# SPACE TIME RAINFALL MODELING USING HIDDEN MARKOV MODEL

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To My Beloved Family

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# ABSTRACT

Statistical modeling of rainfall in space-time scales is essential in providing information on the behavior of the rainfall process at a particular region. One of the conventional ways of rainfall modeling is done through studies of the probabilistic structure of the rainfall. In the last decade, many statistical rainfall models were done regardless of the atmospheric information. A model which succeeded in incorporating atmospheric information will be useful in studies of climate variability or climate change. Therefore, this study applied the hidden Markov model (HMM) and non-homogeneous hidden Markov model (NHMM) to model daily rainfall of 40 stations in Peninsular Malaysia over a period of 34 years during the Northeast monsoon and Southwest monsoon. Four different models for rainfall amounts namely single exponential distribution, mixture of two exponential distributions, single gamma distribution and mixture of two gamma distributions were examined for the non-zero rainfall amount. The relationship between the local rainfall process and the large scale atmospheric process were investigated through the behavior of the composite wind anomalies at 850hPa with omega vertical velocity at 500hPa on each hidden state from HMM. The HMM was then extended to NHMM by including the time-varying covariates (atmospheric variables) into the model. So far, the most popular algorithm used for the parameter estimation of HMM was Baum-Welch algorithm, but it was only guaranteed to find a local maximum with a high dependency on initial parameters chosen. Hence, this study also proposed a parameter estimation, segmental *K*-means algorithm that sacrifices some of Baum-Welch's generality for computational efficiency. The findings here showed that the segmental *K*-means algorithm is able to improve the conventional model with a reduced computational time. The performances of the HMM and NHMM are assessed through the comparison between the observed rainfall data with the simulated rainfall data. For the rainfall occurrences, the HMM is considered as a very well fit for the tropical regions because it can capture fairly well the rainfall data in Peninsular Malaysia. It is found that the rainfall process in Peninsular Malaysia is associated to the atmospheric composites: low rainfall probabilities are characterized by a high pressure system and high rainfall probabilities are accompanied by a low pressure system. The HMM is able to reproduce the wet/dry spells for most of the stations but overestimated on the short duration of the wet/dry spell (one or two days wet/dry spell). For the rainfall amount, the NHMM has exploited all the mechanisms related to the atmospheric information and rainfall data, and able to reproduce and predict the interannual variability during the Northeast monsoon.

# ABSTRAK

Permodelan statistik hujan dalam skala reruang dan masa adalah penting dalam menyediakan maklumat tentang tingkah laku proses hujan di kawasan tertentu. Salah satu cara konvensional pemodelan hujan adalah dilakukan melalui kajian struktur kebarangkalian hujan. Dalam dekad yang lalu, kebanyakan model hujan statistik telah dibina tanpa mengambil kira maklumat atmosfera. Model yang berjaya menggabungkan maklumat atmosfera adalah berguna dalam kajian kebolehubahan iklim atau perubahan iklim. Oleh itu, kajian ini menggunakan Model Markov Terpendam (HMM) dan Model Markov Terpendam Tak Homogen (NHMM) untuk memodelkan hujan harian bagi 40 stesen di Semenanjung Malaysia dalam tempoh 34 tahun semasa Monsun Timur Laut dan Monsun Barat Daya. Empat model yang berlainan untuk amaun hujan iaitu taburan eksponen tunggal, taburan campuran dua eksponen, taburan gamma tunggal dan taburan campuran dua gamma telah dikaji untuk amaun hujan tak sifar. Hubungan antara proses penurunan hujan tempatan dan proses atmosfera berskala luas disiasat melalui kelakuan anomali angin komposit 850hPa dengan halaju menegak omega 500hPa di setiap keadaan terpendam dari HMM. HMM kemudiannya diperluas kepada NHMM dengan memasukkan kovariat masa berubah-ubah (pembolehubah atmosfera) dalam model tersebut. Setakat ini, algoritma yang paling popular digunakan untuk anggaran parameter HMM adalah algoritma Baum-Welch, tetapi ia dijamin hanya untuk mencari maksimum tempatan dengan pergantungan yang tinggi kepada parameter awal yang dipilih. Oleh itu, kajian ini juga mencadangkan suatu anggaran parameter, algoritma bersegmen *K-means* yang menghapuskan beberapa tatacara biasa Baum-Welch untuk kecekapan pengiraan. Dapatan kajian menunjukkan bahawa algoritma bersegmen *K-means* dapat meningkatkan kecekapan model konvensional dengan pengurangan masa pengiraan. Prestasi daripada HMM dan NHMM dinilai melalui perbandingan di antara data hujan cerapan dengan data hujan simulasi. Untuk kejadian hujan, model HMM dianggap sesuai untuk rantau tropika kerana ia dapat merakam data curah hujan dengan baik di Semenanjung Malaysia. Didapati proses penurunan hujan di Semenanjung Malaysia boleh dikaitkan dengan komposit atmosfera: kebarangkalian hujan yang rendah dicirikan oleh sistem tekanan tinggi dan kebarangkalian hujan yang tinggi disertai dengan sistem tekanan yang rendah. HMM dapat menghasilkan semula rentetan basah/kering untuk kebanyakan stesen tetapi terlebih anggaran pada rentetan basah/kering (satu atau dua hari rentetan basah/kering) yang singkat tempohnya. Bagi amaun hujan, NHMM telah menerokai semua mekanisme yang berkaitan dengan maklumat atmosfera dan data hujan, serta dapat menghasilkan semula dan meramal kebolehubahan antara tahunan semasa monsun timur laut.

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| | | |
|---|---|---|
| AIC | - | Akaike's Information Criterion |
| ARMA | - | Autoregressive Moving Average |
| BIC | - | Bayesian Information Criterion |
| C | - | Central |
| CART | - | Classification and Regression Trees |
| CDF | - | Cumulative distribution function |
| E | - | Eastern |
| EM | - | Expectation Maximization |
| ENSO | - | El-Niño Southern Oscillation |
| FO | - | Frequency Occurrence |
| GCM | - | General Circulation Model |
| HMM | - | Hidden Markov Model |
| MCMC | - | Markov Chain Monte Carlo |
| MMD | - | Malaysian Meteorology Department |
| MJJA | - | May-June-July-August |
| NCAR | - | National Center for Atmospheric Research |
| NCEP | - | National Centers for Environmental Prediction |
| NDJF | - | November-December-January-February |
| NHMM | - | Non-Homogeneous Hidden Markov Model |
| NOAA | - | National Oceanic and Atmospheric Administration |
| NW | - | Northwestern |
| PC | - | Principal Component |
| QQ | - | Quantile-Quantile |
| SD | - | Standard Deviation |
| RMSE | - | Root Mean Square Error |
| SST | - | Sea Surface Temperature |

| | | |
|---|---|---|
| SVD | - | Singular Value Decomposition |
| SW | - | Southwestern |
| TPM | - | Transition Probability Matrix |
| W | - | Western |

# LIST OF SYMBOLS

| | | |
|---|---|---|
| $a_{ij}$ | - | Transition probability from *i* to *j* |
| $n_{ij}$ | - | The number of transition from *i* to *j* |
| $M$ | - | The number of stations |
| $N$ | - | The number of the hidden states |
| $S$ | - | The hidden sequence |
| $\pi$ | - | Initial Probability |
| $G$ | - | The emission probability |
| $p_{(is)}$ | - | The probability of occurrence at station *i* for state *s* |
| $c$ | - | Threshold of rainfall |
| $\Gamma(.)$ | - | Gamma function |
| $\alpha$ | - | Shape parameter for Gamma distribution |
| $\beta$ | - | Scale parameter for Gamma distribution |
| $\Sigma$ | - | The variance-covariance matrix |
| $k$ | - | The number of mixture component |
| $g$ | - | The number of grid nodes |
| $T$ | - | The number of observation |
| $f_t$ | - | The forward variable at time *t* |
| $b_t$ | - | The backward variable at time *t* |
| $L$ | - | Likelihood |
| $v$ | - | The number of parameters of the model |

| | | |
|---|---|---|
| $sc$ | - | Scaling coefficient |
| $S_p^2$ | - | Pooled sample variance |
| $Sv(.)$ | - | Survival function |
| $SE(.)$ | - | Standard error of a function |
| $z$ | - | The significance of the log odds ratio |
| $cp$ | - | Cumulative probability |
| $F^{-1}(.)$ | - | The inverse of the cumulative distribution function |
| $\mathbf{R}$ | - | The multivariate rainfall process |
| $\mathbf{W}$ | - | The complete data |
| $\mathbf{I}$ | - | The identity matrix |
| $\boldsymbol{\mu}$ | - | Mean vector |
| $\mathbf{X}$ | - | Vector of atmospheric process |
| $\mathbf{X}_{(std)}$ | - | Standardized atmospheric variable |

# LIST OF APPENDICES

# CHAPTER 1

# INTRODUCTION

## 1.1 Introduction

The weather is the state of the atmosphere such as the wind speed and direction, air temperature and pressure, relative and specific humidity, rainfall occurrence, and rainfall amounts in a particular region over a short-term period. In view of this, climate describes the weather pattern of a particular region over a long-term period and is also defined as the average weather pattern of a region. Malaysia is located in the equatorial zone, situated in the approximate northern latitude between 1°N and 6°45'N and the approximate eastern longitude of 99°36'E to 104°24'E. The atmospheric temperature in Malaysia is uniformly high throughout the year. The climate of Malaysia has a seasonal rhythm because Malaysia experiences the Northeast monsoon from November until February and the Southwest monsoon from May until August. The Northeast monsoon brings about heavier rainfall, especially in the eastern region, whereas the Southwest monsoon contributes towards the dry period in Peninsular Malaysia.

Rainfall has an important impact on human and physical environments because all living things cannot live without water. With the rapid growth of population and industrialization, the management of water resources has become an increasing concern in Malaysia. The analysis of rainfall behavior, particularly regarding rainfall amount and rainfall occurrence, is useful for managing water consumption. This thesis therefore investigates the characteristics of monsoon rainfall associated with the atmospheric circulation in Peninsular Malaysia on the spatial and temporal scale.

## 1.2 Background of the Study

A stochastic rainfall model is always preferred compared to a deterministic model due to the complexity, randomness, and dynamic behavior of rainfall. Stochastic rainfall modeling on a space and time scale is essential in providing information on the probabilistic structure of the rainfall process in certain regions. It is crucial to predict the trend in rainfall for managing water resources and natural disaster planning preventions such as flood or drought. Furthermore, simulations from the stochastic rainfall model can be used as input to stream flow, crop growth, runoff, soil loss, and flooding models. Therefore, knowledge based on analysis of rainfall characteristics is essential in order to understand rainfall patterns for designing, planning, and managing various water resource systems.

Stochastic models on rainfall are mostly developed in isolation, i.e. without reference to atmospheric information. Atmospheric information may be included as part of the rainfall model to produce simulations, which are consistent with the given atmospheric information. In addition, a model, which incorporates atmospheric information, would be useful in studies investigating the impact of climate change. Therefore, the stochastic rainfall model, also known as the "weather state model" or "downscaling model", has been developed for this purpose (Hay *et al.*, 1991; Bardossy and Plate, 1991; Hughes *et al.*, 1993). The weather state model can be used to generate realistic rainfall simulations using historical rainfall data and atmospheric data. Furthermore, the weather state model may also be used to study the impact of climate change and variability of rainfall using atmospheric simulations from a General Circulation Model (GCM).

A weather state model, namely the Hidden Markov model (HMM), has been developed to condition the daily rainfall in regard to the available atmospheric information at multiple sites. The Hidden Markov model is a doubly stochastic process in which the rainfall observation distribution depends on several unobserved discrete states (Rabiner and Juang, 1986). The Hidden Markov models have become popular tools for modeling dependent random variables in such diverse areas such as DNA recognition, speech processing, and rainfall modeling. For rainfall modeling, the hidden (unobserved) states of HMM can be used to interpret the various patterns of circulation anomalies (Robertson *et al*., 2004; Robertson *et al*., 2005; Greene *et al*., 2008). The HMM can be extended to model non-stationary processes by incorporating time-varying atmospheric variables, which is known as the non-Homogeneous Hidden Markov model (NHMM). This model exhibits unobserved weather states and serves as a link between the local rainfall process and large-scale atmospheric information.

This thesis presents the modeling of daily rainfall during 2 monsoon seasons using the Hidden Markov model. The distinct hidden states of HMM are interpreted by relating the local rainfall process with atmospheric circulation. Therefore, the HMM can be extended to NHMM by including the atmospheric variable into the model for which the patterns of the atmospheric variable correspond to the hidden states. The variability of the Northeast and Southwest monsoon over Peninsular Malaysia is analyzed and this variability will then be related to atmospheric information using time scales from daily to multi-decadal.

## 1.3 Statement of the Problem

Malaysia receives a large amount of rain every year. Therefore, floods are the most significant natural hazard in Malaysia in terms of population affected, frequency, flood duration, area extent, and social-economic damage (Youssed *et al.*, 2011). According to Feng and Lu (2010), about 40% of total economic loss caused by natural disasters is due to flooding. Spatial and temporal rainfall modeling have important impacts on physical environments. The trend in future rainfall may be easier to predict when the rainfall characteristics are known. However, a more recent issue that has been of concern is the climate change and global warming impact on rainfall. Nevertheless, most studies so far have only concentrated on stochastic rainfall models, which fail to incorporate atmospheric information. The stochastic rainfall model that does not include atmospheric information will not be so useful in studies investigating the effect of climate change or global warming. Therefore, prediction of the variability of rainfall for future periods under different climate change scenarios is essential in order to provide necessary information for high-quality climate-related studies.

In order to assess the effect of climate change on the rainfall trend in Malaysia, it is necessary to use a stochastic rainfall model that can incorporate atmospheric information. This study will focus on rainfall modeling using a Hidden Markov model by associating synoptic atmospheric patterns to the local rainfall in Peninsular Malaysia. The incorporation of the time-varying atmospheric variable into the NHMM can be used to generate realistic rainfall simulations, for example: an extreme rainfall event.

## 1.4 Objectives of the Study

The objectives of the research are:

1 To model rainfall occurrences and rainfall amounts on multi-site rainfall stations using a Hidden Markov Model (HMM).
2 To compare the parameter estimation of HMM using *K*-means and EM algorithm for rainfall occurrences.
3 To assess the performance of HMM on the atmospheric composite with rainfall probabilities in Peninsular Malaysia.
4 To model rainfall amounts using a non-homogenous Hidden Markov model (NHMM).
5 To assess the performance of NHMM on the interannual and interdecadal variability in Peninsular Malaysia.

## 1.5 Scope of the Study

The scopes of this research consists of the following:

1. Forty rainfall stations from Peninsular Malaysia are selected. The rainfall data is obtained from the Malaysian Meteorology Department *(*MMD*)*. The rainfall record over 34 years (1975-2008) is thus compiled;

2. Thirty-five atmospheric variables with different pressure levels are considered as potential candidates for the NHMM input. Thirty-five grid nodes that cover Peninsular Malaysia and the sea surrounding are reduced via singular value decomposition (SVD). The atmospheric data were obtained from the National Centers for Environment Prediction (NCEP) reanalysis data.

3. Two seasonal monsoons, which are the Northeast monsoon from November until February and the Southwest monsoon from May to August, are considered in this study.

4. The optimum number of hidden states for HMM and the number of atmospheric variables included in NHMM are determined via the Bayesian Information Criterion (BIC).

5. The binomial distribution is used to model the rainfall occurrence at each station in HMM. Four probability distribution functions for non-zero rainfall amount in each station, namely: single exponential distribution, single gamma distribution, the mixture of two exponential distribution, and the mixture of two gamma distribution, were examined and selected based on the BIC.

## 1.6 Significance of the Study

Climate change is undoubtedly one of the most important global environmental issues today. The impact of climate change affects the rainfall pattern, for example, increases the severity of floods and causes longer drought, and heavier thunderstorms. However, little work has been done on rainfall that is associated with the large atmospheric circulations in Malaysia. Therefore, a model called the Hidden Markov model was proposed in this study.

This thesis first fits the rainfall occurrence and rainfall amounts via HMM in Peninsular Malaysia and then identifies the physical definition of each state in the HMM with large-scale atmospheric behaviors and rainfall pattern. The state sequence of the HMM that categorizes each day into a state provides an exhaustive description of the rainfall process and is able to accurately estimate the future rainfall process. This model can provide a useful descriptive analysis of the rainfall in Malaysia.

In general, the EM algorithm is used to estimate the parameters of HMM. Another parameter estimation algorithm called segmental *K*-means for the HMM is also used in this study. The segmental *K*-means may provide more flexibility for the algorithm in the rainfall-modeling framework.

The HMM can be extended to NHMM by including atmospheric variables into the model. The goals of this work are to analyze the subseasonal to multidecadal variability of monsoon rainfall and produce simulations or predictions, which are consistent with the included atmospheric information. The state sequence of the NHMM can be used to classify the ENSO into a few states and the relationship between ENSO and monsoon rainfall can be investigated. The findings from NHMM are useful for the assessment of the impact of climate change and tend to result in a good model for descriptive and predictive modeling of the rainfall in Malaysia.

## 1.7 Structure of the Thesis

This thesis is organized into six chapters. The first chapter begins with the introduction to the research. This chapter also presents the background, the problem statement, the objectives, and the scope of research.

Chapter 2 provides an overview of the literature on rainfall modeling based on rainfall occurrence and rainfall amounts.

Chapter 3 outlines the description of rainfall data. Parameter estimation via EM algorithm and *K*-means algorithm; algorithm for find the optimum states paths; steps to generate the simulations and predictions; and methods for model assessments, are also discussed in detail.

Chapter 4 presents the HMM results in regard to rainfall occurrence. The results estimated from the *K*-means and EM algorithm are then compared. The relationship between the rainfall process and atmospheric circulations are interpreted in detail.

Chapter 5 shows the results of the HMM and NHMM in regard to rainfall amounts. The atmospheric variables are described and the atmospheric variable to be included into NHMM are selected. The comparison between the simulations and predictions from the HMM and NHMM is also discussed.

Finally, Chapter 6 concludes the study. This chapter summarizes the study and conclusion based on the analysis and results of this study. This chapter also suggests some recommendations for future study.

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