FUZZY AUTOCATALYTIC SET OF FUZZY GRAPH TYPE-3 BASED ON FUNCTIONAL ANALYSIS THEORY

UMILKERAM QASIM OBAID

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To those who are the most influential ones in my life

My beloved parents

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ABSTRACT

Fuzzy Autocatalytic Set (FACS) of fuzzy graph Type-3 is one of the newly capable graphs for a breakthrough of the relationship between fuzzy graph and autocatalytic set. It was successfully used to model an incineration process of a regional clinical waste in 2005. In this research, the mathematical structures of FACS and its properties were explored based on the suitable concept of fuzzy and non-fuzzy metric structure and fuzzy and non-fuzzy normed structure with the structure of its graph. In other words, the possible new structures of FACS have been explored via two new insights of its metric fuzziness and its normed fuzziness. Firstly, a fuzzy detour FT3-distance between vertices in FACS was investigated whereby a quasi-pseudo-FT3-metric fuzziness on FACS which depends on this distance was established, followed by an introduction to the concept FT3-cycle space of FACS as a vector space which is then proven as a normed space. These concepts have led to the visualization of the structure of FACS in a fuzzy norm, hence some propositions and theorems were established. In addition, the study on these structures of FACS was exploited, in particular the connection with functional analysis features and the application of these structures to the clinical incineration process.

ABSTRAK

Set Automangkinan Kabur (FACS) bagi graf kabur jenis-3 ialah suatu graf baharu yang menghubungkan di antara graf kabur dan set automangkinan. Ia telah berjaya memodelkan proses satu loji pelupusan bahan buangan klinikal pada tahun 2005. Dalam penyelidikan ini, struktur matematik bagi FACS serta sifat-sifatnya telah diselidiki berdasarkan kepada struktur kabur dan tak kabur yang sesuai dan struktur norma kabur dan tak kabur beserta struktur grafnya. Dengan lain perkataan, kemungkinan struktur baharu bagi FACS telah diselidiki melalui metrik kabur dan norma kabur. Pada mulanya, jarak lencongan FT3 di antara bucu-bucu FACS dikaji sehinggakan kuasi pseudo-FT3-metrik kabur FACS terbina. Ia kemudian diikuti dengan pengenalan kepada ruang kitaran-FT3 FACS sebagai suatu ruang vektor. Seterusnya, ruang yang baharu ini dibuktikan sebagai sebuah ruang norma. Konsep ini membolehkan visualisasi struktur FACS dalam norma kabur, seterusnya beberapa proposisi dan teorem telah diwujudkan. Sebagai tambahan, kajian terhadap FACS dieksploitasi, khususnya hubungannya dengan analisis fungsian beserta aplikasinya dalam proses pelupusan bahan buangan klinikal ini.

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LIST OF SYMBOLS

$\mathbb{C}$	-	Set of complex numbers
$\mathbb{R}$	-	Set of real numbers
$\mathbb{R}^+$	-	Positive real numbers
$\mathbb{Z}$	-	Set of integers
$\mathbb{Z}^+$	-	Positive integers
$\mathbb{Z}_2$	-	Set of integers modulo 2
$\mathbb{N}$	-	Set of natural numbers
$[a, b]$	-	Closed interval of real numbers between a and b
$(a, b]$	-	Interval of real number open in a and closed in b
min	-	minimum
max	-	maximum
$\forall$	-	For all
$\exists$	-	There exists
$\Leftrightarrow$	-	Equivalent
$a \equiv b$ (mod m)	-	a is congruent to b modulo m
V	-	Set of vertices
E	-	Set of edges
$DG(V, E)$	-	Directed graph
$DG(\sigma, \mu)$	-	Fuzzy graph
$\|V\|$	-	The number of vertices
$\|E\|$	-	The number of edges
$\wedge$	-	The minimum of two fuzzy sets
$\vee$	-	The maximum of two fuzzy sets
e or (u, v)	-	Edge between adjacent vertices u, v
$\mu(e)$ or $\mu((u, v))$	-	Membership value of fuzzy edge connectivity for edge e

C or $\left(c_{ij}\right)$	-	Matrix
$C_{F_{ij}}$	-	Adjacency matrix
λ_1	-	Eigenvalue of the matrix
$*$	-	Continuous t-norm on a set X
$\cdot$	-	The usual multiplication on a set X
$\lVert . \rVert$	-	norm
$N_{BS}(x,t)$	-	Fuzzy norm of x with respect to t
ℓ_μ	-	μ-length of path
$d_{NG_\mu}(u,v)$	-	Fuzzy detour μ-distance between vertices u and v
$d_p(x,y)$	-	pseudo-metric on a set X
$d_q(x,y)$	-	quasi-metric on a set X
$d_{qp}(x,y)$	-	quasi-pseudo-metric on a set X
$d_{qp}^{-1}(x,y)$	-	quasi-pseudo-metric on a set X and is equal to $d_{qp}(y,x)$
$d_{qp}^{s}(x,y)$	-	metric on a set X and is equal to the maximum of two functions d_{qp} , d_{qp}^{-1}
$M_{GV}(x,y,t)$	-	fuzzy metric on a set X with respect to t
$M_{GV}^{-1}(x,y,t)$	-	fuzzy metric on a set X with respect to t and is equal to $M_{GV}(y,x,t)$
$M_{GV}^{\#}(x,y,t)$	-	fuzzy metric on a set X with respect to t and is equal to the minimum of two fuzzy sets M_{GV}, M_{GV}^{-1}
ℓ_{FT3}	-	FT3-length of path p in FACS
d_{FT3}	-	Fuzzy detour FT3-distance on FACS
M_{FT3}	-	fuzzy FT3-metric induced by d_{FT3} on FACS
$\{v_k\}$	-	Sequence of vertices in FACS
$B_{M_{FT3}}$	-	Open balls in FACS
$\tau_{M_{FT3}}$	-	Topology on the set of vertices V in FACS
$C^n_{e_1,e_2,\ldots,e_n}$	-	FT3-cycle of FACS for each fuzzy edge connectivity $e_k \in E$ and *n* is the number of edges in this cycle
$\lvert C^n_{e_1,e_2,\ldots,e_n} \rvert$	-	The length of FT3-cycle
$C^n_{e_k}$	-	FT3-fuzzy detour cycle of FACS containing the edge $e_k \in E$ with *n* the number of all edges in this cycle

$\vec{C}_{e_k}^{n}$	-	Vector representation of FT3-fuzzy detour cycle
$S_C(DG_{FT3})$	-	The FT3-cycle space of FACS
$\left\|C_{e_k}^{n}\right\|$	-	Length of FT3-fuzzy detour cycle
$\parallel C_{e_k} \parallel$	-	Norm of FT3-fuzzy detour cycle
N_{FT3}	-	Fuzzy norm induced by a norm $\parallel \parallel$ on $S_C(DG_{FT3})$
$\{C_{e_n}\}$	-	Sequence of FT3-fuzzy detour cycles in $S_C(DG_{FT3})$

LIST OF ABBREVIATIONS

FT3	-	Fuzzy graph of Type-3
ACS	-	Autocatalytic Set
FACS	-	Fuzzy Autocatalytic Set
PFE	-	Perron-Frobenius eigenvector
$DG_{FT3}(V,E)$	-	Digraph of FACS of fuzzy graph Type-3
M_{FT3}	-	Fuzzy FT3-metric induced by d_{FT3} on FACS
$B_{M_{FT3}}$	-	Open balls in FACS
$S_C(DG_{FT3})$	-	The FT3-cycle space of FACS
O_2	-	Oxygen
H_2O	-	Water
CO_2	-	Carbon Dioxide
CO	-	Carbon Monoxide
otp*	-	Other pollutants

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CHAPTER 1

INTRODUCTION

1.1 Research Background

Fuzzy set theory is one of the useful and abstract mathematical theories that utilize the notion of grade membership to characterize a fuzzy set in which was inspired by Professor Lofti Zadeh in 1965. It has been employed in mathematical modelling of many systems in real world applications. Since that time, the characteristics and applications of this theory have been discussed by numerous researchers (Gitman and Levine, 1970; Tamura *et al*., 1971; Kandel and Yelowitz, 1974; Pathak and Pal, 1986; Keller and Tahani, 1992). This is due to the fact that most crisp mathematical models are always unable to model some portions of reality including complexity and vague attitudes. So, it is objectively shown that crisp model is not enough to describe the whole process of a system. Moreover, the fuzziness notion has become invention tool in modelling, predicting ambiguity or uncertainty that occur in everyday life and help us to make decisions about a situation or problem.

Another mathematical theory generated from modeling a set of relationships among elements is called graph theory that started its history with mathematician Leonhard Euler in 1736. In this context, directed graph is one of the particular important types of graphs which the elements are distinguished from one another. Its applications can be found in several domains such as in topology of food webs that is depicted as a map of organisms in ecological community as shown in Figure 1.1 (Dunne *et al*., 2002; Woodward *et al*., 2005).

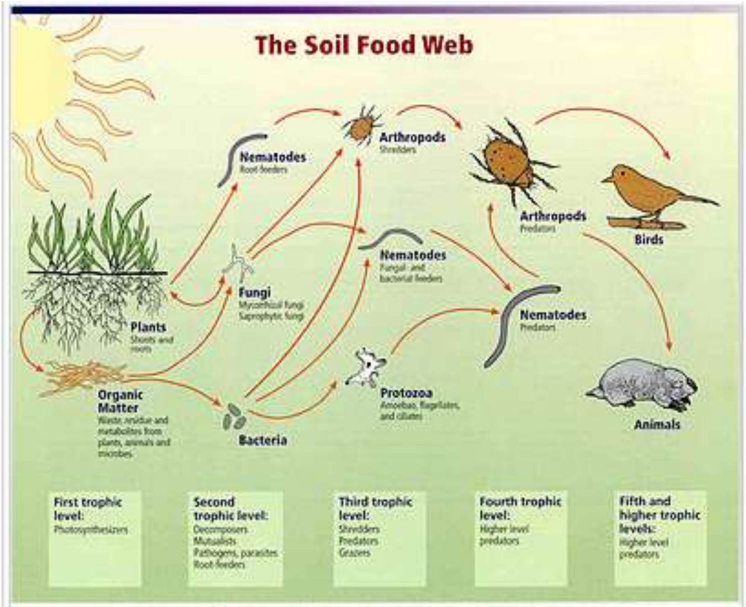

Figure 1.1 An example of topology of food webs (Natural Resources Conservation Service Soils, 2015)

Furthermore, the application of directed graph is also used in technological field like model of air transport webs that composed of airports and linking flights as shown in Figure 1.2 (Barrat *et al*.,2004; Guimera *et al*., 2005). Thus, many models have been developed by using these graph-theoretic ideas in different fields such as industrial, engineering, computer science, management, education and chemistry (Chartrand, 1985; Evans and Minieka, 1992). However, these ideas included uncertainty and unpredictability in many details of a system wholly, like the travel time of automobile or automobile capacity on a road network may not be known precisely (Blue *et al*., 2002). Arguably, some models are inevitably unsuitable in describing the system. So, the notion of fuzzy sets may give a normal approach of dealing with these problems.

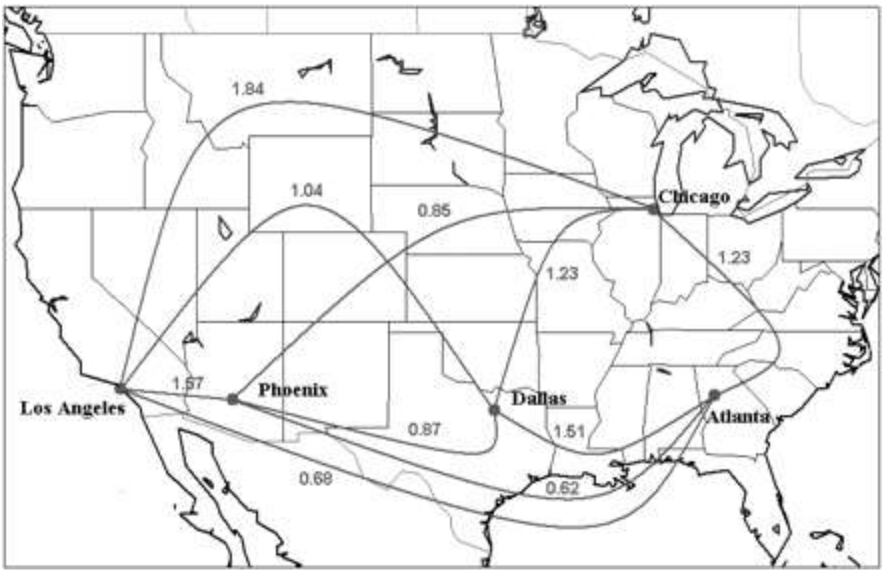

Figure 1.2 An example of a complex weight network (Barrat *et al*., 2004)

The emergence of fuzzy graph theory is to apply the idea of fuzziness to graph theory and express of fuzzy relations on fuzzy sets that initially described by Professor Azriel Rosenfeld in 1975. Ever since that, fuzzy graph has been promptly expanded and implemented in diverse fields. The study on a modeling of clinical waste incineration process in Malacca (Sabariah, 2005; Tahir *et al.*, 2010) is a significant example of the implementation of fuzzy graph theory. Schematic diagram of the incinerator for this process is exhibited in Figure 1.3.

This study has been formally initiated into translating the incineration process to a graphical model and producing a crisp model (see Figure 1.4). In this regard, this model was designed to interpret the association between input-output elements and components of the incineration process. However, a graphical representation in Figure 1.4 was demonstrated to be wholly inadequate (Sabariah, 2005). Researching through various graphical models representing this process was finally yielded model of six variables which described interaction between variables that played a pivotal role in the incinerator namely waste, fuel, oxygen, carbon dioxide, carbon monoxide and other gases including water.

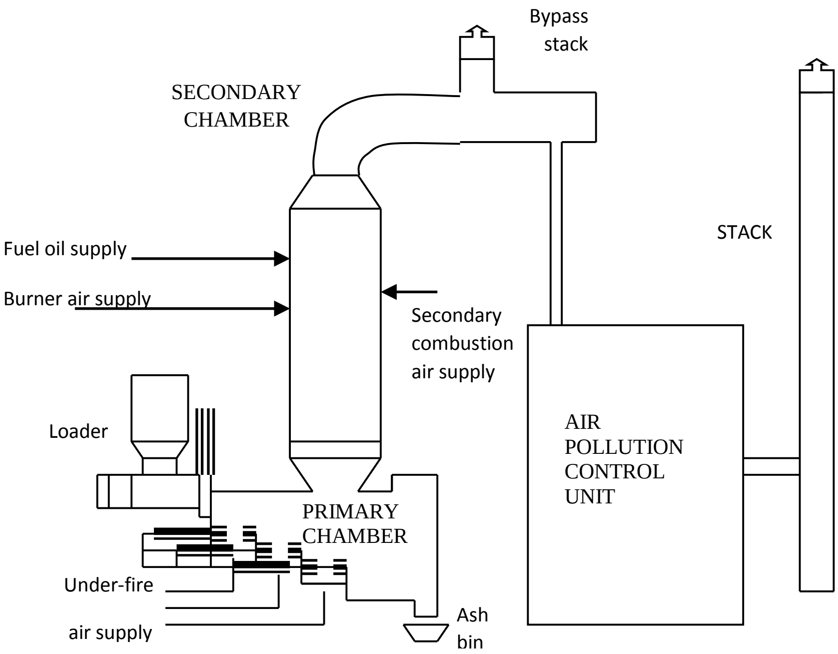

Figure 1.3 Schematic diagram for clinical waste incinerator (Sabariah, 2005)

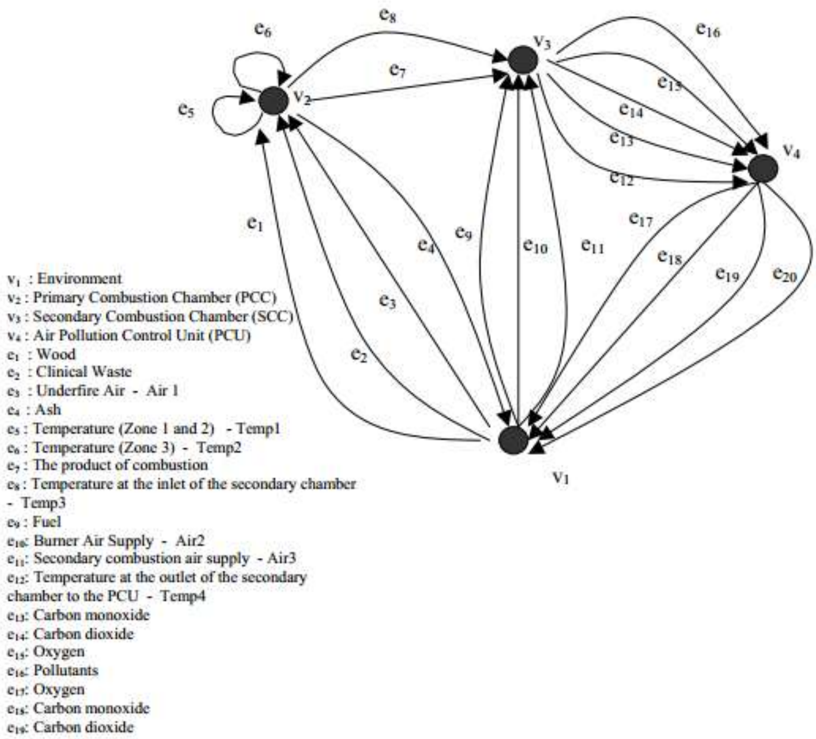

Figure 1.4 A graph showing the association between input-output variables and components of the incineration plant (Sabariah, 2005)

In addition, the notion of autocatalytic set as initiated by Jain and Krishna (1998; 2003) has been integrated with the model of six variables and then it was perfectly capable of distinguishing the dynamics of the incineration process into two major types namely concentration dynamics and graph dynamics. The concentration dynamics characterized the rate of change of the concentration of each of the chemical items in the process while the graph dynamics characterized the change of the network of the process. Nevertheless, both dynamics were not comfortably suitable in explaining the actual rate of change and the evolution of the variables (Sabariah, 2005; Tahir *et al.*, 2010). This led to the development of the modeling work of the process and adapted the fuzzy logic as an integral part of the system. In other words, fuzzy graph had succeeded to apply the modeling of this process.

Representing the process as fuzzy graph made amalgamation three notions which are graph, autocatalytic set and fuzzy. This study was focused on fuzzy graph of type-3 and innovated a fresh notion called Fuzzy Autocatalytic Set (FACS) of fuzzy graph type-3. It was shown that the modeling fuzzy work (FACS) of the incineration process was more precise and suitable in describing the dynamics of this process (Tahir *et al.*, 2010).

Thereby, the study of the algebraic structure for FACS particularly on algebraic properties of adjacency matrix of FACS in explaining the modeling of the incineration process was expatiated by Sabariah (2005). Actually, transformation from graph to matrix was investigated by using adjacency matrix and resulted few propositions and proven theorems. The new notion of FACS would be expected to be employed in other systems and would give great possibilities of creating new theories and structures in mathematical domains to enlighten scientific knowledge.

Therefore, this research is undertaken to further explore and inspect the mathematical structures of FACS of fuzzy graph Type-3 from new two angles of its structure which are its metric fuzziness and its normed fuzziness. More precisely, the structure of the graph FACS with its relation to (fuzzy) metric structure and (fuzzy) normed structure are taken into consideration in more investigation on FACS of fuzzy graph Type-3 and the richness of directed graph through functional analysis advantages. At the same time, the study on these new structures of FACS resulted some propositions and certain theorems that hope to share in developing more results of fuzzy functional analysis. Main properties and characteristics for this notion of FACS have been developed as an interface that will make sense in these new structures of FACS such as convergence, Cauchyness and completeness.

1.2 Problem Statement

This work focuses to design a new approach for representing the structures of FACS of fuzzy graph Type-3 that have a large or complicated graph. The ultimate aim of the research is how to classify the mathematical structures for FACS of fuzzy graph Type-3 as a fuzzy metric structure and a fuzzy normed structure.

1.3 Research Objectives

The main objectives of this research are given as follows:

(a) To explore the mathematical structure of FACS of fuzzy graph Type-3 particularly on the metric structures through their relation on a fuzzy detour distance in FACS.

  i. To construct a fuzzy detour FT3-distance between vertices in FACS.

  ii. To verify that this distance is a quasi (pseudo)-metric.

(b) To convert FACS of fuzzy graph Type-3 to its fuzziness of quasi-(pseudo)-metric space. In other words, this objective will address the following:

  i. To describe the fuzzy metric structure that would represent FACS of fuzzy graph Type-3.

  ii. To find the properties for the structure of the fuzzy metric of FACS (such as convergence, Cauchyness and bicompleteness in this space) that describe behavior some vertices or nodes go through paths in FACS.

(c) To extend the investigation into vertices or nodes go through cycles in FACS and construct the following:

i. To construct a cycle space of FACS as a vector space.

ii. To construct a normed space of FACS. In other words, to describe a crisp norm on FACS.

(d) To develop the structure of the new normed space of FACS to its fuzziness of a crisp norm on FACS.

i. To present the idea of fuzzy norm on the cycle space of FACS that relevant to a crisp norm on FACS.

ii. To find the properties for the structure of the fuzzy normed cycle space of FACS (such as convergence, Cauchyness and completeness in this space) that describe behavior cycles in FACS.

(e) To explain and implement the concepts deduced from (fuzzy) metric structure and (fuzzy) normed structure of FACS in the modeling of the incineration process which serves as example for these current structures.

1.4 Scope of Research

This study is concentrated on the exploration of mathematical structures of FACS of fuzzy graph Type -3 with no loops, particularly on the (fuzzy) metric structures and (fuzzy) normed structures. The structure of its graph is based on the previous results on FACS investigated by Sabariah (2005).

1.5 Significance of Research

The significance of this research would be able to promote and expand the newly qualified graphs of FACS of fuzzy graph Type-3 in mathematics domain. In same time, the successful classifications of FACS as (fuzzy) metric structures and (fuzzy) normed structures can be added to the structural examples in the theory of functional analysis. As well, developments or deep insights into some of their properties of the current structures of FACS highlight the mathematical interpretations of the incineration process.

1.6 Research Framework

This section exposes the methodology which we used for representing graphs into fuzzy metric structures and fuzzy normed structures. Thus, this research includes four main stages. Based on the research framework as shown in Figure 1.5, the study will be performed according to the listed stages.

1) Explore the mathematical structure of FACS of fuzzy graph Type-3 particularly on the metric structure.

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2) Extend the investigation into other structure representation which is the fuzzy metric structure.

3) Explore the mathematical structure of FACS of fuzzy graph Type-3 particularly on the normed structure.

4) Extend the investigation into other structure representation which is the fuzzy normed structure.

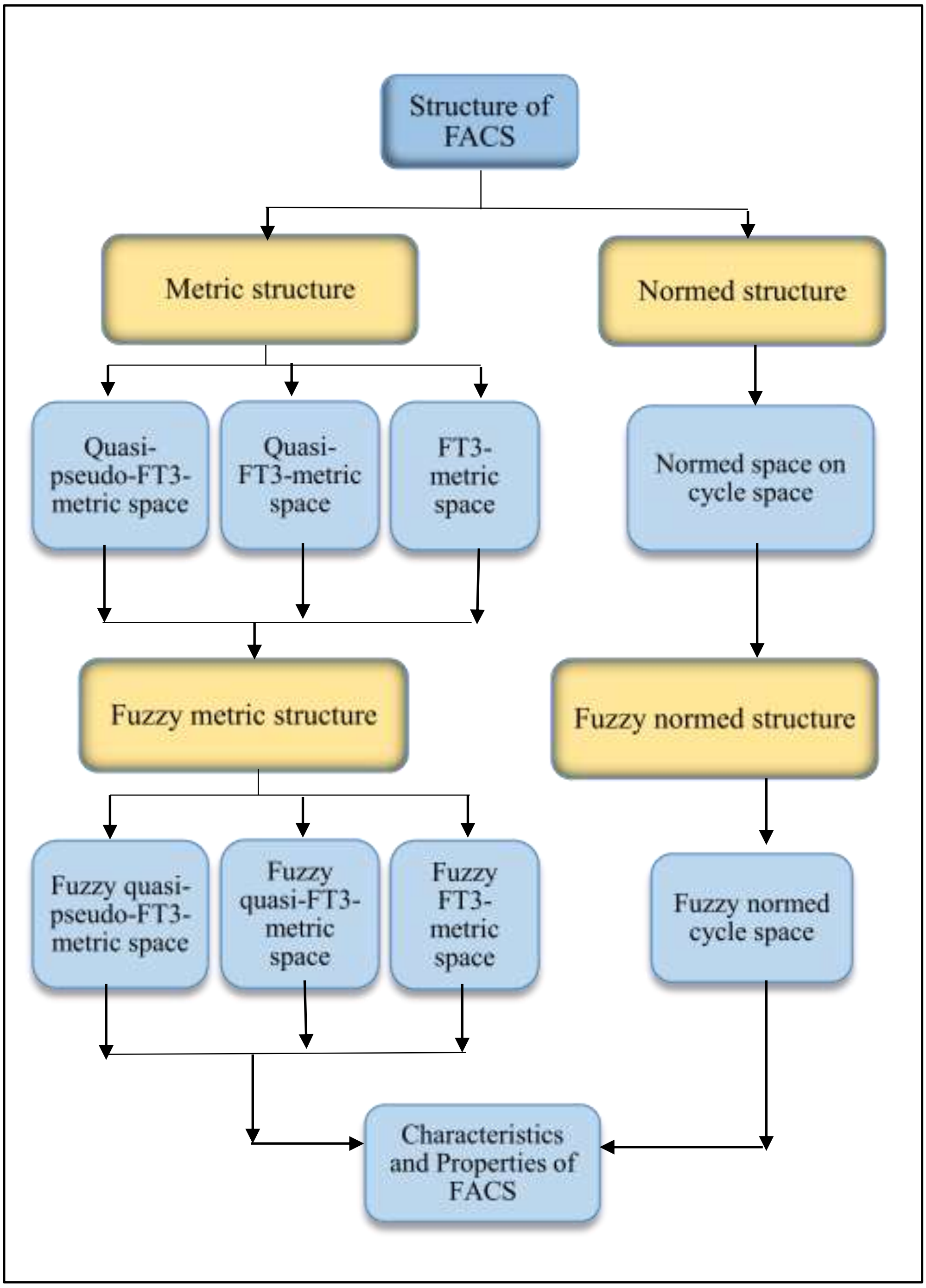

Figure 1.5 Research framework

1.7 Thesis Outline

This thesis includes seven chapters. Its summary is depicted in Figure 1.6. Chapter 1 deals with the introduction to the research. It contains the background of the problem, problem statement, research objectives, scope of research and significance of research.

Chapter 2 offers the literature review of relevant research. In fact, essential mathematical background that will be used throughout the thesis is presented in this chapter. Basically, it includes three main sections. The first section reviews a brief discussion on crisp and fuzzy graphs, followed by reviewing on the notion of FACS in the second section. The third section introduces concepts and facts in fuzziness of metric spaces and normed spaces which assist the study on the visualization of FACS in these existing spaces.

The next four chapters contain the major contributions of the work. Chapter 3 presents the fuzzy detour FT3-distance between two distinct vertices in FACS. Then, this distance will be utilized to present the metric structures of FACS. In fact, structural classification for FACS by the construction of the quasi-pseudo-FT3-metric space of FACS will be shown in this chapter. Beside that the irreducible graph of FACS of fuzzy graph Type-3 is specified by the quasi-FT3-metric space of FACS and applied in the modeling of the incineration process which used as example for this structure.

Chapter 4 focuses on establishing the fuzzy metric structures of FACS and some properties of these fuzzy metric structures of FACS are presented such as convergence, Cauchyness and bicompleteness in FACS. By these properties, the relationship between paths which are not cycles in FACS will be shown in a fuzzy quasi-FT3-metric of FACS and also used in explaining these paths in the incineration process.

In chapter 5, the cycles in FACS is discussed by introduction a modern notion of fuzzy detour FT3-cycle in FACS and a FT3-cycle space of FACS which will be proven as a vector space in this chapter. In addition, these notions present a useful fashion for constructing a new type of normed space of FACS. Then, the normed structure of FACS would analyze and explore the basic cycles with respect to this norm in the incineration process.

Chapter 6 discusses on the representation of FACS in fuzziness of normed structure of FACS by the construction a fuzzy normed cycle space of FACS that will be presented in this chapter. In fact, a notion of a fuzzy norm in a fuzzy graph theory setting is established and interpreted the catalytic chain reaction between two cycles in this structure of FACS and employed in the incineration process that have different catalytic chain reaction because of its fuzzy norm structure. In addition, some important results would be proven involving convergence, Cauchyness and completeness of this structure of FACS.

It must be noted that the physical interpretations of these new structures of FACS for the incineration process in each chapter reflect the application of the findings in this thesis.

Finally, Chapter 7 concludes the findings of this thesis by presenting the summary of every chapter and highlighting some recommendations for future research.

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