

SIMULATION OF AQUEOUS HUMOUR (AH) FLOWS AND DEFORMATION OF
DESCEMENT MEMBRANE DETACHMENT (DMD) IN A THREE DIMENSIONAL
ANTERIOR CHAMBER (AC)

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To my beloved family

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ABSTRACT

This thesis strives the interaction between the aqueous humour (AH) flows and the deformation of Descemet membrane detachment (DMD) in a 3D anterior chamber (AC). Descemet membrane detachment (DMD) is an uncommon condition with a wide range of possible cause of disease. The aqueous humour flow in the anterior chamber is described as flow driven by buoyancy effects due to the existing temperature difference. The mathematical model has been developed and numerical result for finite element method are carried out using COMSOL Multiphysics software by solving the governing equations for the AH flows and the deformation of DMD. The fluid flow behavior and the deformation of the detached Descemet membrane are analysed in order to comprehend the progression of the DMD in the AC due to the AH flows and vice versa. The direction of the gravitational force acting has a great influence to the fluid flow in the AC. Thus, the position of the patient with DMD need to be concerned in order to induce the phenomena of spontaneous reattachment. The spontaneous attachment or re-detachment of the DMD could then be induced in the simulation to better understand its occurrence for a viable treatment to be devised.

ABSTRAK

Tesis ini mensimulasikan interaksi antara aliran humor akues (AH) dan perubahan bentuk Pemisahan Detasmen Membran (DMD) dalam ruang anterior 3D (AC). Pemisahan Detasmen Membran (DMD) ialah satu keadaan luar biasa yang berpunca daripada pelbagai jenis penyakit. Aliran humor akues dalam ruang anterior digambarkan sebagai aliran yang didorong oleh kesan keapungan akibat perbezaan suhu yang ada. Model matematik telah dibangunkan dan hasil berangka untuk kaedah elemen terhingga dijalankan menggunakan perisian COMSOL Multiphysics dengan menyelesaikan persamaan menakluk untuk aliran AH dan perubahan bentuk DMD. Tingkah aliran bendalir dan perubahan bentuk Pemisahan Detasmen Membran dianalisis untuk memahami keadaan DMD dalam AC yang disebabkan oleh aliran AH atau sebaliknya. Arah daya bertindak graviti mempunyai pengaruh yang besar terhadap aliran bendalir di AC. Oleh itu, kedudukan pesakit bersama DMD perlu dititikberatkan dalam memastikan fenomena nyahsambungan secara spontan. Sambungan secara spontan atau penyahsambungan Pemisahan Detasmen Membran DMD kemudiannya dapat diinduksi dalam simulasi untuk memahami dengan lebih jelas kejadiannya untuk mendapatkan rawatan yang sesuai.

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LIST OF SYMBOLS

T	-	Temperature
T_c	-	Temperature by Cornea
T_p	-	Temperature by Pupil
m	-	Mass
α	-	Coefficient of Thermal Expansion
ρ	-	Fluid Density
τ	-	Tolerance
c_p	-	Specific Heat
k	-	Thermal Conductivity
μ	-	Viscosity
U	-	Velocity
g	-	Graviti
β	-	Coefficient of Thermal Expansion
μ	-	Viscosity
h_0	-	Typical Length

LIST OF ABBREVIATIONS

AC	-	Anterior Chamber
AH	-	Aqueous Humour
DM		Descemet Membrane
DMD	-	Descemet Membrane Detachment
3D	-	3 Dimensional
2D	-	2 Dimensional
PDEs	-	Partial differential equations

CHAPTER 1

INTRODUCTION

1.1 Introduction

The eye is one of the most important organs for humans, enabling sight of the reality before them and the world they live in. Any hindrances to its functionality can severely affect cognition. Descemet membrane detachment (DMD) is one such hindrance, a disease of the eye that may cause blindness when left untreated. Its formation involves the fluid flow inside the eye. However, the small dimensions of the human eye and the extremely low velocities of the flow of ocular fluid make it difficult for an in-vivo experiment. Therefore, many aspects of fluid flow within the eye have not yet been fully understood or quantitatively explained. Alternatively, computational simulations of the ocular fluid flow can be used to understand the flow mechanisms in the human eye especially when the eye has DMD. In this chapter, the background of the problem is explained and the anatomy of the human eye is elaborated to serve as illustrations of the basic mechanism of fluid motion.

From this, the statement of the problem and the objectives of this research are highlighted. Limitations and scopes of the research are also stated here to entice further research. Finally, the significance of the study and the outline of the thesis are proposed.

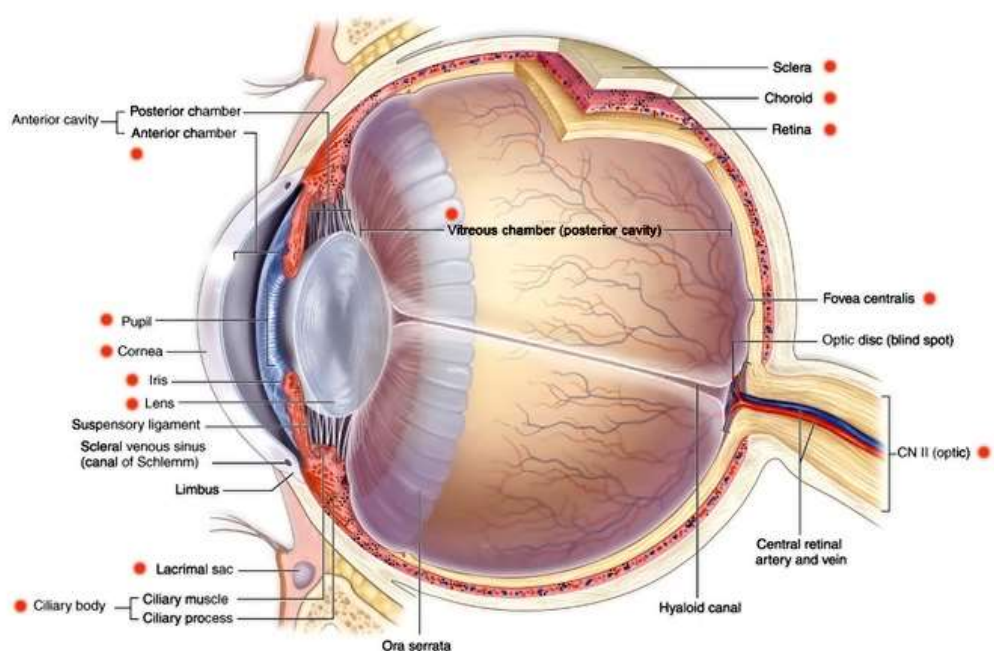


Figure 1.1 Structure of Human Eye (Kara, 2011).

1.2 Research Background

The structure of the eye is shaped like a sphere. The front part forming a translucent dome is named the Cornea which covers the underlying iris that gives colouration and pupils, as show in Figure 1.1. The region that connects the Corneas with this underlying layer is called the anterior chamber (AC). The Corneas themselves contain several layers, three main and two auxiliary as shown in Figure 1.2 that is the epithelium, the Stroma, the endothelium, the Browman layer and the Descemet membrane (DM). DMD occurs when the DM is separated from the Stroma and by the aqueous humour (AH) fluids which flow into the space between through a tear or minute fissure in the DM layer. The condition which causes DMD, have severe effects to vision of the eye.

DMD could also occur through artificial means as in cataract surgery, iridectomy, trabeculectomy, Corneal transplantation, deep lamellar keratoplasty, holmium laser sclerostomy, alkali burn and viscocanalostomy have been reported by Mulhern *et al.* (1996), Potter and Zalatimo (2005), Hirano *et al.* (2002) and Ünlü and Aksünger (2000). Sevillano *et al.* (2008) had reported a technique of curing DMD caused by cataract surgery with sulphur hexafluoride injection. Potter and Zalatimo (2005) had presented the case of treating the scrolled DMD by injecting fourteen percent of intracameral perfluoropropane (C_3F_8) into the AC. Recently, Couch and Baratz (2009), investigated two cases of delayed bilateral DM and in one eye it was fixed surgically and the other eye improved spontaneously. They estimated that the spontaneous reattachment happen because of the buoyancy effect or difference in pressure between the different region of the eyes which cause the AH flow in the AC. The spontaneous reattachment of the DM has been supported by some observational and anecdotal evidences (Marcon *et al.*, 2002; Nouri *et al.*, 2002; Couch and Baratz, 2009; Ismail *et al.*, 2013).

Fitt and Gonzalez (2006) had showed that under normal conditions the buoyancy effects due to temperature gradient in the AC enhance the AH to flow. Ismail *et al.* (2013) intended to explain the phenomena of the spontaneous reattachment and thus, developed a mathematical model to describe the AH flow in the AC with DMD. They concluded that the temperature difference across the eye and the orientation of the patient may control the clinical outcomes for the DMD. Nonetheless, the model developed by Ismail *et al.* (2013) was based on the lubrication theory limit, which includes a lot of simplification. Consequently, the model may only partially illuminate the behaviour of the fluid flow in the AC. Therefore, in this thesis, the fluid mechanical theory is applied to model the fluid flow in the AC with the presence of detached DM in order to study the flow mathematically and then to explain the spontaneous reattachment phenomena as noted by Couch and Baratz (2009) scientifically.

1.3 Statement of the Problem

Data pertaining to the Flow of AC is difficult to obtain due to the miniscule size of both chambers. The complexities of measuring fluid movements inside the living eye also add to the difficulty in obtaining the data. Circumventing this however is possible using computer generated simulations to understand the fluid mechanics of AH inside the AC. These models generated thus enable the better understanding of the fluid dynamics in the AC in cases where the observed party contracts DMD diseases. Currently, most studies present in literature uses the perturbation or asymptomatic methods which is incapable of solving complex fluid flow models used to describe fluid flow in the AC where DMD diseases are present. There is then a need for the numerical computation.

The Finite Element Method is proposed in this study to obtain numerical results of fluid flow in the AC. Beforehand, it is understood that previous studies have been made to observe patient orientation and its effect to the reattachment phenomenon of DMD but they were conducted exclusively in a 2D setting, therein lies the problem of this research. The limitations of data from 2D models could not accurately describe the actual situation of the eyes which is a 3D object thus this paper proposes similar investigations but recreated in a 3D setting. Understanding the effect of DMD by different position of the patient and Flow of AH into the AC may unlock the ability to induce spontaneous reattachment, saving the patient from total vision loss.

1.4 Objectives of the Study

This research aims to understand the characteristics of fluid flow in human eyes experiencing DMD in 3D anterior chamber (AC). The specific objectives are:

1. To derive the equation of motion of AH flow in the AC during DMD.
2. To demonstrate the implementation of COMSOL Multiphysics in model setting of the generalized the AH flow and the deformation of DMD.
3. To investigate the AH flow during the DMD that influenced by the different position of the patient.

1.5 Scope of the Study

The modelling of fluid flow of human eyes with DMD is analysed specifically based on the Navier-Stokes Equation. The fluid flow in the AC that is considered as an incompressible Newtonian and driven by temperature gradient. Boussinesq approximation is applied to allow a more convenient procedure to obtain the solution. Solving the governing equations numerically using the finite element method. The COMSOL Multiphysics software is used for computing the numeric results of the plotting. The temperature gradient that drives the flow of fluids in the AC with DMD is also considered along with the position of the patient. For the purpose of the research, the position of patient discussed are stand and sleep.

1.6 Significant of the Study

DMD although a rare type of disease to the eyes, occurring mostly during cataract extraction, 50% of persons aged 65 - 74 and 70% of persons over 75 are found likely to be inflicted with cataract (Kara, 2011). The symptoms of cataracts include Cloudy vision, glare, colour visual problems and double vision. Surgery is usually required to treat cataracts by removing the offending Lens and replacing it with artificial Lens. Tearing of the DM is still unclear but a popular theory posits the mechanism to be due to the mechanical force applied to the cornea during surgery.

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