Faculty of Electrical Engineering
Universiti Teknologi Malaysia
VOL. 9, NO. 1, 2007, 30-36
ELEKTRIKA
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Computational Complexity Reduction for MIMO-OFDM Channel Estimation Algorithms

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Abstract: Channel estimation algorithms have a key role in signal detection in MIMO-OFDM systems. In this system, the number of channel components which need to be estimated is much more than conventional SISO wireless systems. Consequently, the computational process of channel estimation is highly intensive. In addition, the high performance channel estimation algorithms mostly suffer from high computational complexity. In the other words, the system undergoes intensive computations if high performance efficiency is desired. However, there is an alternative solution to achieve both high performance efficiency and relatively low level of computational complexity. In this solution, high efficient channel estimation is firstly designed, and then it is simplified using alternative mathematical expressions. In this research, QR decomposition (QRD) as an alternative mathematical expression to alleviate the computational complexity of those complex algorithms which need matrix inversion is investigated. Herein, the channel estimation algorithm which is targeted to simplify is Least Square (LS) method. The results show QR decomposition can greatly reduce the complexity of LS channel estimation. As an example, for particular scenario, it achieves reduction of computational complexity as much as 77% while it keeps the performance efficiency of the system at the same level.

Keywords: Channel estimation, MIMO, OFDM.

1. INTRODUCTION

Multiple-input-multiple-output (MIMO) antenna architecture has the ability to increase capacity [1] and reliability [2] of a wireless communication system. Space-time coding and spatial multiplexing are two main categories of MIMO system. Space-time coding is able to increase the reliability of the wireless communication link [2, 4] While, spatial multiplexing or layered space-time coding can linearly increase the data rate of the wireless system [1]. Orthogonal frequency division multiplexing (OFDM) is well-known for efficient high speed transmission and robustness to frequency selective channels. Hence, the integration of these two technologies has the potential to meet the ever growing demands of future communication systems [3]. If space-time coding is used at the transmitter, the channel knowledge is required at the receiver to decode the transmitted symbols. Therefore, accurate channel estimation plays a key role in data detection especially in MIMO-OFDM system where the number of channel coefficients is M×N time more than SISO system. (M and N are the number of transmitted and received antenna respectively).

Technically, there are four types of channel estimation [11]; training-based, blind, semi-blind and data–aided channel estimation. However, training-based channel estimation has a lower computational complexity, since the statistical properties of the receiving data is not needed to be examined. In this method, training symbols or pilot tones which are known to the receiver are multiplexed along the data stream and transmitted into wireless link [5].

However, one drawback of training-based channel estimation is increase the overhead of the transmitted block. This problem can be solved using adaptive algorithm. In adaptive algorithm, channel is estimated using pilot data at the start of transmission, then channel can be tracked using recovered data from previous blocks.

The most well-known training-based channel estimation are; LS (Least square), MMSE (Minimum Mean Square Error), LMS (least mean square), RLS (Recursive Least Square). In this paper the focus is on LS algorithm, which is simple to analysis and efficient in Quasi-static channels [10]. The performance of LS algorithm is the same as MMSE where the SNR of the system is high. In addition, unlike the MMSE it does not need any prior knowledge to upgrade the channel coefficients. Another reason to choose LS channel estimation algorithm is because the aim of this paper is to investigate the effectiveness of QRD (QR decomposition) to reduce the computational complexity of channel estimation algorithms in MIMO-OFDM system, not to design high performance channel estimation for this system.

Nonetheless, QRD can also be applied to high performance adaptive channel estimation in time varying channel in order to reduce its computational complexity. Finally, it should be notified that any matrix decomposition method such as Cholesky, LU (Lower Upper), SVD (Singular Value Decomposition) rather than QRD might be used to reduce computational complexity of any algorithm which needs the matrix inversion.

2. SYSTEM MODEL

Figure 1 shows the basic model of MIMO-OFDM system with M and N_r number of antenna at the transmitter and receiver respectively. In this model, MIMO transmission is assumed to be OSTBC (Orthogonal Space-Time Block Coded). Therefore the block of user information after mapping in MPSK modulator is coded by the MIMO-STBC encoder with the matrix dimension of P×M. Where P is the number of time interval needed to transmit this matrix by M number of transmit antenna. It should be mentioned here that every elements of this coded matrix is an OFDM block with 64 symbols. Every columns of this matrix before transmission is fed to M number OFDM module. In this module, before adding cyclic prefix, IFFT transformation is performed for each element of the encoded matrix.

If a column of encoded matrix which enter to the OFDM block is $(X^1, X^2,\ldots,X^m)^T$ in frequency domain then the output of OFDM module will be $(x^1, x^2,\ldots,x^m)^T$ in time domain. Each element of encoded matrix X^k before OFDM module has a length of N= 64 symbols while after OFDM module change to x^k in time domain with the length of 80 symbols.

The received signal after distortion by frequency selective channel and AWG noise at antenna **j** from antenna **i** can be represented by "Equation (1)".

$$y^{ji}(n)=\sum_{l=0}^{L-1} h_l^{ji}(n).x^j(n-l)+v^j(n)\ ,\ i=1,2,..,M \tag{1}$$

where $h_l^{ji}(n)$ is lth channel coefficient between received antenna j and transmitted antenna i at time n. $V^J(n)$ is AWGN with zero mean and variance one. The "Equation (1)" in vector form can be rewritten by "Equation (2), (3), (4), (5)".

$$y^{ji} = \left(y_0^{ji}, y_1^{ji}, \ldots, y_{N+N_{CP}}^{ji} \right)^T \tag{2}$$

$$\begin{pmatrix} h^{ij}(0) & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\ h^{ij}(1) & h^{ij}(0) & 0 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & h^{ij}(1) & h^{ij}(0) & 0 & \cdots & 0 & 0 & 0 \\ h^{ij}(l-1) & \vdots & h^{ij}(1) & \ddots & 0 & \cdots & 0 & 0 \\ 0 & h^{ij}(l-1) & \vdots & \ddots & \ddots & 0 & \cdots & 0 \\ \vdots & 0 & h^{ij}(l-1) & \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \vdots & \vdots & \ddots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & h^{ij}(l-1) & \cdots & h^{ij}(1) & h^{ij}(0) \end{pmatrix} \tag{3}$$

$$x^i = \left(x_0^i, x_1^i, \ldots, x_{N+N_{CP}}^i \right)^T \tag{4}$$

$$v^j = \left(v_0^j, v_1^j, \ldots, v_{N+N_{CP}}^j \right)^T \tag{5}$$

The received signal at antenna j, is summation of transmitted signal by all transmit antenna i=1,2,…M. Hence it can be represented by as

$$y^j(n) = \sum_{i=1}^{M}\sum_{l=0}^{L-1} h_l^{ji}(n).x^i(n) + v^j(n) \tag{6}$$

Equation (6) in vector form can be rewritten by Equations (3), (7), (8), (9) as

$$y^j = \left(y_0^j, y_1^j, \ldots, y_{N+N_{CP}}^j \right)^T \tag{7}$$

$$x^i = \left(x_0^i, x_1^i, \ldots, x_{N+N_{CP}}^i \right)^T \tag{8}$$

$$v^j = \left(v_0^j, v_1^j, \ldots, v_{N+N_{CP}}^j \right)^T \tag{9}$$

Received signal vector at time interval t by antenna j=1,2,…,N_r can be represented as

$$y = \begin{pmatrix} \left(y_0^1, y_1^1, \ldots, y_{N+N_{CP}}^1 \right)^T \\ \left(y_0^2, y_1^2, \ldots, y_{N+N_{CP}}^2 \right)^T \\ \vdots \\ \left(y_0^{N_r}, y_1^{N_r}, \ldots, y_{N+N_{CP}}^{N_r} \right)^T \end{pmatrix} \tag{10}$$

$$y = \begin{pmatrix} \sum_{i=1}^{M} toeplitz(h^{i1}).[x_0^i, x_1^i \cdots x_{N+N_{CP}}^i]^T \\ \sum_{i=1}^{M} toeplitz(h^{i2}).[x_0^i, x_1^i \cdots x_{N+N_{CP}}^i]^T \\ \vdots \\ \sum_{i=1}^{M} toeplitz(h^{iN_r}).[x_0^i, x_1^i \cdots x_{N+N_{CP}}^i]^T \end{pmatrix}$$

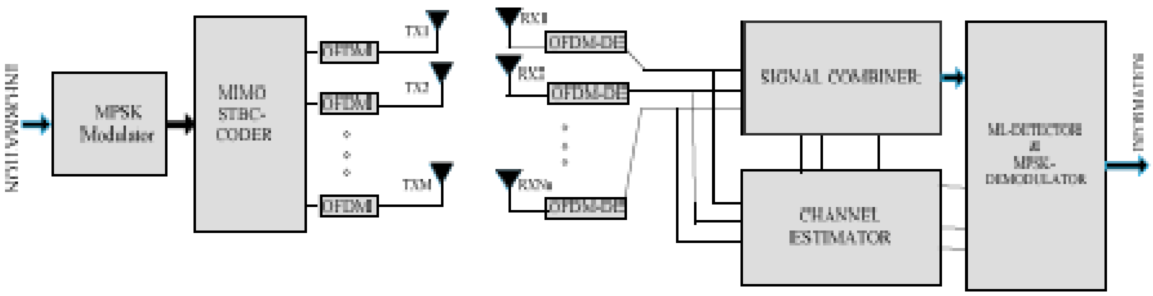

Figure 1. MIMO-OFDM system model

Toeplitz function is a channel matrix function which can be defined as

toeplitz(h) =

$$\begin{pmatrix} h^{ij}(0) & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\ h^{ij}(1) & h^{ij}(0) & 0 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & h^{ij}(1) & h^{ij}(0) & 0 & \cdots & 0 & 0 & 0 \\ h^{ij}(l-1) & \vdots & h^{ij}(1) & \ddots & 0 & \cdots & 0 & 0 \\ 0 & h^{ij}(l-1) & \vdots & \ddots & \ddots & 0 & \cdots & 0 \\ \vdots & 0 & h^{ij}(l-1) & \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \vdots & \vdots & \ddots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & h^{ij}(l-1) & \cdots & h^{ij}(1) & h^{ij}(0) \end{pmatrix} \quad (11)$$

After removing cyclic prefix and FFT transformation by OFDM demodulator, received signal in frequency domain can be represented as

$$Y_k^j = \sum_{i=1}^{M} H_\kappa^{ji} . S_K^i + V_k^j \quad (12)$$

V_k^j is AWGN in frequency domain and it can be calculated using

$$V_k^j = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} v^j . e^{-j2\pi kn/N} \quad (13)$$

Receive signal in vector form can also be represented as

$$\begin{aligned} Y^j &= \sum_{i=1}^{M} S^i H_\kappa^{ji} + V^j \\ &= \mathrm{S} \cdot H^J + \mathrm{V}^J \end{aligned} \quad (14)$$

In more detail each of variable in Equation (14) can be written as in Equations (15),(16),(17) and (18)

$$Y^j = \left[Y_1^j(n), y_2^j(n), ..., y_N^j(n) \right]^T \quad (15)$$

$$H_\kappa^{ji} = \begin{pmatrix} H_1^{(1,J)} \\ H_2^{(1,J)} \\ \vdots \\ H_N^{(1,J)} \\ \vdots \\ H_1^{(M,J)} \\ H_2^{(M,J)} \\ \vdots \\ H_N^{(M,J)} \end{pmatrix} \quad (16)$$

$$S = \begin{pmatrix} S_1^1(n) & 0 & \cdots & 0 & \cdots & S_1^M(n) & 0 & \cdots & 0 \\ 0 & S_2^1(n) & \cdots & 0 & \cdots & 0 & S_1^M(n) & \vdots & 0 \\ \vdots & 0 & \ddots & \vdots & \vdots & \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & 0 & S_N^1(n) & \cdots & 0 & \cdots & 0 & S_1^M(n) \end{pmatrix} \quad (17)$$

$$V^j = \left(V_1^j(n), V_2^j(n), \cdots, V_N^j(n) \right)^T \quad (18)$$

Received signal by antenna [1, 2 …N_r] infrequency domain after removing cyclic prefix and FFT transformation can be written as

$$Y = S \cdot H + V \quad (19)$$

In more detail each of variables in Equation (19) can be written as in Equations (17), (20), (21) and (22)

$$Y = \begin{pmatrix} Y_1^1(n) & Y_N^2(n) & \cdots & Y_N^{N_r}(n) \\ Y_2^1(n) & Y_N^2(n) & \cdots & Y_N^{N_r}(n) \\ \vdots & \vdots & \vdots & \vdots \\ Y_N^1(n) & Y_N^2(n) & \cdots & Y_N^{N_r}(n) \end{pmatrix} \quad (20)$$

$$H = \begin{pmatrix} H_1^{(1,1)} & H_1^{(1,2)} & \cdots & H_1^{(1,N_r)} \\ H_2^{(1,1)} & H_2^{(1,2)} & \cdots & H_2^{(1,N_r)} \\ \vdots & \vdots & \vdots & \vdots \\ H_N^{(1,1)} & H_N^{(1,2)} & \cdots & H_N^{(1,N_r)} \\ \vdots & \vdots & \vdots & \vdots \\ H_1^{(M,1)} & H_1^{(M,2)} & \vdots & H_1^{(M,N_r)} \\ H_2^{(M,1)} & H_2^{(M,2)} & \vdots & H_2^{(M,N_r)} \\ \vdots & \vdots & \vdots & \vdots \\ H_N^{(M,1)} & H_N^{(M,2)} & \cdots & H_N^{(M,N_r)} \end{pmatrix} \quad (21)$$

$$V = \begin{pmatrix} V_1^1(n) & V_N^2(n) & \cdots & V_N^{N_r}(n) \\ V_2^1(n) & V_N^2(n) & \cdots & V_N^{N_r}(n) \\ \vdots & \vdots & \vdots & \vdots \\ V_N^1(n) & V_N^2(n) & \cdots & V_N^{N_r}(n) \end{pmatrix} \quad (22)$$

In this research block type method to estimate the subcarrier has been used. Therefore, all the elements in matrix H in Equation (20) have to be estimated completely. Detection of the OFDM block can be done using Alamouti scheme [2].User information can be detected after M-array remapping.

3. LS CHANNEL ESTIMATION IN FREQUENCY DOMAIN FOR MIMO-OFDM

From Equation (12), it can be seen that for estimation of channel component between receive antenna j and transmit antenna i=1,2,…M, the number of subcarriers which has to be estimated is M×N. where N is the number of subcarriers. In the other words for every receive antenna j=1,2,…N_r vector H_κ^{ji} in Equation(16) has to be estimated.

If one OFDM training block with N subcarriers transmitted from every of transmitted antenna, then from the model for every receive antenna there will be N equation with N×M unknown, hence these equations are under determined and can not be solved. For solving this problem there are two solutions, first solution is transmitting M OFDM blocks which in practical case is not applicable. Second solution is reducing the unknown elements by looking at an alternate representation of the received signal, called the transform-domain estimator that was first proposed by van de Beek in [9] for OFDM systems and well explained in [10] for MIMO-OFDM system. Base on this method CFR (Channel Frequency Response) can be expressed in terms of the CIR (Channel Impulse Response) through the Fourier transformation. Hence, the received signal model in “Equation (14)” can be expressed in terms of the CIR. The benefit of this representation is that usually the length of the CIR is much less than the number of subcarriers of the system. CIR representation can be achieved using following transformation

$$\mathrm{H}^{(j,i)} = F \cdot \mathrm{h}^{(j,i)} \tag{23}$$

where $\mathrm{h}^{(j,i)}$ is the (L×1) channel impulse vector and F is Fourier transform in vector form, and it can be represented as

$$F = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{\frac{-j2\pi(1)(1)}{N}} & \cdots & e^{\frac{-j2\pi(1)(L-1)}{N}} \\ \vdots & \vdots & \vdots & e^{\frac{-j2\pi(2)(L-1)}{N}} \\ 1 & e^{\frac{-j2\pi(N-2)(1)}{N}} & \cdots & \vdots \\ 1 & e^{\frac{-j2\pi(N-1)(1)}{N}} & \cdots & e^{\frac{-j2\pi(N-1)(L-1)}{N}} \end{pmatrix}_{N\times L} \tag{24}$$

To extend the matrix Fourier transform to operate on multiple channels following matrix in Equation (25) can be defined as

$$\phi = \begin{pmatrix} F & 0 & \cdots & 0 \\ 0 & F & \cdots & 0 \\ \vdots & \cdots & \ddots & \vdots \\ 0 & \cdots & 0 & F \end{pmatrix}_{M\times DIM(F)} \tag{25}$$

By using this definition, transformation of CFR to CIR in Equation (14) can be done as

$$\begin{aligned} Y^j &= S^j H_k^j + V^j = X^j \cdot \phi \cdot h^j + V^j \\ &= W \cdot h^J + V^j \end{aligned} \tag{26}$$

By applying LS algorithm on Equation (26), channel component can be estimated using Equation (27). By this transformation one OFDM block is enough to estimate the channel. The only condition is N≥M×L.

$$\tilde{\mathrm{h}}^j = (\mathrm{W}^H \cdot W)^{-1} \cdot W^H \cdot Y \tag{27}$$

4. QR DECOMPOSITION SOLUTION

QR decomposition is just an alternative for calculating matrix inversion. There are different methods for QR decomposition. Here Householder algorithm is used. The steps of QRD algorithm to solve LS problem can be presented as follow:

1. Making the LS error function for Equation (27) as represented in Equation (28).

$$\varepsilon = \mathrm{Y} - \mathrm{W}\tilde{\mathrm{h}} \quad \text{and if } \varepsilon = 0 \Rightarrow Y = \mathrm{W}\tilde{\mathrm{h}} \tag{28}$$

2. Decompose W into Hermitian matrix Q and upper triangular matrix R using Householder algorithm as Equation (29).

$$\mathrm{Y} = \mathrm{W}\tilde{\mathrm{h}} = \mathrm{Q}_{\mathrm{M}\times\mathrm{M}} \cdot \begin{bmatrix} \mathrm{R} \\ 0 \end{bmatrix}_{M\times N} \cdot \tilde{h} \tag{29}$$

3. Multiply Hermitian of Q to both side of Equation (29). The result can be represented as in Equation (30).

$$\begin{bmatrix} \mathrm{R} \\ 0 \end{bmatrix}_{M\times N} \cdot \tilde{h} = \mathrm{Q}_{\mathrm{M}\times\mathrm{M}}{}^{\mathrm{H}} \cdot \mathrm{Y} \tag{30}$$

4. Finally, solve the channel using back substitution.

There are two classes of block-based QR decomposition [7]: Householder method and Gram-Schmidt method. In this paper Householder is chosen over the Gram- Schmidt as the Gram-Schmidt is not numerically stable. The details of Householder QR decomposition can be found in reference [7].

5. THE PRINCIPLES OF COMPLEXITY COMPARISON BETWEEN LS AND QRD ALGORITHM

The advantage of using QR decomposition is to reduce the computational complexity of the LS channel estimation. In this research, the computational complexity in terms of number of mathematical operations has been measured. The derivations are based on an Mt-by-Mr MIMO-OFDM system with *N* subcarriers and a channel length of L. The known matrix **W** has dimensions (N ×L.Mt). For simplicity in notation L.Mt is denoted by M. For a consistent comparison, the complex operations are converted to real operation equivalents.

Table 2 shows the real equivalent operations for the various complex operations. In addition, each type of real operations has different levels of complexity when implemented in the hardware. For example multiplications, additions, and subtractions can be set to 1 FLOPs (Floating Point Operations), divisions to 6 FLOPs, and square roots to 10 FLOPs (table-1). It should be emphasized that counting of the number operations is only an estimate of the computational complexity of the algorithms. A more exact measure would be to implement the algorithm in hardware and count the number of instructions and processing time required. However, in computer simulations, FLOP counts can give a good indication of the relative complexity of different algorithm.

Table 1. Number of flops in every real operation

Operation	# of Flops
Multiplication, addition and subtraction	1
Division	6
Square root	10

Table 2. Number of real operations in every complex operation

Complex operation	Number of Real Operation		
	Multiplication	Division	Subtraction and Addition
Multiplication	4	2	0
Division	6	3	2
Subtraction / Addition	0	0	2
Complex magnitude	2	0	1

The complexity equations are derived by counting each type of operations in the various algorithms. The equations to count the number of operation in LS algorithm are shown in Table 3, and for QR-LS in Table 4.

Table 3. The number of mathematical operation in LS

Operation	# of Complex Multi	# of Complex Add/sub	# of Complex Division	# of Square Root
$A = W^H \bullet W$	NM^2	$NM^2 - M^2$	0	0
A	M^3	M^3	$3M\ 2/2 + M/2$	0
$B = A^{-1} \bullet W^H$	NM^2	$NM^2 - NM$	0	0
$h = B \bullet Y$	NM	$NM - M$	0	0

Table 4. The number of mathematical operation in H-QRD

Operation	# of Comp. Multi	# of Comp. Add/sub	# of Comp. Div	# of Sq root	# of Comp. Mag
Determine $\|v\|$	0	$NM/2 - M^2/4 - M/4$	0	0	$NM - M^2/2 - 2$
Update $v(1)$	M	M	M	M	M
Determine	0	0	M	0	0
Update W	$NM^2 + 2NM - M^3/3 - M^2/2 + 5M/6$	$NM^2 + NM - M^3/3 - M^2/2 + M/6$	0	0	0
Update Y	$2NM - M^2 + M$	$2NM - M^2$	0	0	0
Back substitution	$M^2/2 - M/2$	$M^2/2 + M/2 - 1$	0	0	0

6. SIMULATION RESULTS

The system specification for this simulation can be summarized in Table 5. For this simulation the channel has $L=16$ paths where the amplitude of each path varies independently according to the Rayleigh distribution with an exponential power delay profile [8], and can be represented as in Equation (31).

The results can be classified into two parts; performance comparison and complexity comparison results between QR and LS channel estimations. These are presented in the next sections.

Table 5. Simulation parameters for MIMO-OFDM system

System	MIMO(STBC)-OFDM
# receive antenna	2
# transmit antenna	2
Channel	Frequency selective, Rayleigh fading
Noise	AWGN
# subcarrier	64
# cyclic prefixes	16
Channel length	16
T_{rms} (RMS delay spread)	25 ns
T_S -sampling frequency	1/80 MH

$$h_l = N(0, 1/2\sigma_l^2) + jN(0, 1/2\sigma_l^2) \quad l=1,2,...,L-1 \qquad (31)$$

where $\sigma_l^2 = (1 - e^{\frac{-T_S}{T_{RMS}}}) \times e^{\frac{-LT_S}{T_{RMS}}}$ And for normalization $\sum_{l=0}^{L-1} \sigma_l^2 = 1$ and L approximated by L= $\frac{10\ T_{rms}}{T_s}$.

6.1 Performance Comparison Between LS and QRD Algorithm

The performance of two channel estimation algorithms in the system was investigated using BER and MSE. Figure 2 and 3 show the performance comparison of both algorithms.

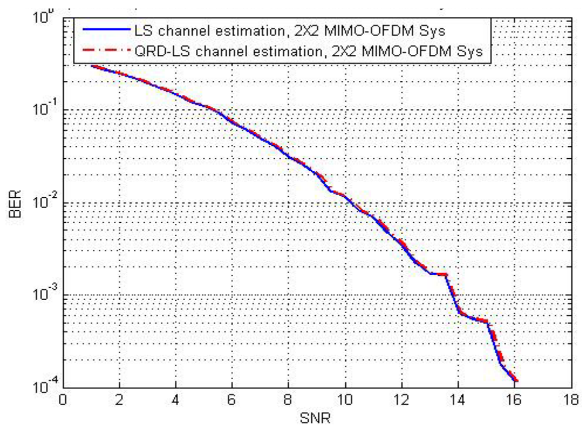

Figure 2. Performance comparisons in terms of system bit error rate

Figures 2 and 3 show that the BER and MSE graphs of the LS and QRD estimation algorithm are completely overlap on each other. Hence it can be concluded that the QRD and LS channel estimation algorithms have the same performance efficiency. These results was expected because the QRD method essentially solves for the least square solution, but it achieves it through matrix decomposition whereas in standard LS channel estimation, a pseudoinverse is used to solve for the channel unknowns.

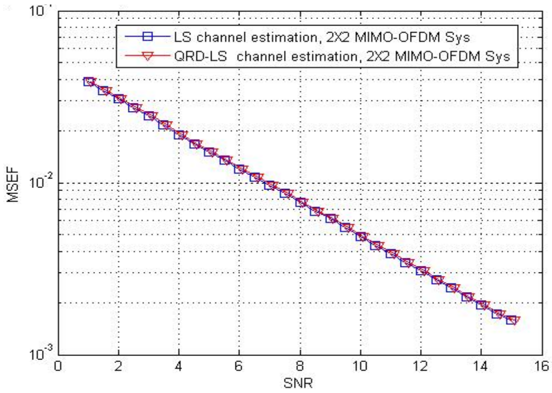

Figure 3. performance comparisons in terms of Mean Square Error

The use of QRD methods does not improve the performance in terms of lower channel estimation error. However, in the next section the benefit of QRD, which is the significant reduction of the complexity of the system, is portrayed.

6.2 Complexity Comparison Between LS And QRD Algorithm

Using the system parameters for the MIMO-OFDM system specified in Table 5, the number of operations for a 2 transmit antenna system with a channel length of 5 and 16 was calculated for the two algorithms. In this section, the complexity comparison in terms of FLOPs count is performed for two algorithms. Figures 4 and 5 show the complexity comparison of both algorithms.

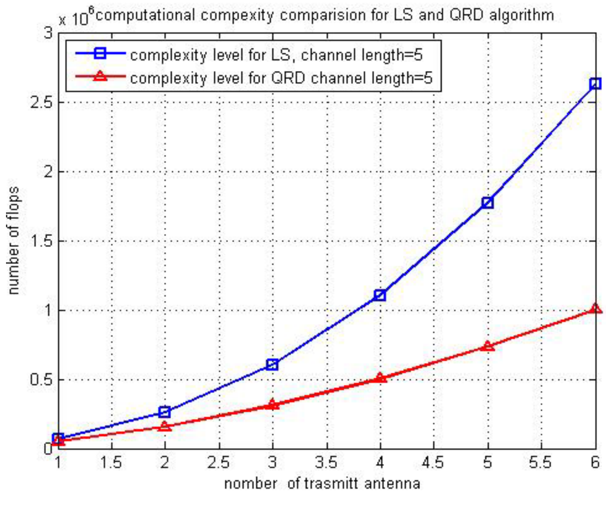

Figure 4. Complexity comparisons between LS and QRD with channel length=5

In Figure 4, the number of channel length is hold at 5 while the number of transmit antenna vary from 1 to 6. The results show the computational complexity of LS algorithm is higher than QRD.

The results in Figure 5 is more highlighted which the number of channel length increase to 16. Increasing the channel length increases the number of unknown parameters, thereby will increase the complexity of the channel estimation. It shows that the LS increases exponentially as the channel length increases and has much higher complexity than the H-QRD for long channel lengths.

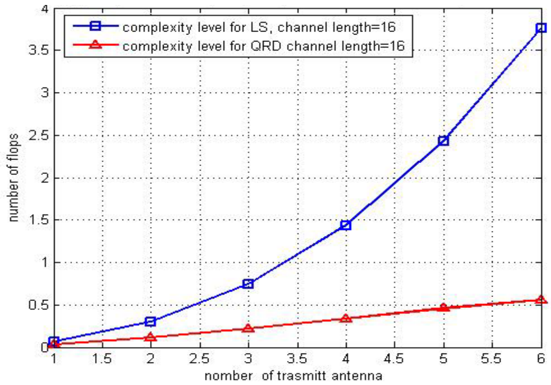

Figure 5. Complexity comparisons between LS and QRD with channel length=16

Figure 6 shows the simulation result using 2 transmit antenna while the number of channel component vary from 1 to 16. The previous conclusion for computational complexity can be made here. In Figure 7, the number of transmit antenna is increased to 8 while the channel is changed from 1 to 16. As expected, when the number of antennas increases, both estimation techniques increase in complexity because the size of the unknown matrix **W** increases. The general trend of the H-QRD method is that it increases almost linearly with the number of transmits antennas of the system. The LS method increases exponentially at a considerably higher rate than the H-QRD methods. Therefore, the H-QRD is especially preferable for higher number of transmit antennas since it does not explode in complexity as the LS solution. Finally the Numerical example for computational complexity comparison between two channel estimation algorithms is provided in Table 6.

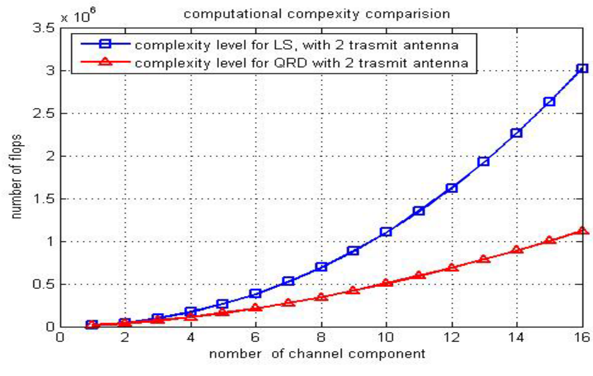

Figure 6. Complexity comparisons between LS and QRD with T_X=2

Table 6. Numbers of complex operation and FLOPs in two algorithms with L=16 and Tx =4 (A: # of complex multiply, B: # of add/sub, C: # of complex division, D: # of square root, E: # of complex magnitude, F: # of flops).

Algorithm	A	B	C	D	E	F
LS	790528	782272	6176	0	0	14373568
QRD	187200	184047	192	2144	128	3376126

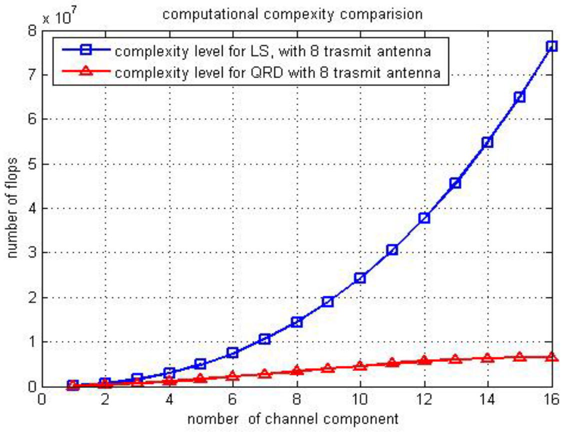

Figure 7. Complexity comparisons between LS and QRD with T_X=8

The results prove that the H-QRD method is lower in complexity than the LS method. The results in Table 6 show that the total number of operation for the LS method is much higher than the H-QRD method. For this simulation scenario using H-QRD achieves a complexity reduction by approximately 77%. This verifies that the H-QRD has significantly lower complexity than that of direct LS estimation via the pseudoinverse, hence a better option for channel estimation.

7. CONCLUSION

The simulation results prove that LS and QRD channel estimation algorithm has the same performance efficiency. However the computational complexity of the QRD channel estimation is much lower than LS algorithm. In addition, computational complexity for QRD channel estimation is approximately linearly proportional with number of transmit antenna and channel length, whereas for LS algorithm is exponentially proportional with the number of transmit antenna and channel length. As finding indicate; using QRD channel estimation, computational complexity of the system for above particular scenario which mentioned in table-6 can dramatically decrease by 77 %.

Finally it can be concluded that QR decomposition can be as an ultimate solution for reduction computational complexity for such algorithms like Recursive Least Square (RLS) when designing high performance algorithm with desirable computational complexity is not possible.

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