THE SCHUR MULTIPLIER AND CAPABILITY OF PAIRS OF SOME FINITE GROUPS

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To my beloved parents,

Nawi Abdul Rahman and Nik Zainun Nik Abdul Rahman.

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ABSTRACT

The homological functors of a group have its origin in homotopy theory and algebraic K-theory. The Schur multiplier of a group is one of the homological functors while the Schur multiplier of pairs of groups is an extension of the Schur multiplier of a group. Besides, a pair of groups is capable if the precise center or epicenter of the pair of groups is trivial. In this research, the Schur multiplier and capability of pairs of groups of order p, p^2, p^3, p^4, pq, p^2q and p^3q (where p and q are distinct odd primes) were determined. This research started with the computation of the normal subgroups of the groups. The generalized structures of the normal subgroups have been found by the assistance of Groups, Algorithms and Programming (GAP) software. By using the Sylow theorems and the results of the nonabelian tensor products, derived subgroups, centers of the groups, abelianization of the groups and the Schur multiplier of the groups, the Schur multiplier of pairs of groups of order p, p^2, pq, p^2q, p^3, p^4 and p^3q were then determined. The classification of the groups had also been used in the computation of the Schur multiplier and capability of pairs of groups. The order of the epicenter of pairs of groups of order p, p^2, p^3, p^4 and pq were also computed by using GAP software to determine the capability of pairs of the groups. The Schur multiplier of pairs of group is found to be trivial or abelian. All pairs of groups where G is isomorphic to the elementary abelian groups of order p^2, p^3 and p^4, nonabelian group of order p^3 of exponent p, direct product of two cyclic groups of order p^2, semi-direct product of two cyclic groups of order p^2, and direct product of cyclic group of order p and nonabelian group of order p^3 of exponent p are capable. For other groups, only certain pairs of groups are capable depending on their normal subgroups.

ABSTRAK

Fungtor homologi bagi suatu kumpulan berasal dari teori homotopi dan teori-K aljabar. Pendarab Schur bagi suatu kumpulan adalah satu daripada fungtor homologi sementara pendarab Schur bagi kumpulan berpasangan adalah suatu perluasan daripada pendarab Schur bagi suatu kumpulan. Di samping itu, suatu kumpulan berpasangan adalah berupaya jika pusat kumpulan tepat atau epi pusat kumpulan bagi kumpulan berpasangan tersebut adalah remeh. Dalam penyelidikan ini, pendarab Schur dan keberupayaan untuk suatu kumpulan berpasangan berperingkat p, p^2, p^3, p^4, pq, p^2q dan p^3q (dengan p dan q adalah nombor perdana yang berbeza) telah ditentukan. Penyelidikan ini bermula dengan pengiraan subkumpulan normal bagi kumpulan tersebut. Struktur umum subkumpulan normal telah ditentukan dengan bantuan perisian Groups, Algorithms and Programming (GAP). Dengan menggunakan teori Sylow dan keputusan bagi hasil darab tensor tak abelan, subkumpulan terbitan, pusat kumpulan, keabelanan dan pendarab Schur bagi kumpulan tersebut, pendarab Schur bagi kumpulan berpasangan berperingkat p, p^2, pq, p^2q, p^3, p^4 dan p^3q kemudiannya ditentukan. Pengelasan bagi kumpulan tersebut juga telah digunakan dalam pengiraan pendarab Schur dan keberupayaan untuk suatu kumpulan berpasangan. Peringkat bagi epi pusat kumpulan bagi kumpulan berpasangan berperingkat p, p^2, p^3, p^4 dan pq juga telah dikira dengan menggunakan perisian GAP bagi menentukan keberupayaan untuk suatu kumpulan berpasangan. Pendarab Schur bagi kumpulan berpasangan tersebut telah ditemui remeh atau abelan. Semua kumpulan berpasangan di mana G isomorfisma kepada kumpulan permulaan abelan berperingkat p^2, p^3 dan p^4, kumpulan tak abelan berperingkat p^3 dengan p eksponen, hasil darab langsung dua kumpulan kitaran berperingkat p^2, hasil darab semi-langsung dua kumpulan kitaran berperingkat p^2, dan hasil darab langsung kumpulan kitaran berperingkat p dan kumpulan tak abelan berperingkat p^3 dengan p eksponen adalah berupaya. Bagi kumpulan yang lain, hanya kumpulan berpasangan tertentu sahaja adalah berupaya bergantung kepada subkumpulan normalnya.

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LIST OF SYMBOLS

$G \otimes_{\mathbb{Z}} G$	-	Abelian tensor square of G
G^{ab}	-	Abelianization of G
$a \mid b$	-	a divides b
A_n	-	The alternating group
$Z(G)$	-	Center of the group G
$C_G(A)$	-	Centralizer of A in G
$[g, h]$	-	The commutator of g and h
G'	-	The commutator subgroup of G
${}^g h$	-	The conjugate of h by g
$\mathbb{Z}_n$	-	Cyclic group of order n
D_n	-	The dihedral group of order $2n$
$G \times H$	-	Direct product of G and H
$G \oplus H$	-	Direct sum of G and H
$\square$	-	End of proof
$Z^*(G)$	-	Epicenter of G
$Z^*(G, N)$	-	Epicenter of a pair of G
$\exp(G)$	-	Exponent of G
$Z^\wedge(G)$	-	Exterior center of G
$C_G^\wedge(x)$	-	Exterior centralizer of x in G
$Z_G^\wedge(N)$	-	Exterior G-center of N
$G \wedge G$	-	Exterior square of G
G	-	A group
$\langle x \rangle$	-	Group generated by the element x
$G \cong H$	-	G is isomorphic to H

$H \triangleleft G$	-	H is a normal subgroup of G
$H \leq G$	-	H is a subgroup of G
$1_{\wedge}$	-	Identity of exterior square
$\|G : H\|$	-	Index of the subgroup H in the group G
$\cap$	-	Intersection
g^{-1}	-	The inverse of g
$\ker(\kappa)$	-	Kernel of the homomorphism κ
gH	-	Left coset of H with coset representative g
$\|G\|, \|x\|$	-	Order of the group G, the order of the element x
(G, N)	-	A pair of G where N is a normal subgroup of G
$\mathbb{Q}_n$	-	The quaternion group of order $4n$
${}^{G}/_{H}$	-	Quotient group for G of H
Hg	-	Right coset of H with coset representative g
$M(G)$	-	Schur multiplier of G
$M(G, N)$	-	Schur multiplier of a pair of G
$G \rtimes H$	-	Semi-direct product of G and H
S_n	-	The symmetric group of order $n!$
$G \tilde{\otimes} G$	-	Symmetric square of G
$G \otimes H$	-	Tensor product of G and H
$G \otimes G$	-	Tensor square of G
1	-	Trivial group
$\wedge$	-	Wedge product

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CHAPTER 1

INTRODUCTION

1.1 Introduction

The homological functors of a group have its origin in homotopy theory and algebraic K-theory. One of the homological functors is the Schur multiplier of a group. The Schur multiplier of a group G, denoted by $M(G)$, was first introduced by Schur [1] while studying projective representations of groups in 1904. Since then, many researchers interested to study on the Schur multiplier of a group.

In 1998, Ellis [2] extended the notion of the Schur multiplier of groups to the Schur multiplier of a pair of groups. Let (G, N) be an arbitrary pair of finite groups where N is a normal subgroup of G. Then the Schur multiplier of the pair, denoted by $M(G, N)$ is a functorial abelian group whose principal feature is a natural exact sequence

$$\begin{aligned} H_3(G) \xrightarrow{\eta} H_3\left({}^{G}\!/\!_{N}\right) \to M(G,N) \to M(G) \xrightarrow{\mu} M\left({}^{G}\!/\!_{N}\right) \\ \to {}^{N}\!/\!_{[N,G]} \to (G)^{ab} \xrightarrow{\alpha} \left({}^{G}\!/\!_{N}\right)^{ab} \to 1 \end{aligned} \tag{1.1}$$

in which $H_3(-)$ denotes some finiteness-preserving functor from groups to abelian groups (to be precise, $H_3(-)$ is the third homology of a group with integer coefficients). The homomorphisms η, μ, α are those due to the functorial of $H_3(-)$, $M(-)$ and $(-)^{ab}$.

The study of capability of groups was initiated by Baer [3] in 1938 who determined all capable abelian groups. Following Hall and Senior in [4], a group G is capable if it is a group of inner automorphisms of some group or equivalently if there exists a group H such that $G \cong {}^{H}/_{Z\,(H)}$. Hall in [5] remarked that characterization of capable groups is important in classifying groups of prime power order.

In 1996, Ellis [6] extended the theory of the capability of groups to a theory for a pair of groups. He defined the capability of a pair of groups in terms of Loday's notion of a relative central extension. Given a normal subgroup N in G, a relative central extension of the pair (G, N) consists of a group homomorphism $d : M \rightarrow G$ and action $(g, m) \rightarrow {}^{g}m$ of G on M satisfying:

(i) $d({}^{g}m) = gd(m)g^{-1}$ for g in G and m in M;

(ii) $mm'm^{-1} = {}^{d(m)}m$ for m and m' in M;

(iii) $N = \text{Image}(d)$;

(iv) the action of G on M is such that G acts trivially on the kernel of d.

The pair of group, (G, N) is said to be capable if it admits a relative central extension with the property that Ker(d) consists precisely of those elements in M on which G acts trivially. Note that M is an arbitrary subgroup of G.

In this research, the Schur multiplier of pairs of groups of order p^n for $n = 1, 2, 3, 4$ and p^nq for $n = 1, 2, 3$ where p and q are distinct primes, and the capability of pairs of groups of order p^n for $n = 1, 2, 3, 4$ and pq for $n = 1$ where p and q are distinct primes are determined.

1.2 Research Background

The study of classification of groups have been of interest of many authors.

These include the classification of groups of order p, p^2, p^3, p^4 and p^2q by Burnside in [7] and groups of order p^3q by Western in [8]. Based on the given classifications, this research focuses on the groups of order p^n for $n = 1, 2, 3, 4$ and p^nq for $n = 1, 2, 3$ where p and q are distinct primes. Throughout this research, G denotes a finite group.

The Schur multiplier of a group is the second integral homology group $H_2(G, \mathbb{Z})$. In 1956, Green [9] proved that the order of the Schur multiplier of a finite group G of order p^n is at most $p^{\frac{1}{2}n(n-1)}$, that is specifically for any group G of order p^n there exists an integer $t(G) \geq 0$ such that $|M(G)| = p^{\frac{1}{2}n(n-1)-t(G)}$. After many years, Berkovich [10], Zhou [11], Ellis [12], Salemkar *et al.* [13], and Niroomand [14, 15] classified the structure of G for the cases $t(G) = 0$ and 1, $t(G) = 2$, $t(G) = 3$, $t(G) = 4$ and $t(G) = 5$ respectively. Recently, Jafari *et al.* [16] determined the Schur multiplier of groups of square free order and groups of order p^2q and pq^2 where p and q are primes and $p < q$ while Rashid [17] determined the Schur multiplier of groups of order p^2 and p^3q for $p < q$. In 2011, Rashid *et al.* [18] computed the Schur multiplier of groups of order p^2q where p and q are primes (with no restriction on p and q). In 2013, Zainal *et al.* [19] computed the Schur multiplier of groups of order p^3 and abelian groups of order p^4 while Ok [20] computed the Schur multiplier of some nonabelian groups of order p^4.

A pair of groups (G, N) is a group G with a normal subgroup N. The Schur multiplier of a pair of groups, $M(G, N)$, is defined to be the abelian group which appears in the natural exact sequence as in (1.1). In 1998, Ellis [2] proved that the order of the Schur multiplier of a pair of finite group G with normal subgroup N of order p^n and quotient group ${}^{G}\!/_{N}$ of order p^m is bounded by $p^{\frac{1}{2}n(2m+n-1)}$ and hence equals to $p^{\frac{1}{2}n(2m+n-1)-t(G,N)}$ for integer $t(G, N) \geq 0$. Salemkar *et al.* [21], Hokmabadi *et al.* [22], Moghaddam *et al.* [23] and Mohammadzadeh *et al.* [24] characterized the structure of such a pair of groups (G, N) when $t(G, N) = 0, 1, 2, 3, 4, 5$.

In 1938, Baer [3] started a research on the capability of groups by determining all capable abelian groups. Following Hall and Senior in [4], a group is capable if it is a central factor group. In 1979, Beyl *et al.* [25] established a special condition for a group to be capable. They showed that a group is capable if and only if the epicenter of the group is trivial. In 1995, Ellis [26] introduced the exterior center of the group and proved that a group is capable if and only if the exterior center of the group is the trivial group.

In 1996, Ellis [6] introduced the exterior G-center of N for a pair of groups and proved that the pair of groups is capable if and only if the exterior G-center of N is a trivial group. Later in 2011, Pourmirzaei *et al.* [28] introduced the precise center for a pair of groups with the property that the precise center for a pair of groups is a subgroup of the exterior G-center of N for a pair of groups. They found that the pair of groups is capable if and only if the precise center for a pair of groups is the trivial group. Besides, they determined all capable pairs of finitely generated abelian groups.

In this research, the classifications of groups of order p^n for $n = 1, 2, 3, 4$ and $p^n q$ for $n = 1, 2, 3$ given in [7] and [8] are used to compute the Schur multiplier of pairs of groups. After that, the capability of pairs of some of these groups are determined.

1.3 Problem Statement

Given the groups of order p^n for $n = 1, 2, 3, 4$ and $p^n q$ for $n = 1, 2, 3$ where p and q are distinct primes, then the following questions are addressed and answered:

(i) What are the Schur multiplier of pairs of groups of order p^n for $n = 1, 2, 3, 4$?

(ii) What are the Schur multiplier of pairs of groups of order $p^n q$ for $n = 1, 2, 3$ where p and q are distinct primes?

(iii) Which pairs of groups of order p^n for $n = 1, 2, 3, 4$ are capable?

(iv) Which pairs of groups of order pq where p and q are distinct primes, are capable?

1.4 Research Objectives

The objectives of this study are:

(i) to compute the Schur multiplier of pairs of groups of order p^n for $n = 1, 2, 3, 4$,

(ii) to compute the Schur multiplier of pairs of groups of order $p^n q$ for $n = 1, 2, 3$ where p and q are distinct primes,

(iii) to determine the capability of pairs of groups of order p^n for $n = 1, 2, 3, 4$,

(iv) to determine the capability of pairs of groups of order pq where p and q are distinct primes.

1.5 Scope of the Study

This research focuses on the determination of the Schur multiplier of pairs of groups of order p^n for $n = 1, 2, 3, 4$ and $p^n q$ for $n = 1, 2, 3$ where p and q are distinct primes, and the capability of pairs of groups of order p^n for $n = 1, 2, 3, 4$ and pq where p and q are distinct primes. For the group of order p, there is only one group and the group is cyclic. For the groups of order p^2, there are two groups in the classification, one is a cyclic group and another one is an elementary abelian group. For the groups of order p^3, there are five groups in the classification which includes three abelian groups and two nonabelian groups which are also known as an extra-special p-groups. All groups in the classifications of groups of order p, p^2 and p^3 are considered. For the groups of order p^4, there are four abelian groups and 11 nonabelian groups in the classification. All four abelian groups are considered in this research, but only six nonabelian groups out of 11 which satisfy the conditions $|Z(G)| = p^2$, $|G'| = p$, and $G' \subseteq Z(G)$ are considered in this research. For the groups of order pq, the computation only focuses on the case when $p < q$ while for the groups of order p^2q, the computation focuses on both cases when $p < q$ and $p > q$. For the groups of order p^3q where $p < q$, there are 27 nonabelian groups. However, only 11 of them are considered in this research. The groups are from the case when p is an odd prime.

1.6 Significance of Findings

The contribution of this research consists of new theoretical findings on determining the Schur multiplier and capability of pairs of groups. The findings obtained can be used for further determination of the Schur multiplier and capability of pairs of a family of groups. Thus, the findings of this research contributes new theories in Group Theory.

Besides that, the Schur multipliers have proved to be a powerful tool in other areas such as algebraic number theory, block theory of group algebras and classification of finite simple groups. In addition, the Schur multiplier is originated from homological algebra. Equally important, homological algebra has indeed found a large number of applications in many different fields, ranging from finite and infinite group theory to representation theory, number theory, algebraic topology and sheaf theory.

1.7 Research Methodology

This research begins with examining the classifications of groups of order p^n for $n = 1, 2, 3, 4$ and $p^n q$ for $n = 1, 2, 3$. As mentioned earlier, the classifications of groups of order p, p^2, p^3, p^4 and p^2q are given by Burnside in [7] and the classifications of groups of order p^3q are given by Western in [8]. Jafari *et al.* [16] determined the Schur multiplier of groups of square free order and groups of order p^2q and pq^2 where p and q are primes and $p < q$ while Rashid [17] determined the Schur multiplier of groups of order p^2 and p^3q for $p < q$. In 2011, Rashid *et al.* [18] computed the Schur multiplier of groups of order p^2q where p and q are primes. In 2013, Zainal *et al.* [19] computed the Schur multiplier of groups of order p^3 and abelian groups of order p^4 and Ok [20] computed the Schur multiplier of some nonabelian groups of order p^4. In the first step of this research, the Groups, Algorithms and Programming (GAP) software has been used to identify the structure of the groups and the normal subgroups of the groups. Next, by using the Sylow theorems and existing results of the nonabelian tensor products, derived subgroups, centers of the groups, abelianization of the groups and the results of the Schur multiplier found in [16–20], the Schur multiplier of pairs of groups of order p^n for $n = 1, 2, 3, 4$ and $p^n q$ for $n = 1, 2, 3$ are computed. The order of the epicenter of pairs of groups of order p^n for $n = 1, 2, 3, 4$ and pq are also computed by using GAP software to determine the capability of pairs of the groups. The research methodology is illustrated in Figure 1.1.

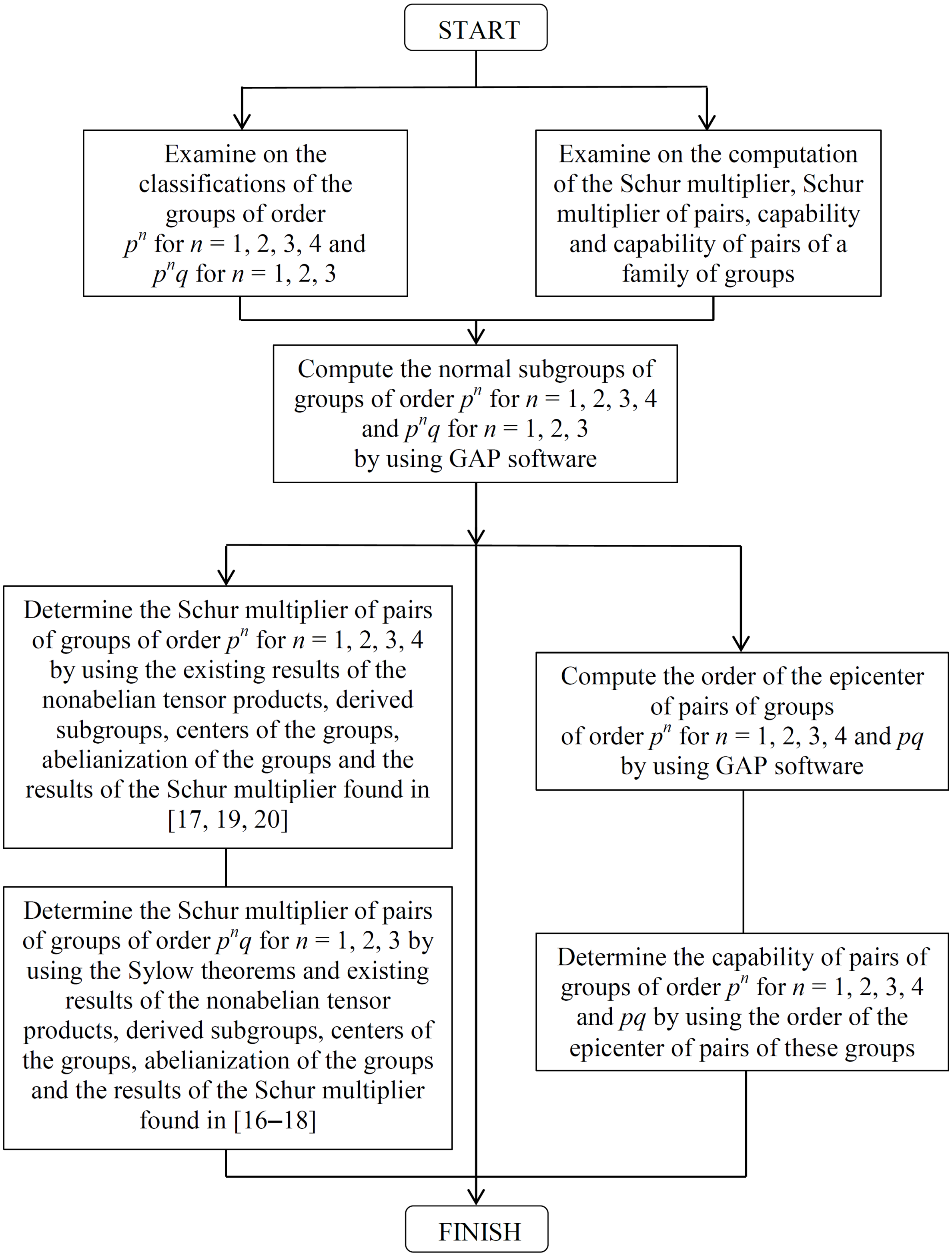

Figure 1.1 Research methodology

1.8 Thesis Organization

The first chapter is the introduction to the whole thesis. This chapter contains a brief introduction on the Schur multiplier of a group, Schur multiplier of a pair of groups, capability of a group and capability of a pair of groups, research background, problem statement, research objectives, scope of the study, significance of the study and research methodology.

The literature review of this research is provided in Chapter 2. The contribution of many researchers on the Schur multiplier of a group, nonabelian tensor product, Schur multiplier of a pair of groups, capability of a group and capability of a pair of groups are also included. Some concepts and established results from previous researchers that are used throughout this research are presented in this chapter. Besides, a brief introduction on Groups, Algorithms and Programming (GAP) software is also given.

Chapter 3, Chapter 4 and Chapter 5 contains the main results of this research. In Chapter 3, the computations of the Schur multiplier of pairs of groups of order p^n for $n = 1, 2, 3, 4$ are presented while in Chapter 4, the computations of the Schur multiplier of pairs of groups of order $p^n q$ for $n = 1, 2, 3$ where p and q are distinct primes are shown. In Chapter 5, the determination of the capable pairs of groups of order p^n for $n = 1, 2, 3, 4$ and pq where p and q are distinct primes are presented.

Chapter 6 presents the summary and conclusion of this research. Some useful suggestions for future research are also stated in this chapter. Figure 1.2 illustrates the contents of this thesis.

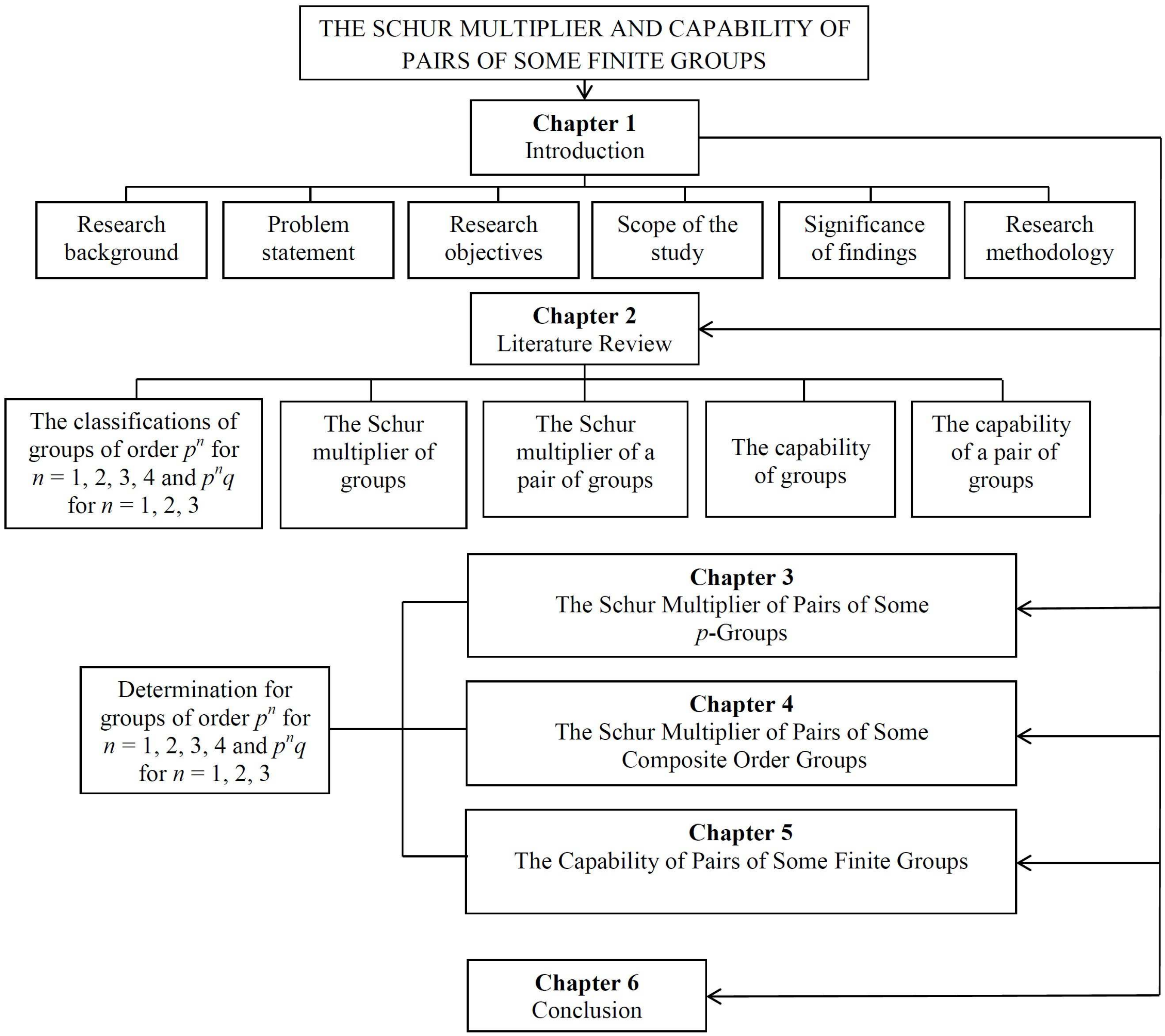

Figure 1.2 Thesis organization

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