STREAMLIKE FUNCTION FORMULATION OF ENTRY FLOW

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ABSTRACT

A numerical study of fluid flow in a rectangular duct is presented. The flow of the working fluid is assumed to be steady and laminar. It is also assumed that the flow field is rotational. The flow analysis is carried out in three dimensional space using Cartesian coordinate. The problem formulation results in a non-linear type partial differential equation for the through flow velocity, which is solved numerically using marching technique. The other formulation is an elliptic type partial differential equation, for which the streamlike function is solved using successive over relaxation method. The results obtained are investigated to determine the velocity distribution across a constant duct cross section and the development of through flow velocity in the duct.

1.0 INTRODUCTION

1.1 General

The development of a parabolic Poiseuille profile downstream of entry flow into plane channel is one of the standard problems in laminar flow theory. It has also attracted more alternative than is warranted by its intrinsic practical importance. This is because it exemplifies certain features of viscous flow.

The importance of this study on entry flow of a viscous fluid is mainly to investigate the velocity distribution in the entrance region. Most approximate analysis of the problem involve some forms of Prandtl's boundary layer approximation and the exact solution of the Navier Stokes equation which is to illustrate certain qualitative aspects of viscous flow. The boundary concept as introduced by Prandtl [1] and the resulting approximations only involve the boundary layer for two dimensional flow. By two dimensional boundary layer flow it means that, a boundary layer which is formed over plane surface, infinite in lateral extent, where the projections of the streamlines of the outer flow on this surface (that is the geometrical surface) are straight lines perpendicular to the leading edge. In three dimensional flow, calculations will constitute a complex mathematical problem. This is due to the fact that three velocity and vorticity components are involved. The partial differential equations governing the fluid motion are complicated and it is hardly surprising that their analytical solution becomes difficult or even impossible unless considerable simplifications are made. In an attempt to overcome these difficulties and thereby extend the range of possible solutions, a finite difference technique is used. The primary reason for this development is, of course, the advent of electronic digital computers featuring both high speed and high capacity.

1.2 Previous Work in Entry Flow

Historically, a numerical solution of an entrance flow in a rectangular duct has been the subject of extensive research. In 1942, Langhaar [1] has postulated a linearization of the Navier Stokes equation of fluid motion which enables him to solve the laminar flow problem for an incompressible fluid in a circular pipe. Mohanty and Asthana [2] have divided the entrance region into two parts, the inlet region and the filled region. Their objective is to re-examine analytically the flow in the pipe entrance region and to verify salient results by experiments.

The result which they obtained shows that, the experimental result agrees well with the analytical one.

Han [3] employed Langhaar's linearization assumption to solve the development of flow problem in a rectangular duct. Sparrow, Lin and Lungren [4] devised an approximate technique for two dimensional entrance flow which was the basis for a recent solution by Wigninton and Dalton [5] for entrance flow in a rectangular duct. Rubin, Khosla and Saari [6] studied the entrance region in two parts. In the first part, the entry region is evaluated by a boundary layer/potential core analysis and in the second part, a numerical solution is obtained for the viscous flow equation which is derived earlier in the first part. They solved a two stream functions, velocity and vorticity systems which are independent of the Reynolds number, with a combined Alternating Direction Implicit Method (ADI) with a point relaxation numerical procedure. The results of axial flow behavior in the first and second parts seem to agree fairly well with the experimental data.

Many recent investigations have centered on the numerical solution of the finite different equation. Hornbeck's [7] finite difference analysis for a circular pipe yielded velocity distributions somewhat different from those of Langhaar but agree very well with regard to the entrance length and the pressure distributions. Experimental investigation of the flow development in a rectangular duct by Sparrow, Hixon and Shavit [8], and Beavers, Sparrow and Magnuson [9] indicate that Han's solution underestimates the entrance length over the estimates of the entrance pressure drop.

2.0 METHOD OF ANALYSIS

2.1 Exact Numerical Method

Laminar incompressible flow in a straight two dimensional axisymmetric rectangular channel have also been investigated by a variety of analytical and numerical techniques. These are typified by the linear boundary layer (Oseen) approximation [3] for evaluating the axial velocity and pressure distribution downstream of an initial entry region. There are more exact numerical analysis using boundary layer [10,11] or Navier Stokes equation [12,13,14] and finally a boundary layer/potential core expansion method [15,16,17] that gives a better models of flow in entry region. Basically, the governing equations used in this analysis include:

Continuity equation :

$$\nabla.\bar{V}=0 \tag{1}$$

Momentum equation :

$$\frac{D\bar{V}}{Dt}=\frac{1}{\rho}\nabla p+\nu\nabla^2\bar{V} \tag{2}$$

2.2 Stream Function Method of Analysis

This method of analysis is only applicable for two dimensional flow system. The introduction of stream function in two dimensional flow system will allow a relatively simple mathematical solution. The mathematical solution is those of a complex variable is given in [18,19]. In order to solve the boundary layer equation, as in the of steady flow, it is much easier to introduce a stream function that satisfies the continuity equation. By introducing this function into the Navier Stokes equation, we will obtain a partial differential equation of the third order. This method of analysis can be understood better from the recent study of entry flow problem [6]. Thus, in general, the governing equations involved are :

Continuity equation :

$$\nabla.\bar{V}=0 \tag{3}$$

Vorticity equation :

$$\nabla x\bar{V}=\bar{\Omega} \tag{4}$$

Momentum equation :

$$\frac{D\overline{V}}{Dt} = \frac{1}{\rho}\nabla p + \nu\nabla^2\overline{V} \tag{5}$$

A defined stream function : $u = -\dfrac{\partial\psi}{\partial y}$

$$v = \frac{\partial\psi}{\partial x} \tag{6}$$

2.3 Streamlike Function Method of Solution

From the previous method of analysis, it is seen that several problems were encountered. The direct method which solved the momentum equation together with the continuity equation constitutes a large system. They are computationally inefficient and suffer from round-off error accumulation. The second method which utilised the vorticity definition besides momentum and continuity equations finally evolve a Poisson type of equation. This requires a considerable programming effort and also limited to two dimensional flow problems.

The streamlike function method of analysis first proposed and used by Abdallah and Hamed [20] seems an advantage. It has successfully been used by Darus [21] in his channel flow problem. However, the fluid model is assumed inviscid. Basically in this method, the governing equations involve are :

Continuity equation :

$$\nabla.\overline{V} = 0 \tag{7}$$

Momentum equation :

$$\frac{D\overline{V}}{Dt} = \frac{1}{\rho}\nabla p + \nu\nabla^2\overline{V} \tag{8}$$

Vorticity equation :

$$\nabla x\overline{V} = \overline{\Omega} \tag{9}$$

A defined streamlike function :

$$w = \frac{\partial \psi}{\partial y}$$

$$v = -\frac{\partial \psi}{\partial z} - \int \frac{\partial u}{\partial x} dy \tag{10}$$

3.0 MATHEMATICAL FORMULATION

3.1 Governing Equation

Assuming steady laminar flow of an incompressible viscous fluid with rotational flow field and constant physical properties, the flow in the entrance region of the rectangular duct may be described by the following equations:

Continuity equation :

$$\nabla . \overline{V} = 0 \tag{11}$$

Momentum equation :

$$\frac{D\overline{V}}{Dt} = \frac{1}{\rho} \nabla p + \nu \nabla^2 \overline{V} \tag{12}$$

where $\overline{V}$ is the velocity vector, P is the total pressure divided by the density and $\overline{\Omega}$ is the vorticity vector which is defined as the curl of $\overline{V}$

$$\nabla x \overline{V} = \overline{\Omega} \tag{13}$$

By substituting equation (13) and assuming steady state, the momentum equation becomes:

$$\left(\overline{V}.\nabla\right)\overline{\Omega} - \left(\overline{\Omega}.\nabla\right)\overline{V} = \nu \nabla^2 \overline{\Omega} \tag{14}$$

The Cartesian coordinate system is used in this analysis to simplify the boundary conditions. Referring to Figure 1 and 2, the x-axis is taken along the duct primary direction while y and z axes are taken in the cross sectional.plane. Equation (11), (12) and (13) are written in this coordinate as,

Continuity equation,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{15}$$

Vorticity equation,

x-component,

$$\xi=\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z} \tag{16a}$$

y-component,

$$\eta=\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x} \tag{16b}$$

z-component,

$$\zeta=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y} \tag{16c}$$

Equation (14) then becomes,

x-component,

$$u\frac{\partial \zeta}{\partial x}+v\frac{\partial \zeta}{\partial y}+w\frac{\partial \zeta}{\partial z}-\xi\frac{\partial u}{\partial x}-\eta\frac{\partial u}{\partial y}-\zeta\frac{\partial u}{\zeta}=\nu\left(\frac{\partial^2 \xi}{\partial x^2}+\frac{\partial^2 \xi}{\partial y^2}+\frac{\partial^2 \xi}{\partial z^2}\right) \tag{17a}$$

y-component

$$u\frac{\partial\eta}{\partial x}+V\frac{\partial\eta}{y}+w\frac{\partial\eta}{\partial z}-\xi\frac{\partial V}{\partial x}-\eta\frac{\partial V}{\partial y}-\zeta\frac{\partial V}{\partial z}=\nu\left(\frac{\partial^2\eta}{\partial x^2}+\frac{\partial^2\eta}{\partial y^2}+\frac{\partial^2\eta}{\partial z^2}\right) \quad (17b)$$

z-component

$$u\frac{\partial\zeta}{\partial x}+V\frac{\partial\zeta}{y}+w\frac{\partial\zeta}{\partial z}-\xi\frac{\partial w}{\partial x}-\eta\frac{\partial w}{\partial y}-\zeta\frac{\partial w}{\partial z}=\nu\left(\frac{\partial^2\zeta}{\partial x^2}+\frac{\partial^2\zeta}{\partial y^2}+\frac{\partial^2\zeta}{\partial z^2}\right) \quad (17c)$$

where u, v and w are the velocity components in the x, y and z direction respectively and ξ,η and ζ are the vorticity components in the x, y, and z direction respectively.

3.2 Simplification of the Governing Equation

An order of magnitude analysis which is valid only for the average values of each term of the flow field, may be applied to equation (17). Representative distances in the axial direction are taken to be in the order of the development length (entrance length) z, while the distances in the transverse direction y are taken in the order of the duct half width "a", and the direction z are taken in the order of the duct half breadth "b". The basic assumption underlying this simplifying approximation is that the entrance length is much greater than the half width and the half breadth of the duct, that is,

$$Y >> a,$$
$$Z >> b$$

From simplicity, we assume the following orders of magnitude:

$$x = O(Z)$$
$$y = O(a)$$
$$z = O(b)$$
$$u = O(U_o)$$
$$v = O\left(\frac{aU_o}{Z}\right)$$

and assuming that b = O(a) that is, the duct has a finite aspect ratio, thus,

$$wO\left(\frac{aU_o}{Z}\right)$$

The final governing equations are reduced to the following forms,

$$\xi = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \tag{18}$$

$$\eta = \frac{\partial u}{\partial z} \tag{19}$$

$$\zeta = -\frac{\partial u}{\partial y} \tag{20}$$

$$u\frac{\partial \xi}{\partial x} + v\frac{\partial \xi}{\partial y} + w\frac{\partial \xi}{\partial z} - \xi\frac{\partial u}{\partial x} - \eta\frac{\partial u}{\partial y} - \zeta\frac{\partial u}{\partial z} = \nu\left(\frac{\partial^2 \xi}{\partial y^2} + \frac{\partial^2 \xi}{\partial z^2}\right) \tag{21a}$$

$$u\frac{\partial \eta}{\partial x} + v\frac{\partial \eta}{\partial y} + w\frac{\partial \eta}{\partial z} - \xi\frac{\partial v}{\partial x} - \eta\frac{\partial v}{\partial y} - \zeta\frac{\partial v}{\partial z} = \nu\left(\frac{\partial^2 \eta}{\partial y^2} + \frac{\partial^2 \eta}{\partial z^2}\right) \tag{21b}$$

$$u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} + w\frac{\partial \zeta}{\partial z} - \xi\frac{\partial w}{\partial x} - \eta\frac{\partial w}{\partial y} - \zeta\frac{\partial w}{\partial z} = \nu\left(\frac{\partial^2 \zeta}{\partial y^2} + \frac{\partial^2 \zeta}{\partial z^2}\right) \tag{21c}$$

3.3 Dimensionless Form of the Governing Equation

Equation (21) may now be expressed in dimensionless form by defining suitable dimensionless variables. The following dimensionless variable are used [22],

$$X = \frac{x}{a\,\mathrm{Re}}, \qquad Y = \frac{y}{a}, \qquad Z = \frac{z}{a}, \qquad P = \frac{(p - p)}{\rho U_o^2}$$

$$U = \frac{u}{U_o}, \qquad V = \frac{v \text{Re}}{U_o}, \qquad W = \frac{w \text{Re}}{U_o}, \qquad \text{Re} = \frac{\rho a U_o}{\mu}$$

Using the above variables, the following expressions are than obtained:

$$\frac{\partial u}{\partial x} = \frac{U_o}{a\,Re}\frac{\partial U}{\partial X}, \qquad \frac{\partial u}{\partial y} = \frac{U_o}{a}\frac{\partial U}{\partial Y}, \qquad \frac{\partial u}{\partial z} = \frac{U_o}{a}\frac{\partial U}{\partial Z}$$

$$\frac{\partial v}{\partial x} = \frac{U_o}{a\,Re}\frac{\partial V}{\partial X}, \qquad \frac{\partial u}{\partial y} = \frac{U_o}{a\,Re}\frac{\partial V}{\partial Y}, \qquad \frac{\partial u}{\partial z} = \frac{U_o}{a\,Re}\frac{\partial V}{\partial Z}$$

$$\frac{\partial w}{\partial x} = \frac{U_o}{a\,Re}\frac{\partial W}{\partial X}, \qquad \frac{\partial w}{\partial y} = \frac{U_o}{a\,Re}\frac{\partial W}{\partial Y}, \qquad \frac{\partial w}{\partial z} = \frac{U_o}{a\,Re}\frac{\partial W}{\partial Z}$$

On applying the above expressions to equations (15), (18) and (21), one obtains the following:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} + \frac{\partial W}{\partial Z} = 0 \tag{22}$$

$$\xi = \frac{\partial W}{\partial Y} - \frac{\partial V}{\partial Z} \tag{23}$$

$$\eta = \frac{\partial U}{\partial Z} \tag{24}$$

$$\zeta = -\frac{\partial U}{\partial Y} \tag{25}$$

$$X\frac{\partial \xi}{\partial X} + V\frac{\partial \xi}{\partial Y} + W\frac{\partial \xi}{\partial Z} - \xi\frac{\partial u}{\partial X} - \eta\frac{\partial u}{\partial Y} - \zeta\frac{\partial u}{\partial Z} = \nu\left(\frac{\partial^2 \xi}{\partial Y^2} + \frac{\partial^2 \xi}{\partial Z^2}\right) \tag{26a}$$

$$U\frac{\partial \eta}{\partial X}+V\frac{\partial \eta}{\partial Y}+W\frac{\partial \eta}{\partial Z}-\xi\frac{\partial v}{\partial X}-\eta\frac{\partial v}{\partial Y}-\zeta\frac{\partial v}{\partial Z}=\nu\left(\frac{\partial^2 \eta}{\partial Y^2}+\frac{\partial^2 \eta}{\partial Z^2}\right) \tag{26b}$$

$$U\frac{\partial \zeta}{\partial X}+V\frac{\partial \zeta}{\partial y}+W\frac{\partial \zeta}{\partial Z}-\xi\frac{\partial w}{\partial X}-\eta\frac{\partial w}{\partial Y}-\zeta\frac{\partial w}{\partial Z}=\nu\left(\frac{\partial^2 \zeta}{\partial Y^2}+\frac{\partial^2 \zeta}{\partial Z^2}\right) \tag{26c}$$

3.4 Derivation of the Streamlike Function Formulation

Equations (22) through (26) do not represent a form entirely suited to finite difference computation. There exists a dimensionless streamlike function. Abdallah and Hamed [20] define a streamlike function ψ which identically satisfies the continuity equation and having the source term. According to this definition, the cross flow velocity components W and V are related through the following expressions :

From definition,

$$W=\frac{\partial \psi}{\partial Y} \tag{27}$$

using continuity equation

$$\frac{\partial V}{\partial Y}=-\frac{\partial W}{\partial Z}-\frac{\partial U}{\partial X}$$

substituting equation (27) into the above equation,

$$\frac{\partial V}{\partial Y}=-\frac{\partial^2 \psi}{\partial Y \partial Z}-\frac{\partial U}{\partial X}$$

integrating with respect to Y, one obtains,

$$V=-\frac{\partial \psi}{\partial Z}-\int\frac{\partial U}{\partial X}dY \tag{28}$$

Substituting equations (27) and (28) into equation (23)

$$\xi = \frac{\partial W}{\partial Y} - \frac{\partial V}{\partial Z}$$

$$\xi = \frac{\partial}{\partial Y}\left(\frac{\partial \psi}{\partial Y}\right) - \frac{\partial}{\partial Z}\left(-\frac{\partial \psi}{\partial Z} - \int \frac{\partial U}{\partial X} dY\right)$$

$$\xi = \frac{\partial^2 \psi}{\partial Y^2} + \frac{\partial^2 \psi}{\partial Z^2} + \frac{\partial}{\partial Z}\int \frac{\partial U}{\partial X} dY \tag{29}$$

or

$$\frac{\partial^2 \psi}{\partial Y^2} + \frac{\partial^2 \psi}{\partial Z^2} = \xi - \frac{\partial}{\partial Z}\int \frac{\partial U}{\partial X} dY \tag{30}$$

equation (30) is first solved for the streamlike function ψ. Then equations (27) and (28) are used to get the cross flow velocities V and W.

3.5 Derivation of Through Flow Velocity Equation

Equation (26) is used in the derivation of the through flow velocity equation. Eliminating the kinematic viscosity term from these equations, two equitions are obtained. Substituting equation (26b) into equation (26a), the following equation is obtained,

$$U\frac{\partial \xi}{\partial X} + V\frac{\partial \xi}{\partial Y} + W\frac{\partial \xi}{\partial Z} - \xi\frac{\partial U}{\partial X} - \eta\frac{\partial U}{\partial Y} - \zeta\frac{\partial U}{\partial Z} = \left(U\frac{\partial \eta}{\partial X} + V\frac{\partial \eta}{\partial Y} + W\frac{\partial \eta}{\partial Z} - \xi\frac{\partial V}{\partial X} - \eta\frac{\partial V}{\partial Y} - \zeta\frac{\partial V}{\partial Z}\right)\left(\frac{\dfrac{\partial^2 \xi}{\partial Y^2} + \dfrac{\partial^2 \xi}{\partial Z^2}}{\dfrac{\partial^2 \eta}{\partial Y^2} + \dfrac{\partial^2 \eta}{\partial Z^2}}\right) \tag{31}$$

substituting equation (26c) into equation (26a), the following equation is obtained,

$$U\frac{\partial\xi}{\partial X}+V\frac{\partial\xi}{\partial Y}+W\frac{\partial\xi}{\partial Z}-\xi\frac{\partial U}{\partial X}-\eta\frac{\partial U}{\partial Y}-\zeta\frac{\partial U}{\partial Z}=\left(U\frac{\partial\zeta}{\partial X}+V\frac{\partial\zeta}{\partial Y}+W\frac{\partial\zeta}{\partial Z}-\xi\frac{\partial V}{\partial X}-\eta\frac{\partial V}{\partial Y}-\zeta\frac{\partial V}{\partial Z}\right)\left(\frac{\dfrac{\partial^2\xi}{\partial Y^2}+\dfrac{\partial^2\xi}{\partial Z^2}}{\dfrac{\partial^2\zeta}{\partial Y^2}+\dfrac{\partial^2\zeta}{\partial Z^2}}\right) \tag{32}$$

Equation (31) and (32) are used to solved separately the through flow velocity U.

3.6 Boundary Condition

Referring to Figure 1 and 2, the following boundary conditions are obtained:

At all walls,

$(y = a,\ z = b)$: $\quad U = V = W = 0 \qquad (33)$

At the entrance : $\quad U(0,0,0) = U_o \qquad (34)$

$\psi(0,0,0) = 0 \qquad (35)$

4.0 METHOD OF SOLUTION

The approach used in the solution of the governing equations is similar to that of those developed by Abdallah and Hamed [23] and Darus [21] for the secondary flow constant area duct. The basic idea behind the mathematical solution is to manipulate the governing equations in order to arrive at a certain first order parabolic type partial differential equations for the through flow velocity. The two equations used for solving the through flow velocity are given by equations (31) and (32). The through flow velocity is then solved by using marching technique [22].

The streamlike function formulation which is given by equation (30) is in the form of an elliptic type partial differential equation. The terms on the right hand side of equation (30) are considered as the source term. This equation is solved for the streamlike function ψ by using successive over relaxation method [24]. For this study, a relaxation factor equal to 1.73 is used. If convergence is not reached after 100 iterations, the program is stop. The cross flow velocity W and V are computed by using equation (27) and (28). Finally the vorticity components ξ , η and ζ are determined by using equation (23), (24) and (25). The iterative procedure can be summarized as follows:

1. All variables are initialized at the entrance
2. Calculate $U(2,J,K)$ from equation (31) or (32) using marching technique.
3. Calculate $\psi(J,K)$ from equation (30) using successive over relaxation.
4. Calculate $W(2,J,K)$ and $V(2,J,K)$ from equation (27) and (28).
5. Calculate $\xi(2,J,K)$, $\eta(2,J,K)$ and $\zeta(2,J,K)$ from equation (23), (24) and (25).
6. Return to step 2

This procedure continues until the flow becomes fully developed. The reader is requested to refer to Figure 11.

4.1 Finite Difference Form of Governing Equation

4.1.1 System of Grid Point

In obtaining the finite difference approximation to the solution of any partial differential equation, it is first necessary to establish a system of grid points in the region occupied by the independent variables. The present problem lends itself to the automatic adoption of grid points located at the intersection of a series of equally spaced horizontal and vertical lines. Such a scheme, known as rectangular grid, is shown in Figure 3. The indexes I, J and K indicate positions in the X, Y and Z directions respectively. The axial mesh spacing is ΔX, while the transverse mesh spacing are $\Delta Y = 1/M21$ and $\Delta Z = Q/M31$. M2 and M3 are integers showing the number of mesh spaces in Y and Z directions respectively and Q which is the aspect ratio of the channel is equal to b/a.

4.1.2 Finite Difference from of the Streamlike Function

Equation (30) is used to solve for the streamlike function. This is solved by using successive over relaxation technique which is the standard method used to solve elliptic type partial differential equation. The finite difference form of this equation is,

$$\psi(J,K)^* = \frac{ORF * (\psi(J,K+1) + \psi(j,K-1) + \psi(J+1,K) + \psi(J-1,K) + (DZ) * XN(J,K))}{4 + (1-ORF) * \psi(J,K)} \tag{36}$$

where,

ORF = over relaxation factor

$\psi(J,K)*$ = new value of streamlike function

$XN(J,K)$ = source term

4.2 Finite Difference from of the Cross Flow Velocity

The cross flow velocities are calculated by using equation (27) and (28). Their finite difference forms are as follows:

$$W(2,J,K) = \frac{(\psi(J+1,K) - \psi(J-1))}{(2*DY)} \tag{37}$$

$$V(2,J,K) = V1(2,J,K) - V2(2,J,K) \tag{38}$$

where,

$$V1(2,J,K) = \frac{-(\psi(J,K+1) - \psi(J,K-1))}{(2*DZ)}$$

$V2(2,J,K)$ is obtained by using the trapezodial rule for the integration, that is,

$$V2(2,J,K) = V2(2,J,K) + \frac{(PR(J-1,K) + PR(J,K) * (DY))}{2} \tag{39}$$

where,

$$PR(J,K) = \frac{(U(2,J,K) - U(1,J,K))}{(DX)}$$

5.0 DISCUSSION OF RESULTS

A computer program has been developed to solve the governing equations for the laminar entrance flow in a rectangular duct. The governing equations were first non-dimensionalized and then recast into finite difference before the numerical solutions were obtained. All the results are thus presented in the non-dimensional form. Stable computational behaviour have been observed throughout.

The grid points used in the numerical computations are (11x21), in the Y and Z directions respectively. The downstream marching axial mesh ΔX equal to 0.03 is used throughout the calculation. The transverse mesh sizes used for the constant Y-Z plane were $\Delta Y = 0.1$ and $\Delta Z = 0.1$. The channel aspect ratio was Q = 2. Figure 3 shows the finite difference grid for the rectangular duct. The inlet velocity profile used was constant in magnitude in the axial direction as shown in Figure 4.

The numerical results are presented at sections of constant X in the form of the through flow velocity in magnitude (computer output) and its contours. These contours are shown in Figure 5 through 10, at ¼ section of duct. In all results presented here, the computation time did not exceed 30 seconds.

The result for the axial velocity development were compared with the analytical solution by Han [3] which is based on an integral technique using a linearized form of the axial momentum equation and the experimental data by Goldstein and Kreid [25] using a Laser Doppler flowmeter. Carlson and Hornbeck numerical solution [11] are also included in the comparison. At the duct center line in Figure 5, the numerical solution for axial velocity development are in good agreement with the solutions obtained by [3], [11] and [25]. The numerical results for X less than 0.08 agrees very well with [3] and [11]. However, after X = 0.2, the numerical solutions were closer to experimental data.

The entrance length L is defined as the dimensionless axial position at which the center line velocity reaches 99 percent of its fully developed value [11]. The numerical solution yielded L = 0.3. Carlson and Hornbeck's numerical solutions yielded L = 0.278 for the first model and L = 0.266 for the second model. Han's analytical value was L = 0.301 while the experimental value found by Goldstein and Kreid was L = 0.36. These large discrepancies, as mentioned by Goldstein and Kreid, are highly sensitive to very small variation in the center line velocity as the fully developed value is approached and even these large variation is of relatively little significance. Table 1 shows the comparisons for the entrance length.

Figures 6, 7 and 8 show the axial velocity contours for different X planes taken at Z = 0.0, Z = 0.5 and Z = 0.75 respectively. the axial velocity profile at Z = 0.00 assembles the center line velocity. At X = 0.33, the axial velocity has reached its fully developed value. It can be observed from these figures that the fluid flow actually characterized the Poiseuille flow which is parabolic. It can also be observed that the effect of vorticity is more significant on the flow especially nearer to the wall (Z = 0.75).

Figure 9 and 10 illustrate the axial velocity profile at constant of X = 0.12 and X = 0.3. It can be observed from these figures that the maximum value of the through flow velocity is initially closer to the wall. However, the velocity contours move downward and towards the center of the duct as the flow moved further downstream. It appear therefore that the progressive change in the velocity contours are due to the effect of denser circulation occurs which starts at the wall and moves towards the center of the duct.

6.0 CONCLUSION

Numerical results have been presented for the laminar entrance flow in a rectangular channel. The adequacy of the results obtained for the entrance length is shown by comparing the solution of this simpler model to that of a more exact model. The choice of equation (31) and (32) at least and qualitatively verifies the entrance flow solution.

From the solutions obtained, it can be concluded that the formulation of the streamlike function is possible for the entry flow problem at least in rectangular duct. Finally, the present study centers only for the velocity distribution in a rectangular duct. It may be suggested that the present study can be also applied to the study of the pressure distribution in a rectangular duct which is more of engineering interest especially in turbomachines.

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Table 1 Entrance Length, L

Investigation	Dimensionless entrance length, L
Carlson and Horbeck (4) :	
First model	0.278
Second model	0.266
Han (5)	0.301
Goldstein and Kreid (6)	
(experimental)	0.360
Present solution	0.300

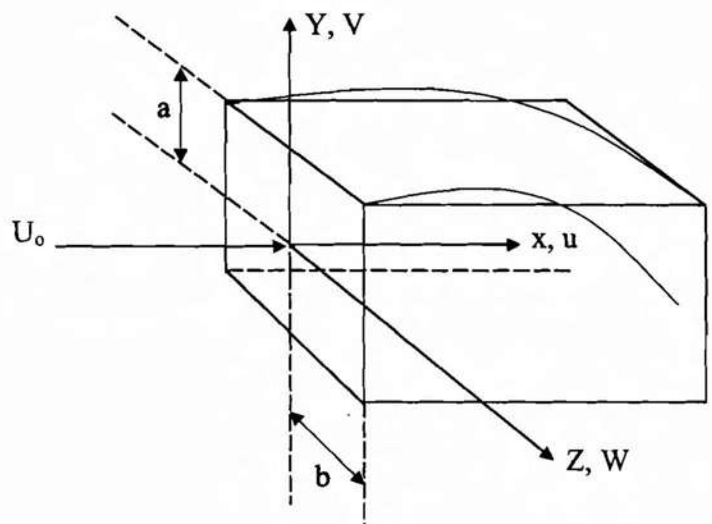

Figure 1 Rectangular channel in Cartesian coordinate

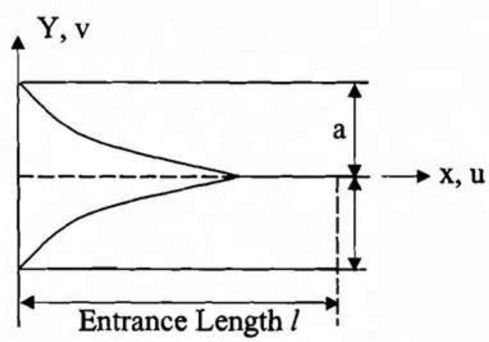

Figure 2 Physical model of entrance region in X-Y plane

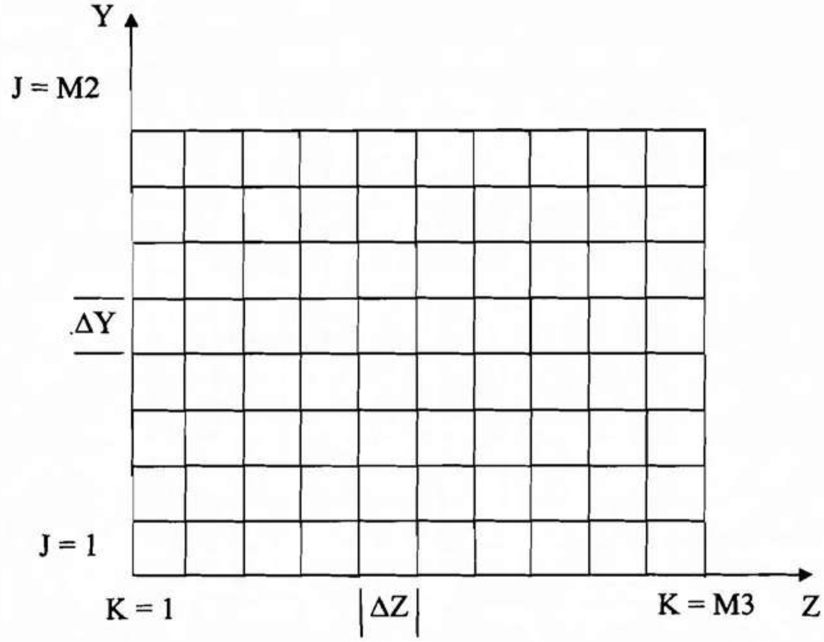

Figure 3 Finite difference grid for rectangular channel cross section (for constant value of I)

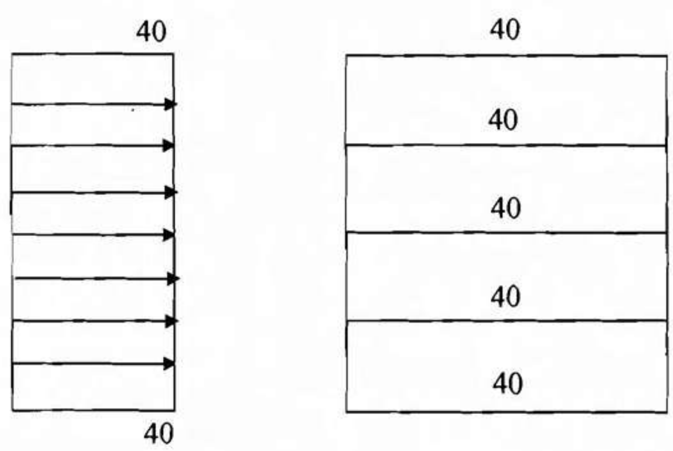

Figure 4 Inlet velocity profile

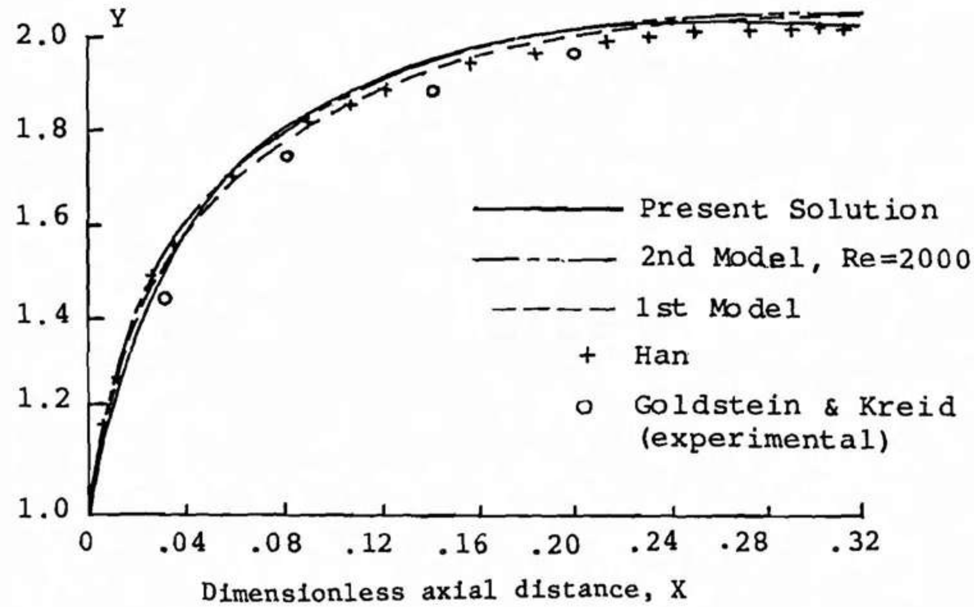

Figure 5 Axial velocity development at duct center line

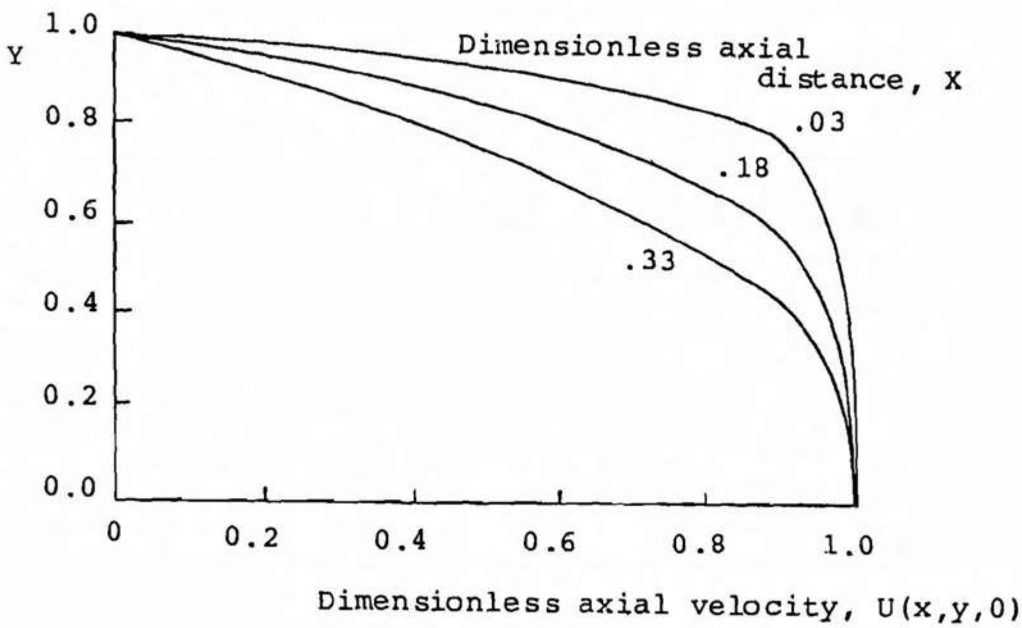

Figure 6 Axial velocity profiles on center plane Z = 0

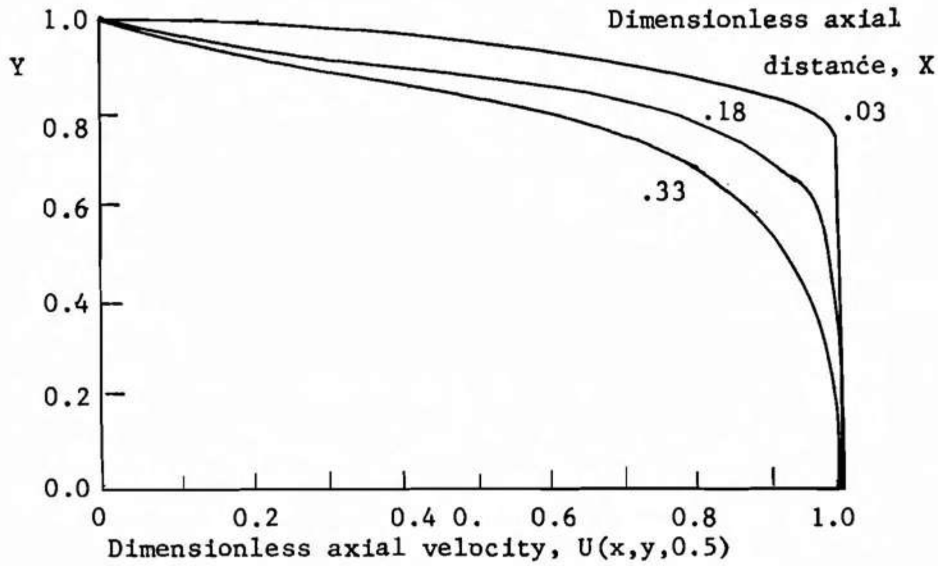

Figure 7 Axial velocity profiles on the plane Z = 0.5

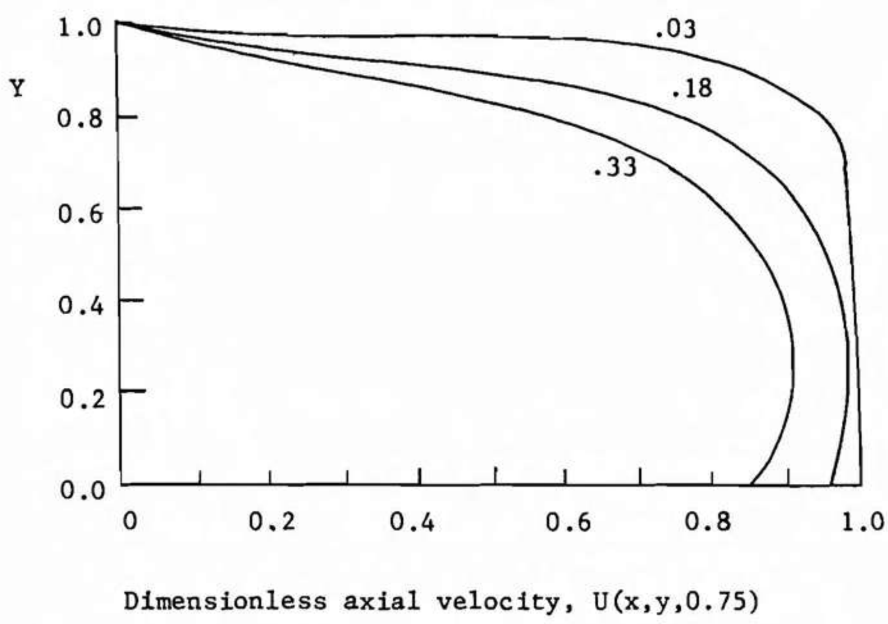

Figure 8 Axial velocity profiles on the plane Z = 0.75

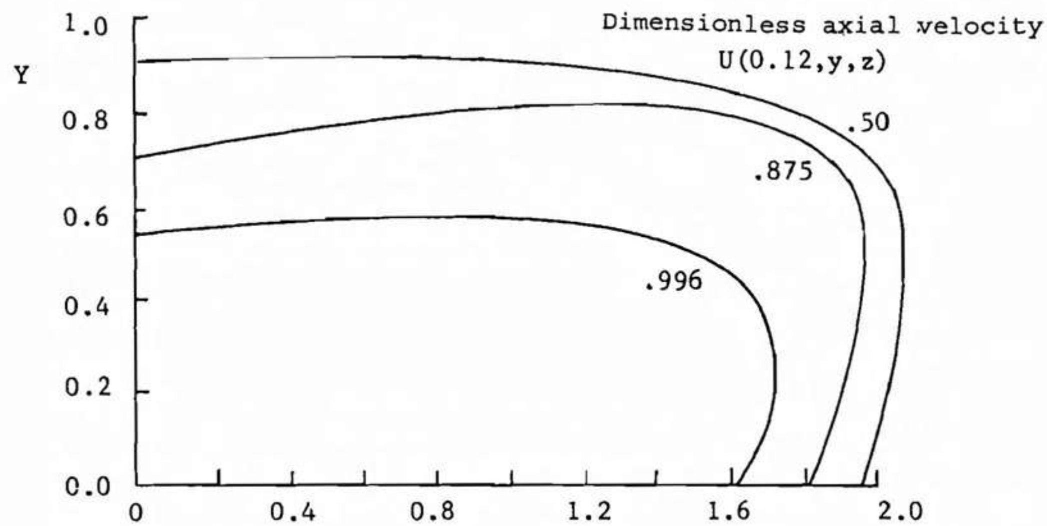

Figure 9 Axial velocity profiles on Y – Z plane at X = 0.12

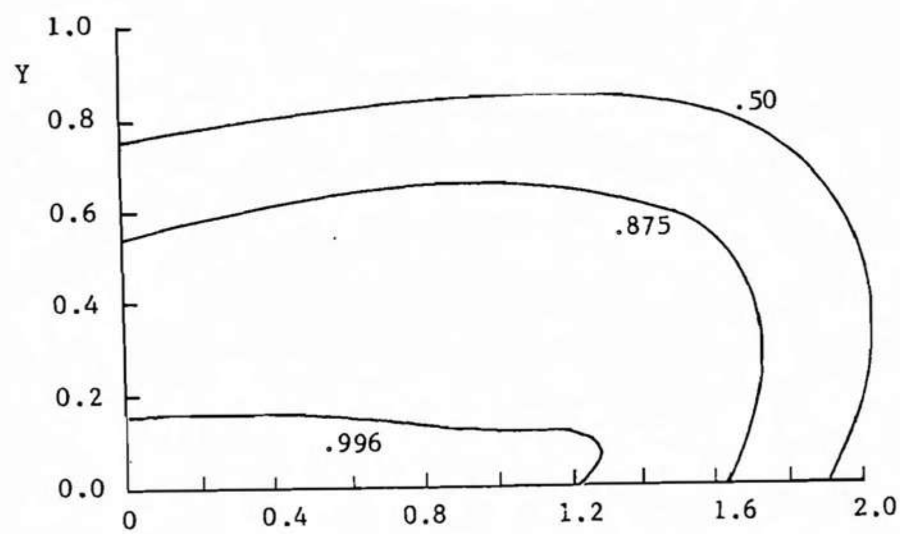

Figure 10 Axial velocity profiles on Y – Z plane at X = 0.3