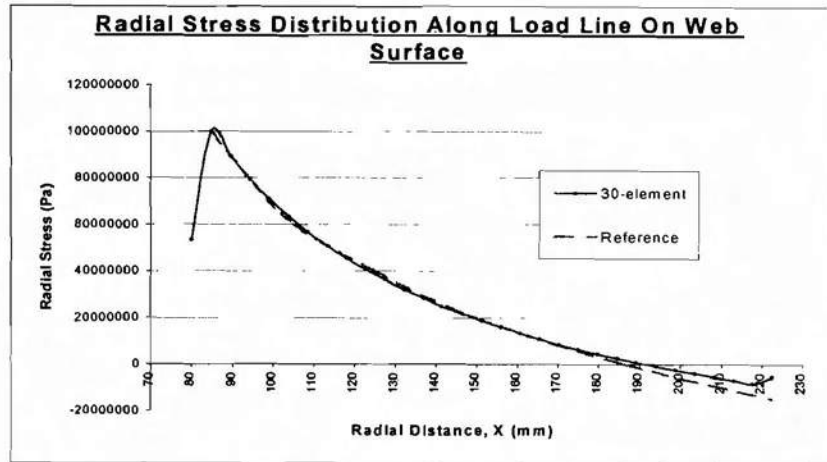


Fig. 7 Graph of Radial Stress and Reference

It can be concluded that the convergence test is successfully achieved as the element refinement of web distance gives a more reliable and accurate solution. The line initially increases with great gradient, inclination reduces and finally the line converges to a stable line. The variance also decreases when numbers of element are added along the web radius. Comparison of convergence test and reference result shows the radial stress distributions are very close to each other and there is variance near the rim-web junction. However, largest VM Stress at the hub-web junction will be different as FEM solution takes this point as rigidly constrained. Element refinement is certainly required at the hub-web junction.

### 3.0 CASE STUDY

A helical gear is applied by 5000 N axial, 5321 N radial and 13740 N tangential load and these forces are transmitted to the helical gear wheel. Results indicate that helical gear wheel has the highest Von Mises stress (close to the hub web junction) due to these loads. However, the significant VM stress is mainly caused by the axial

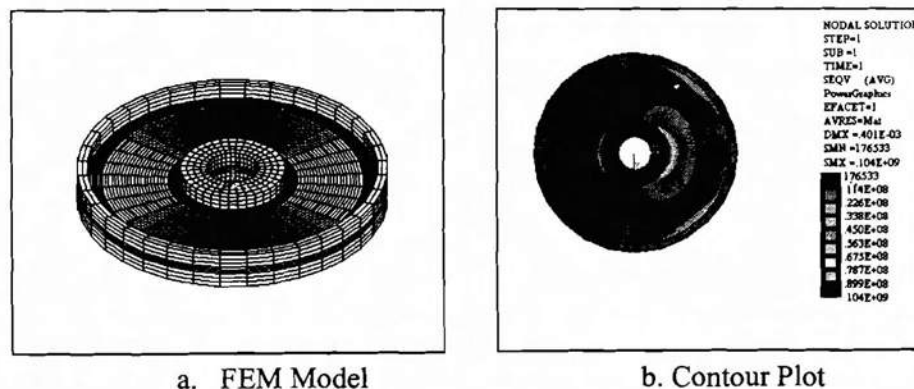
load. The axial component force is predominant over other loads (both radial and tangential load) and it becomes the main stress contributor that we intend to reduce. Although the axial load has the smallest amplitude, its effect on critical stress due to bending moment exerted on gear wheel should be more concerned. Therefore, axial load is applied to analyse the shape optimisation on wheel as this force creates predominant stress.

#### **4.0 FEM RESULTS**

##### **(Constant Web Thickness)**

##### **4.1 Web Inclination Effect**

6 wheel models are generated with 0 (straight web wheel), 5, 8, 10, 12, 15 degree inclined web and the boundary condition (inner hub surface is rigidly constrained) and loading condition (axial load 5000 N exerts on rim) are determined before it is used to generate the desired solution. Stress contour plot indicates the critical part due to such axial loading occur at the hub web junction. For instant, the straight web wheel (zero inclination) has the red colour contour (highest Von Mises Stress) at the hub- web junction. These are shown in the following figure.



**Fig. 8 FEM Model and Contour Plot of Straight Web Wheel**

Query results are used to acquire the solution and the effective stress (Von Mises Stress) is plotted in graphs. This is should as below for these 6 models:

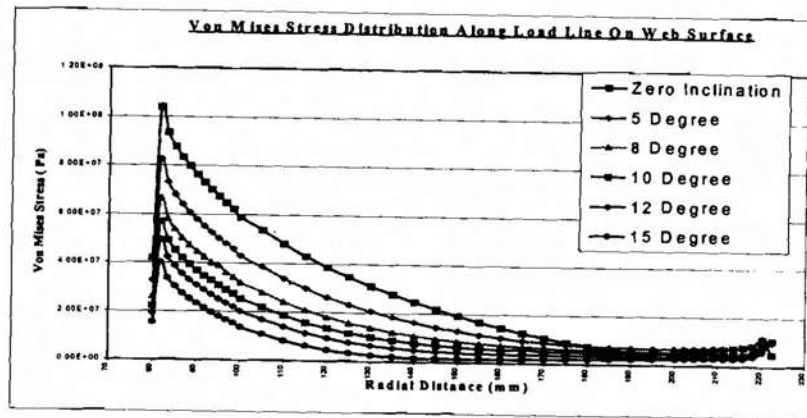


Fig. 9 Von Mises Vs Radial Distance Of Inclination Effect

Results show the Von Mises stress increases to highest point at the hub-web junction and reduces gradually along the radial distance of on the web surface and fluctuate at the rim-web junction. The effective stress reaches its peak at the second point of the node on web surface and the first point has the low stress as the web is assumed rigidly constrained at the inner radius. The significant highest Von Mises stress (tensile stress) caused by the axial load reduces when the web inclination is increased. Although the bottom surface of web has the higher effective stress compares to the upper surface of the web, the bottom surface has the stress in compressive form and this will not cause failure to wheel. The tensile stress normally causes failure prior to compressive stress and compressive stress is much higher compares to the tensile strength. Investigation on the highest Von Mises stress reveals the relationship of the web inclination effect on the stress reduction on wheel, the graph of maximum Von Mises stress of these models is indicated as below:



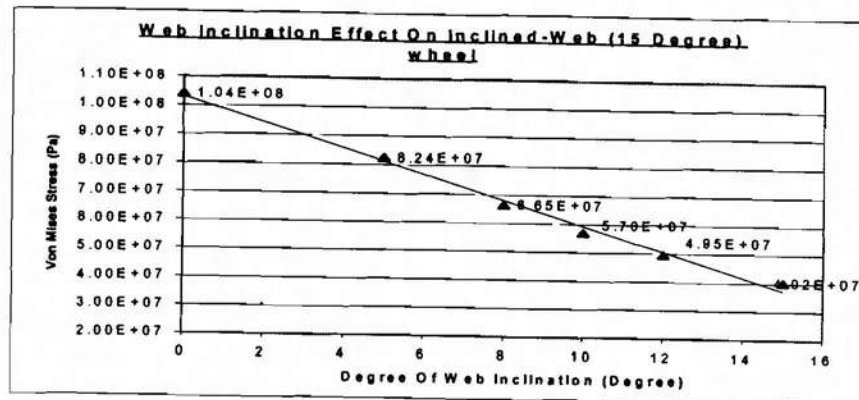


Fig. 10 Von Mises Stress Vs Web Inclination On Wheel

According to Figure 10, straight web wheel has the highest Von Mises Stress 104 M Pa and the stress is reduced to 82.4 M Pa by 5-degree inclination and the stress is decreased by larger web inclination. The stress decreases linearly to the web inclination and shape optimisation is achieved as the stress is reduced by larger web inclination under the axial loading condition.

Apart from these, the web inclination has also decrease the deflection of the wheel and the largest inclination has the minimum deflection in loading direction. As a result, inclination as the geometric parameter of wheel possesses vital importance in the shape optimisation of wheel in order to reduce the critical stress when the wheel is loaded statically. The web is inclined against the loading condition and this increases the stiffness of the wheel which contributes to lower stress distribution as well as deflection under the loading condition. However, the inclination of wheel should not be excessive, as the size and dimension of the wheel need to meet the design requirements and specifications. Both critical stress and deflection of wheel are successfully reduced with inclination and these criteria will determine the performance of wheel when the wheel is loaded.

## 4.2 Fillet Effect

The second parameter involved in the investigation and research is the implementation of fillet effect at the connection of hub-web and rim-web. Since the design of inclined web wheel without fillet shows quite high stress and deflection, fillets at the junctions are required to reduce the stress. As the continuous improvement for the shape optimisation, the previous 15° inclined web wheel without fillet is used for the following investigation. The wheel model are modified by fillet 5mm, 10mm, 12mm and 15 mm.

Results are obtained through FEM solution and the Von Mises stress are plotted in graph shown below:

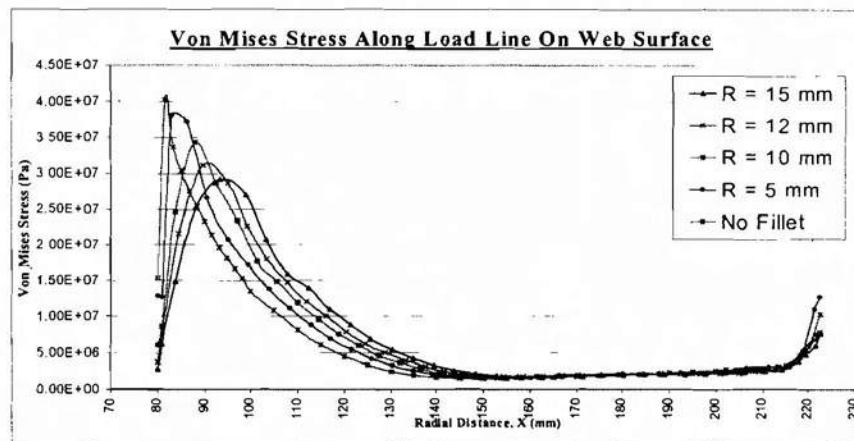


Fig. 11 Von Mises Stress Vs Radial Distance of Fillet Effect

The effective stress (VM Stress) is the stress we are interested to reduce because the effective stress will cause the failure and deformation of wheel. If the effective stress exceeds the yield strength of the wheel material in tensile form, web surface will deformed and crack. The 15-degree inclined web wheel has Von Mises stress 40 M Pa and the graph line arise to highest VM Stress after the first point at hub. The stress distribution is decreased gradually along web radial and the stress increase to about 10 M Pa at the rim-hub junction. This trend happens to all other models and the investigation indicates that the larger fillet the hub-web junction the

lower stress will be obtained. The highest VM stress happens at the point of connection between fillet and web instead of connection of fillet and hub.

An interesting feature of the graphs is the radial distance of the highest stress will be extended to outer from the hub-web junction. The highest stress is shifted from the hub and the value is smaller when the fillet at hub-web junction is increased. This is mainly because of the fillet region has spread the concentrated critical stress over the fillet portion and the highest Von Mises stress is reduced. This is important to optimised the fillet by prolonging the connection point of fillet and web and the stress will be far away from the hub-web junction and reduces. The highest VM stresses are plotted in Figure 11 and the effective stress is reduced exponentially when the fillet at hub-web junction is increased for the models examined here. Further filler at the hub and fillet portion is infeasible because of limitation of hub height, but shape optimisation can be done at the fillet and web portion in order to reduce the effective stress.

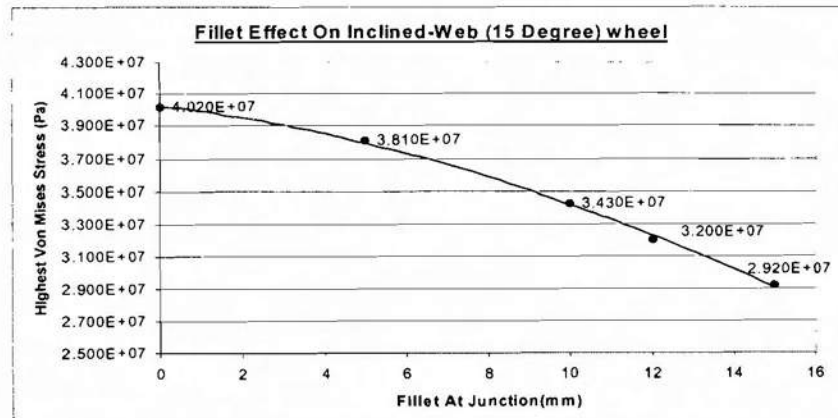


Fig. 12 Highest Von Mises Vs Fillet at Hub-web Junction

Apart from the stress investigation, the deflections of the wheel due to fillet effect are analysed. The deflections of web in load direction (Y direction) are plotted in graph below:

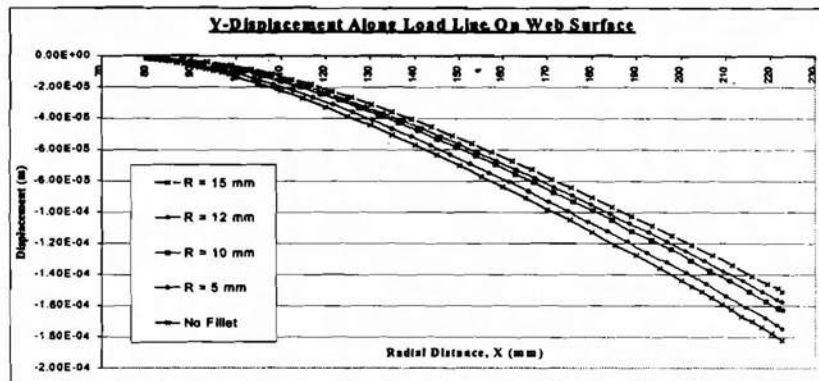


Fig. 13 Y Deflection Vs Radial Distance of Fillet Effect

Figure 13 shows the deflection of web along the radial distance. The deflection is reduced as the fillet of the hub-web junction is increased. The maximum deflection happens at the rim-web junction and this is predicted. The deflection of web increase gradually from the hub-web junction and maximum deflection at the rim-web junction of all graphs are plotted as below:

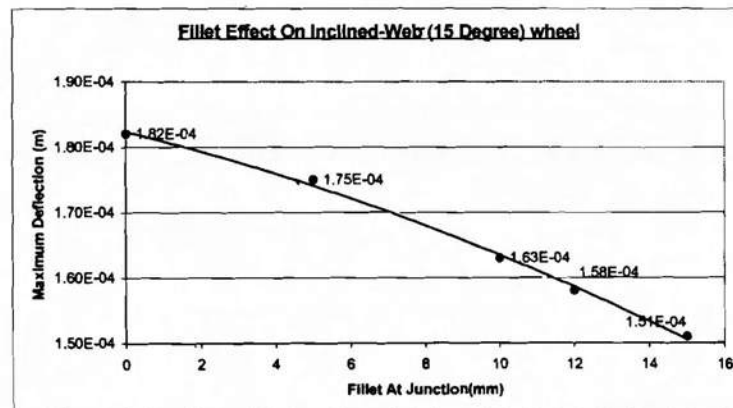


Fig.14 Maximum Deflection Vs Fillet at Junction

The deflection of wheel is very small due to the axial loading condition at rim. The maximum deflection at web outer radius of the 15-degree inclined web

wheel without fillet is only 182 microns. The deflection is reduced proportionally to the additional fillet at hub-web junction. The effectiveness of fillet of reducing the deflection of wheel is increases gradually. The modification at hub-web junction by using fillet has successfully reduced the effective stress distribution and deflection of wheel. Larger fillet can be applied by increase the distance of connection between fillet and web in order to reduce the stress and deflection.

#### 4.3 Rim Volume Effect

Rim volume effect is investigated as another parameter that contributes to reduction of stress and deflection of wheel. The previous modified wheel model (with 15-degree inclined web and 15 mm fillet at hub-web & rim-web junction) are added extra rim volume to determine the rim volume effect towards stress reduction. The investigation consists of three parts and the Von Mises stresses are shown respectively:

- i. Rim Width Effect ( $t$  = rim width thickness, 15mm)

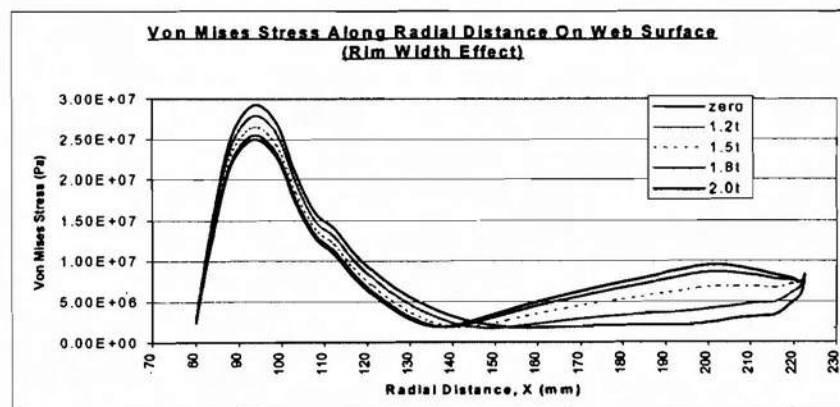


Fig. 15 Von Mises Stress Vs Radial Distance of Rim Width Effect

ii. Rim Depth Effect

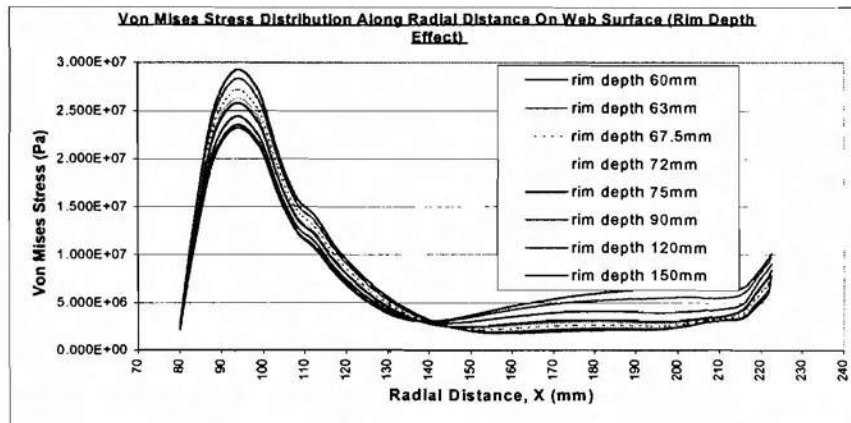


Fig. 16 Von Mises Stress Vs Radial Distance of Rim Depth Effect

iii. Rim Constant d/w Ratio Effect (constant ratio of depth / width = 4 )

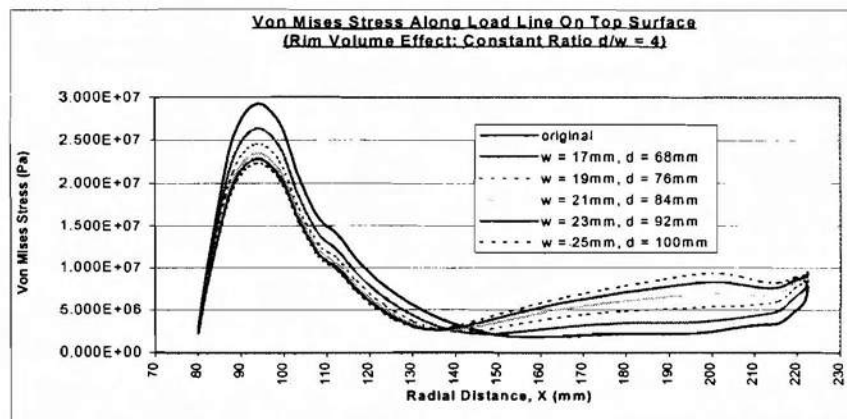


Fig. 17 VM Stress Vs Radial Distance of Constant d/w Ratio

Figures 15, 16 and 17 show that VM Stress is reduced when the rim volume is added either in rim width direction, rim depth direction and constant d/w ratio. There is a similarity of these three graphs: the highest

Von Mises Stress takes place at radial distance 93.43 mm on the web surface and the effective stress decreases gradually outwards to the rim after the peak. However, the reductions of VM stress are different against the shape optimisation. All three rim effects have shown different answer of highest VM stress when the volume is added to the original rim volume. The additional volume added to the rim is treated as extra material on the original rim. The stress fluctuates at region on web surface when the distance is closer to the rim-web junction. The stress reduction percentage and plotted against the increase of rim volume and the graph is indicated in Figure 18 below:

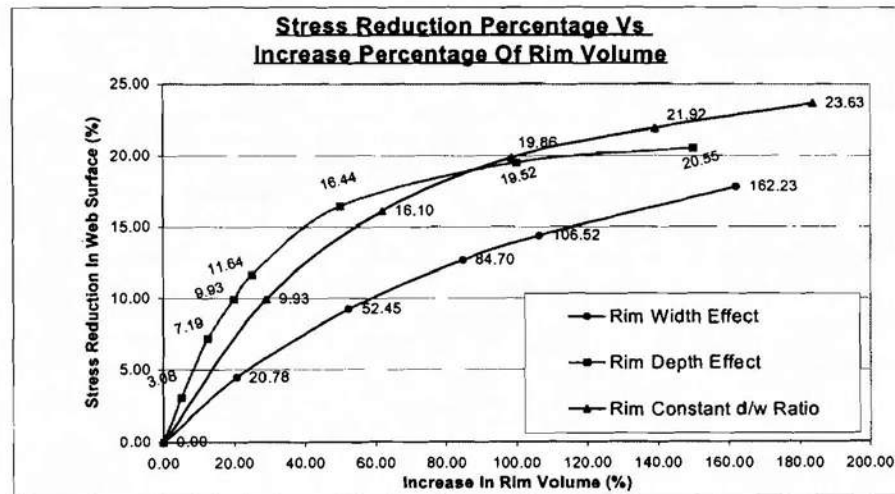


Fig.18 Highest VM Stress Reduction Percentage Vs Extra Rim Volume Percentage

Interesting features of the graph of the model examined here:

- 1) Effectiveness of rim volume of stress reduction decreases when the additional rim volume is added to the original rim. However, the effectiveness of stress reduction is different for these 3 rim width, rim depth and constant d/w effects. Therefore, it is necessary to determine lower extra rim volume is sufficient to

reduce VM stress, as the further stress reduction is not obvious with over sizing the rim volume in view of economic material utilisation.

- 2) Rim depth effect initially has the highest effectiveness of stress reduction by low increase in rim volume compares to others. But the effectiveness gradient decreases rapidly when the increase in rim volume is about 90 % and rim constant d/w ratio has best effectiveness of stress reduction after the 90% increase in rim volume. It is sensible to increase rim volume in rim depth direction in order to reduce effective stress because stress reduction of constant d/w ratio is not significant after almost double rim volume is used.
- 3) The graph also reveals information that effectiveness of stress reduction is lowest for the rim width effect. Results shows gradient of stress reduction due to rim width effect is much smaller than rim depth and constant d/w ratio. Within the range of the proportions examined here, the shape of rim affect the stress reduction effect.

## **5.0 CONCLUSION**

Results apparently show an inclined web wheel can support a larger axial load compared to a straight web wheel. Moreover, optimising the wheel design by modification of geometric parameter such as web inclination, fillet and rim volume effect enable stress reduction and improve wheel strength under static-load analysis. The web inclination has the significant effect on deflection and stress reduction on the straight web wheel when it needs to support the axial load. But web inclination is restricted to design specification and dimensions and proper inclination degree is applied within allowable limitation.

Apart from these, fillet radius plays an important role to replace hub-web and rim-web junction in order to reduce undesirable stress at these critical locations. Rim volume effect like inclination and fillet, reduces the effective stress distribution when the rim volume is increased. However, rim proportions examined by

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investigation reveals effectiveness of rim volume decreases when rim volume is increased and additional rim material should be in depth direction compared to rim volume increase in width and constant depth/ width ratio.

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