TRIVARIATE COPULA FOR FLOOD FREQUENCY ANALYSIS IN JOHOR RIVER BASIN

NORIZZATI BINTI SALLEH

UNIVERSITI TEKNOLOGI MALAYSIA

TRIVARIATE COPULA FOR FLOOD FREQUENCY ANALYSIS IN JOHOR RIVER BASIN

NORIZZATI BINTI SALLEH

A thesis submitted in fulfilment of the
requirements for the award of the degree of
Master of Philosophy

Faculty of Science
Universiti Teknologi Malaysia

AUGUST 2017

To

my beloved mother, father,

sisters and brothers

who have always given me love,

care and cheer

and my fiancée who has always given me endless support,

whenever I need

and whose prayers have always been a source of great inspiration

for me.

May Allah bless you all…

ACKNOWLEDGEMENT

Primarily and foremost, all praise for Almighty ALLAH, who blessed me, the ability to fulfill the requirement for this thesis. I offer my humblest words of thanks to the Holy Prophet Mohammed (Peace be upon Him) who is forever a torch of guidance for humanity.

I would like to express my sincere appreciation and gratitude to my supervisor, Assoc. Prof. Dr. Fadhilah Binti Yusof and co-supervisor, Prof. Dr. Zulkifli Bin Yusop who guided me through this challenging year with amazing patience and perseverance. I also wish to thank the academic and support staff at the Department of Mathematical Science, Universiti Teknologi Malaysia for their kind assistance.

I also wish specially thank to my great parents, family and friends who either directly or indirectly involved in bringing this successful. I am grateful for their support, assistance and ideas in completing this thesis. Finally, I also acknowledge my fiancée for his endless support emotionally and physically for me in completing this thesis.

ABSTRACT

Flood variables are generally random in nature and mutually correlated, consist of peak flow, flood duration and flood volume. Flood frequency analysis defines the severity of a flood event by finding out their mutual dependence structure of flood variables. Copula is presented for flood frequency analysis through this study. Copula is a bivariate statistical method for constructing dependency structure and joint probabilistic distribution. Copula relaxes the restriction of traditional flood frequency analysis by selecting marginal from different families of probability distribution functions for flood variables. Thus, the trivariate copula function is developed in order to assess flood frequency. The analysis used 34 years hourly discharge data from year 1965 until year 2010 from Johor river basin from which the annual maximum were derived. On evaluation of various probability distributions for representation of flood variables, it is found that the peak flow can be fitted as Generalized Pareto (GP) distribution while flood duration and flood volume are well represented as Generalized Extreme Value (GEV). The joint distribution is modeled using five trivariate copulas namely Clayton, Gumbel, Frank, Gaussian and Student-t copulas. Based on the performance measure and simulation, it is found that Clayton copula is the best copula to represent the trivariate dependency structure of flood properties as compared to copulas. The obtained copula based joint distributions are used to calculate the return period of flood risks. The study concludes that the trivariate copula based methodology is a suitable choice for effective risk assessment of flood frequency analysis as it is able to consider the entire characteristics of flood variables.

ABSTRAK

Pembolehubah banjir umumnya rawak dalam alam semula jadi dan saling berkait rapat, terdiri daripada aliran puncak, tempoh banjir dan jumlah banjir. Analisis frekuensi banjir mentakrifkan tahap peristiwa banjir berdasarkan pembolehubah banjir, dengan mengetahui struktur pergantungan antara mereka. Dalam kajian ini, copula digunakan untuk menganalisis frekuensi banjir. Copula adalah kaedah statistik dwi-pembolehubah bagi membina struktur pergantungan dan taburan kebarangkalian bersama. Copula melonggarkan kekangan terhadap analisis frekuensi banjir tradisional dengan memilih marginal dari keluarga yang berbeza fungsi taburan kebarangkalian untuk pembolehubah banjir. Oleh itu, fungsi tiga pembolehubah copula dibangunkan untuk menilai kekerapan banjir. Kajian menggunakan 34 tahun data pelepasan setiap jam dari tahun 1965 sehingga tahun 2010 daripada lembangan sungai Johor di mana maksimum tahunan diperoleh. Penilaian ke atas pelbagai taburan kebarangkalian untuk setiap pembolehubah banjir ditentukan, didapati bahawa aliran puncak boleh disesuaikan sebagai taburan jenis Pareto Teritlak (GP) manakala tempoh banjir dan jumlah banjir diwakilkan dengan baik sebagai taburan Nilai Ekstrim Teritlak (GEV). Pengabungan bersama taburan dimodelkan menggunakan lima jenis tiga pembolehubah copula iaitu copula Clayton, Gumbel, Frank, Gaussian dan Pelajar-*t*. Berdasarkan ukuran prestasi dan simulasi, didapati bahawa copula Clayton adalah copula terbaik untuk mewakili struktur tiga pembolehubah banjir berbanding dengan jenis copula yang lain. Copula yang diperolehi berdasarkan camtunan taburan digunakan untuk mengira tempoh pulangan risiko banjir. Kajian ini menyimpulkan bahawa metodologi tiga pembolehubah copula merupakan pilihan yang sesuai untuk penilaian risiko yang berkesan bagi analisis frekuensi banjir kerana ia dapat mempertimbangkan ciri-ciri pembolehubah banjir secara keseluruhan.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

α	-	Corresponding parameter of $f_X(x)$
β	-	Corresponding parameter of $f_Y(y)$
$C(u,v)$	-	Copula Function
$D_k(x)$	-	Debye Function
$F_n(x_i)$	-	Empirical Distribution Function
k	-	Continuous shape parameter for Generalized Pareto and Generalized Extreme Value distribution
k_c	-	Distribution of Copula function
N	-	Number of observations
μ	-	Continuous location parameter for Generalized Pareto and Generalized Extreme Value distribution
σ	-	Continuous scale parameter for Generalized Pareto and Generalized Extreme Value distribution
ρ	-	Spearman’s rho
τ	-	Kendall’s tau
θ	-	Copula Parameter
$\phi(t)$	-	Generator of Archimedean Copula
$\phi^{-1}(s)$	-	Inverse generator

LIST OF ABBREVIATIONS

AIC	-	Akaike Information Criterion
CDF	-	Cumulative Distribution Function
GOF	-	Goodness-of-fit
IFM	-	Inference Function of Margins
MBP		Likelihood Maximization by Parts Approach
ML		Maximum Likelihood Approach
MLE	-	Maximum Likelihood Estimation
MPL	-	Maximum Pseudo Likelihood
PDF	-	Probability Density Function

CHAPTER 1

INTRODUCTION

1.1 Background of Study

Flooding hazard contributes the greatest economic impact on society among natural risks. It is a natural disaster which impacts to the environment, economics and social sector globally. Flood frequency analysis is one of the most important and widely studied technique in the fields of hydrology and water resources in order to predict the occurrence frequency of extreme flood events (Reddy and Gangguli 2012a; 2012b). Many hydrological models have been used in order to predict floods, (Zhang, 2005; Razi *et al*., 2010; Goodarzi *et al.*, 2013). At the drainage basin scale, judgement of flood risk is important for the planning of water infrastructure projects. In the designs of hydraulic structure (e.g., dam spillways, diversion canals, dikes and river channels), urban drainage systems, cross drainage structures (e.g., culverts and bridges), reservoir management, and flood hazard mapping, risk analysis of floods is critical as design criteria. As such, more effective flood risk assessment method is important due to limited hydrological data and uncertainties involved.

In general, flood is defined as inundation that caused by rivers overflowing their banks on account of heavy rainfall and also caused by a large concentration of runoffs. Flood is generally categorized with respect to mutually dependent random variables such as peak flow, flood duration and flood volume.

Many researchs have been established to implement conventional univariate on flood variables by Cunnane (1987), Linsley (1986), Zhang (2005), and Goodarzi *et al*. (2013). Multivariate flood frequency analysis has been conducted by Singh and Singh (1991) and Yue *et al*. (2001). According to Zhang and Singh (2006), both conventional univariate and multivariate techniques require restrictive assumption that suppose all variables are independent. However, the variables are actually interrelated in reality. Thus, this study aims to adapt multivariate statistical model in flood analysis by using a statistical method called Copula to construct the dependency structure and joint probabilistic distributions.

The great performance of copula has been performed by Favre *et al*. (2004) and also De Michele *et al*. (2007). Copula can be applied even for complex marginal distributions like a finite mixture, (Titterringtone *et al*., 1985). This means that by using Copula, the multivariate joint cumulative distribution function can be described incomprehensively where the dependency structure of variables in which univariate marginal properties can be analysed separately.

Bivariate analysis using Copula function in flood risk assessment have been studied by Salvadori and De Michele (2004), Favre *et al.* (2004), Grimaldi and Serinaldi (2006), Zhang and Singh (2007), Salvadori *et al.* (2011), Chowdhary *et al*. (2011), and Goodarzi (2014). The application of bivariate copula in flood modelling is recognized as a useful method to reveal the significant relationship between the flood characteristics (Favre *et al*., 2004; Grimaldi and Serinaldi, 2006; Zhang and Singh, 2006). However, bivariate copulas are only able to model the dependent variable between two flood variables. In reality all the three flood variables are mutually correlated. Thus, a more complete analysis is possible by considering all the mutually correlated flood properties and then modelling them by using trivariate copulas function.

This thesis aims to develop a trivariate Copula function for flood frequency analysis in order to concede a joint probability distribution model regarding to specified univariate marginal distributions. This can be attained by carrying out the bivariate analysis that undergoes two stages. First stage is to analyse the bivariate copula pair for the first and second variables and for second stage is to rerun the bivariate copula pair of the paired variables with the other third variable.

1.2 Problem Statement

The inspirations to carry on this study arise from the need to have a better flood occurrence modelling. Previous study has worked on modelling the bivariate Copula in flood frequency analysis. However, there were some limitations in bivariate case where it only considered two flood variables and made assumption on the third variable. Thus, the third variable can be arbitrary. More accurate or better modelling can be conducted by considering all the three variables in formulating the flood frequency analysis. The three variables of flood variables consist of peak flow, flood duration and flood volume.

1.3 Objectives of the Study

The objectives are:

1. To find the best fitted bivariate copula model of flood variables by using simulation process before going into trivariate copula analysis.
2. To develop a trivariate copula distribution in a flood frequency analysis based on the result obtained from the bivariate copula.
3. To estimate the conditional probability, conditional return period and joint return periods based on the dependency of flood properties using the trivariate copula for future flood occurrence prediction.

1.4 Scope of the Study

The focus of this research is to use Copula method for flood frequency analysis. The scopes of this study in details are as follows:

1. The Sungai Johor watershed was chosen as the study area

2. The analysis used 34 years hourly discharge data which the annual maximum were derived starting at year 1965 and end at year 2010.

3. For bivariate analysis, the determination of the dependency between all the possible pairs of flood variables which are peak flow-flood duration, peak flow-flood volume and flood duration-flood volume were determined by using Kendall’s tau.

4. Simulations on the best fitted Copula distribution for bivariate case was determined by using Cramer von-mises method.

5. Five copula distributions which are Clayton, Frank, Gumbel, Gaussian and *t*-Copula were employed in trivariate analysis.

6. Inversion of Kendall’s tau, Inversion of Spearman’s rho, Maximum Likelihood Approach and Inference Function of Margins were used for parameter estimation.

7. The best fitted Copula distribution for this study was selected based on the Akaike Information Criterion (AIC) method.

1.5 Significance of Study

Flood is the most desolating natural disaster experienced in Malaysia. Malaysia has experienced the floods that lead to significant impacts to the environment, economics and social sector of the whole nation. Heavy rainfall and large concentration of runoffs are the causes of flooding in Malaysia. Due to that, various flood predicting and warning systems with sophisticated hydraulic and hydrological models have been operated to predict floods (Razi *et al.*, 2010).

Flood frequency analysis is crucial to forecast the probability of having the flood in the future and this present study manage to control flood, lessen the consequences of flood, advocate public on flood influence and plan for upcoming flood risks. Therefore, it is important to model the flood by using the flood properties as it can model the correlation and dependency between the variables. Trivariate flood frequency analysis is much recommended because it can model the flood frequency which involves all the flood variables and can be done through the concept of distributions. The concept of Copula is still recent in Malaysia but it is manage to overcome the restriction of univariate models and serve a bigger choice for dependency function. This study will establish trivariate Copulas as an eligible choice for effective risk assessment of floods as trivariate Copulas are able to determine the measurement of each particular flood variable at once and consider the dependency between the flood variables at the same time.

1.6 Organization of the Thesis

This thesis begins with Chapter 1 where the discussion on the background and problem statement being discussed. Then, it is followed by highlighting on the objectives and also scope of the study and finally interpretation on the significant of the study.

Next, followed by Chapter 2 where the literature review on flood frequency analysis and copula function development and its application in many fields are being discussed. The classical flood frequency analysis and advantages of Copula function are also highlighted in this chapter.

In Chapter 3, the research methodology which describes about the steps to do in adding mathematical interpretation and simulation on bivariate Copula modelling and also steps in performing trivariate Copula modelling are being presented.

Chapter 4 presents the results of analysis on mathematical interpretation and simulation added on the result of bivariate Copula analysis conducted by previous study.

While in Chapter 5, the analysis of joint trivariate Copula distribution, conditional probability, conditional return period and joint return period are being showed. The choice of the most fitted Copula distribution is also discussed in this chapter.

Finally, Chapter 7 wraps up about the main objectives of this study with some suggestion and recommendation for upcoming study.

REFERENCES

Accioly, R.M.S. and Chiyoshi, F.Y. (2004).Modeling Dependence With Copulas: A Useful Tool for Field Development Decision Process. *Journal of Petroleum Science and Engineering* 44(1-2), 83-91.

Akaike, H. (1974). A new look at the statistical model identification. *Automatic Control, IEEE Transactions on*. 19(6), 716-723.

Ang, A. H. S., and Tang, W. H. (1975). *Probability Concepts in Engineering Planning and Design.Volume I- Basic Principles*. (2^{th}ed.) New York: John Wiley and Sons, Inc.

Ashkar, F., and Rousselle, J. (1982). A multivariate statistical analysis of flood magnitude, duration and volume.*Statistical Analysis of Rainfall and Ronoff*, 659-669.

Azzalini, A., and Capitanio, A. (2003). Distributions generated by perturbation of symmetry with emphasis on a multivariate skew-t distribution. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*. 65(2), 367-389.

Chakravarti, Laha, and Roy, (1967). Handbook of Methods of Applied Statistics, Volume I, John Wiley and Sons, pp. 392-394.

Cherubini, U., Luciano, E., and Vecchiato, W. (2004). *Copula methods in finance*. (1^{th}ed.). The Atrium England: John Wiley and Sons Ltd.

Chen, L., Singh, V. P., Shenglian, G., Hao, Z., and Li, T. (2011). Flood coincidence risk analysis using multivariate copula functions. *Journal of Hydrologic Engineering*. 17(6),742-755.

Choros, B., Ibragimov, R., and Permiakova, E. (2010). *Copula estimation*.In Jaworski, P., Durante, F., Hardle, W. K., Rychlik, T. (Ed.) *Copula theory and its applications*. (pp. 77-91). Berlin: Springer Heidelberg.

Chowdhary, H, Escobar, L.A., and Singh, V.P. (2011).Identification of suitable copulas for bivariate frequency analysis of flood peak and flood volume data.*Hydrology Research*. 42(3),193-216.

Clayton, D. G. (1978).A model for association in bivariate life tables and its application in epidemiological studies of familial tendency in chronic disease incidence. *Biometrika*. 65(1), 141-151.

Clemen, R.T. and Reilly,T. (1999).Correlations and Copulas for Decision and Risk Analysis. *Management Science*. 45(2), 208-224.

Cook, R. D., and Johnson, M. E. (1981). A family of distributions for modelling non-elliptically symmetric multivariate data. *Journal of Royal Statistical Society Series B(Methodological)*. 43(2), 210-218.

Cunnane, C. (1987a). Methods and merits of regional flood frequency analysis.*Journal of Hydrology*. 100(1), 269-290.

Cunnane, C. (1987b). *Review of Statistical Models for Flood Frequency Estimation*.In Singh, V. P. (Ed.) *Hydrologic Frequency Modeling*. (pp. 49-45). Springer Netherlands.

De Michele, C., Salvadori, G. (2003). A generalized pareto intensity-duration model of storm rainfall exploiting 2-copulas. *Journal of Geophysical Research Atmosphere*. 108(1), 1884-2012.

De Michele, C., G. Salvadori, M. Canossi, A. Petaccia, and R. Rosso,(2005).Bivariate Statistical Approach to Check Adequacy of DamSpillway. *Journal of Hydrologic Engineering.* 10(1), 50-57.

Embrechts, P., McNeil, A. and Straumann, D. (2002).Correlation and Dependence in Risk Management: Properties and Pitfalls, *Risk Management: Value at Risk and Beyond*, 176-223.

Fadhilah, Y., Foo, H. M., Suhaila, J., Zulkifli, Y., and Kong, C. Y. (2013). Characterisation of Drought Properties with Bivariate Copula Analysis. *Water Resources Management*. 27(12), 4183-4207.

Favre, A. C., El Adlouni, S., Perreault, L., Thiemonge, N., and Bobbe, B. (2004). Multivariate hydrological frequency analysis using copula. *Water Resources Research*. 40(1):

Foo, H. M., Fadhilah, Y, Suhaila, J., and Zulkifli, Y. (2013). Trivariate Copula in Drought Analysis .*Seminar Kebangsaan ISM ke-7 (SKism VII) dan Mesyuarat Agung ISM 2013*. 28 Mac 2013. Hotel De Palma Sek.19 Shah Alam.

Frank, M. J. (1979). On the simultaneous associativity of F(x,y) and x+y – F(x,y). *Aequationes Mathematicae*. 19(1), 194-226.

Frees, E.W. and Valdez, E.A. (1998).Understanding Relationships Using Copulas. North *American Actuarial Journal*. 2(1), 1-25.

Frees, E.W., J. Carriere, and Valdez, E.A. (1996).Annuity Valuation With Dependent Mortality. *Journal of Risk and Insurance*. 6(2), 229-261.

Ganguli, P., and Reddy, M. J. (2013a). Analysis of ENSO-based climate variability in modulating drought risks over western Rajasthan in India. *Journal of Earth System Science.* 122(1), 253-269.

Ganguli, P., and Reddy, M. J. (2013b). Probabilistic assessment of flood risks using trivariate copulas. *Theoretical and Applied Climatology*. 111(1-2), 341-360.

Genest, C., and Favre, A. C. (2007). Everything you always wanted to know about copula modelling but were afraid to ask. *Journal of Hydrologic Engineering*. 12(4), 347-368.

Genest, C., Goudhi, K., and Rivest, L. (1995). A semiparametric estimation procedure of dependence parameters in multivariate families of distributions. *Biometrika*. 82(3), 543-552.

Genest,C., MacKay, R. J. (1986). Copulesarchimedienneset families de loisbidimensionelles don't les margessontdonnees. *Journal of Statisitics*. 14, 145-159.

Genest, C., Neslehova, J., and Ghorbal, N. B. (2011). Estimation based on Kendall's tau in multivariate copula models. *Australian and New Zealand Journal of Statistics*. 53(2), 157-177.

Ghizzoni, T., Roth, G., and Rudari, R. (2010).Multivariate Skew-t approach to the design of accumulation risk scenarios for the flooding hazard. *Advances in Water Resources*. 33(10), 1243-1255.

Gumbel, E. J. (1960). Distributions des Valeurs Extremes en Plusieurs Dimensions. *Publications de l'Institute de Statist'ique de l'Universit'e de Paris.* 9, 171-173.

Grimaldi, S., and Serinaldi, F. (2006). Asymmetric copula in multivariate flood frequency analysis. *Advances in Water Resources*. 29(8),1155-1167.

Goodarzi, E., Mirzaei, M., and Ziaei, M. (2012). Evaluation of dam overtopping risk based on univariate and bivariate flood frequency analyses. *Canadian Jounal of Civil Engineering*. 39(4), 374-387.

Goodarzi, E., Ziaei,M., and Shui, L.T. (2013). *Evaluation of Damn Overtopping Risk Based on Univariate and Bivariate Flood Frequency Analyses*. In Goodarzi, E., Ziaei,M., and Shui, L.T. (Ed.) *Introduction to Risk and Uncertainty in Hydrosystem Engineering* (pp. 123-141). Netherlands:Springer.

Goodarzi, S. M. (2014). *Flood Freequency Analysis Based on Bivariate Copula Functions.*Doctoral dissertation. University of Technology Malaysia.

Haff, I. H. (2013). Parameter estimation for pair-copula construction .*Bernoull*. 19(2), 462-491.

Hiirlimann, W. (2004). Fitting Bivariate Cumulative Returns WithCopulas. *Computational Statistics and Data Analysis*. 45(2), 355-372.

Joe, H. (1997). *Multivariate models and dependence concepts*, (1th ed.), volume 73 of Monographs on Statistics and Applied Probability, London: Chapmann and Hall.

Kao, S. C., and Chang, N. B. (2011). Copula-Based Flood Frequency Analysis at Ungauged Basin Confluences: Nashville, Tennessee. *Journal of Hydrologic Engineering*. 17(7), 790-799.

Kao, S. C., and Govindaraju, R. S. (2008). Trivariate statistical analysis of extreme rainfall events via the Plackett family of Copulas. *Water Resources Research*. 44(22), Kao, S. C., and Govindaraju, R. S. (2010).A copula-based joint deficit index for droughts. *Journal of Hydrology*. 380(1), 121-134.

Karmakar, S., and Simonovic, S. P. (2009). Bivariate flood frequency analysis. Part 2: a copula-based approach with mixed marginal distributions. *Journal of Flood Risk Management*. 2(1),32-44.

Kite, G. W., and Stuart, A. (1977). Frequency and risk analysis in hydrology. (1th*ed.*) Fort Collins: Water Resources publication.

Klein, B., Pahlow, M., Hundecha, Y., and Schumann, A. (2010). Probability analysis of hydrological loads for the design of flood control systems using copulas. *Journal of Hydrologic Engineering.* 15(5), 360-369.

Klugman, S.A. and Parsa, R. (1999). Fitting Bivariate Loss Distributions With Copulas. *Insurance Mathematics and Economics.* 24(1-2),139-148.

Kojadinovic, I., and Yan, J. (2010). Comparison of three semiparametric methods for estimating dependence parameters in Copula models. *Insurance: Mathematics and Economics*. 47,52-63.

Kolev, N., Anjos, U. D., and Mendes, B. V. D. M. (2006). Copulas: a review and recent developments. *Stochastic Models*. 22(4),617-660.

Krstanovic, P. F., and Singh, V. P. (1987).*A multivariate Stochastic Flood Analysis Using Entropy*.In Singh, V. P. (Ed.).*Hydrologic Frequency Modeling*. (pp. 515-539). Netherland: Springer.

Lian, J. J., Xu, K., and Ma, C. (2013). Joint impact of rainfall and tidal level on flood risk in a coastal city with a complex river network: A case study of Fuzhou City, China. *Hydrology and Earth System Sciences*. 17(2), 679-689.

Linsley, R. K. (1986). Flood estimates: how good are they?.*Water Resources Research*. 22(9S), 1595-1645.

Louie, H. (2012). Evaluation of bivariate Archimedean and elliptical copulas to model wind power dependency structures. *Wind Energy*. Advance online publication. 17(2), 225-240.

McNeil, A., Frey, R. and Embrechts,P.(2005). Quantitative Risk Management: Concepts, Techniques, and Tools. *Princeton University Press*.

Mendes, B.V.M. and Souza, R.M.(2004). Measuring Financial Risks With Copulas. *International Review of Financial Analysis*13(1):27-45.

Michele, C. D. and Salvadori, G. (2003). A Generalized Pareto intensity-duration model of storm rainfall exploiting 2-Copulas. *Journal of Geophysical Research: Atmospheres*. 108(D2).

Nelsen, R. B. (1999). *An introduction to copulas*.Springer. Lectures Notes in Statistics Springer New York.

Nelsen, R. B. (2006). *An Introduction to Copula*.Springer. (2nded). New York: SpringerScience Business Media.

Papadimitriou, Z. G., Sagias, N. C., Bithas, P. S., Mathiopoulus, P. T., and Merakos, L. (2006). The trivariateWeibull distribution with arbitrary correlation. *International Workshop on Satellite and Space Communications.*14-15 September. Leganes, Spain:IEEE, 249-253.

Rao, A. R. and Hamed, K. H., (2000). *Flood frequency analysis*. New York: CRC Press.

Reddy, M. J. and Ganguli, P. (2012a).Application of copulas for derivation of drought severity-duration-frequency curves. *Hydrological Processes*. 26(11), 1672-1685.

Reddy, M. J. and Ganguli, P. (2012b).Risk assessment of hydro-climatic variability on groundwater levels in the Manjara basin aquifer in India using Archimedean copulas. *Journal of Hydrology Engineering*. 17(12), 1345-1357.

Requena, A. I., Mediero, L., and Garrote, L. (2013). Bivariate return period based on copulas for hydrologic dam design: comparison of theoretical and empirical approach. *Hydrology and Earth System Sciences Discussions*. 10(1), 557-596.

Salvadori, G., and Michele, C. D. (2010). Multivariate multiparameter extreme value models and return periods: A Copula Approach. *Water Resources Research*. 46(10), W10501.

Salvadori, G., De Michele, C., Kottegoda, N. T., Rosso, R. (2007). Extremes in Nature: An approach using Copulas. *Water Science and Technology*. 56. Springer. New York.

Schweizer, B., and Wolff, E. F. (1981).On nonparametric measures of dependence for random variables. *The Annals of Statistics*. 9(4), 879-885.

Serinaldi, F., and Grimaldi, S. (2007).Fully nested 3-copula: Procedure and application on hydrological data. *Journal of Hydrologic Engineering*. 12(4), 420-430.

Shiau, J. T., Wang, H. Y., and Chang, T.T. (2006). Bivariate Frequency Analysis of Floods Using Copulas.JAWRA *Journal of the American Water Resources Association*. 42(6), 1549-1564.

Singh, K., Singh, V.P. (1991). Derivation of bivariate probability density functions with exponential marginal. *Stochastic Hydrology and Hydraulics*. 5(1), 55-68.

Sklar, A. (1959). Fonctions de repartition a n dimensions et leursmarges. *Paris Institute of Statistics*. 8, 229-231.

Song, S., and Singh, V. P. (2010).Frequency analysis of droughts using the placket copula and parameter estimation by genetic algorithm.Stochastic *Environmental Research and Risk Assesment*. 24(5), 783-805.

Stedinger, J. R., Vogel, R. M. and Foufoula-Georgiou, E. (1993). Frequency analysis of extreme events, in *Handbook of Hydrology*, D.R. Maidment (ed.), McGraw-Hill, Chap. 18, 66.

Titteringtone, D. M., and Rossi, C. (1985).Another look at a Bayesian direct deconvolution method. *Signal processing*. 9(2),101-106.

Varin, C., Reid, N., and Firth, D. (2011).An overview of composite likelihood methods. *StatisticaSinica*. 21(1),5-42.

Volpi, E., and Fiori, A. (2012). Design event selection in bivariate hydrological frequency analysis. *Hydrological Sciences Journal*. 57(8), 1506-1515.

Wang, C., Chang, N. B., and Yeh, G. T. (2009).Copula-based flood frequency (COFF) analysis at the confluences of river systems. *Hydrological Processes.* 23(10), 1471-1486.

Wong, G. H. (2010). *Drought Predictions: Applications in Australia*. Doctoral dissertation, University of Adelaide

Wong, G., Lambert, M. F. and Metcalfe, A. V. (2008). Trivariate Copulas for Characterisation of Droughts. *Austral. Mathematical Society*. 49, 306-323

Yan, J. (2007). Enjoy the joy of copulas: With a package copula. *Journal of Statistical Software*. 21(4), 1-21.

Yanmaz, A. M., and Gunindi, M. E. (2008).Assesment of overtopping reliability and benefits of a flood detention dam. *Canadian Journal of Civil Engineering*. 35(10), 1177-1182.

Yoo, J., Kim, U., and Kim, T. W. (2013). Bivariate drought frequency curves and confidence intervals: a case study using monthly rainfall generation. *Stochastic Environmental Research and Risk Assesment*. 27(1), 285-295.

Yoon, P., Kim, T. W., and Yoo, C. (2012). Rainfall frequency analysis using a mixed GEV distribution: a case study for annual maximum rainfall in South Korea. *Stochastic Environmental Research and Risk Assesment*. 27(5), 1143-1153.

Yue, S. (2001), A bivariate gamma distribution for use in multivariate flood frequency analysis. *Hydrological Processes*. 15(6), 1033-1045.

Yue, S., Ouarda, T. B. M. J., and Bobee, B. (2001). A review of bivariate gamma distributions for hydrological application, *Journal of Hydrology*. 246(1), 1-18.

Yue, S., Ouarda, T. B. M. J., Bobee, B., Legendre, P., and Bruneau, P. (1999). The Gumbel mixed model for flood frequency analysis. *Journal of Hydrology*. 226(1), 88-100.

Zhang, L. (2005). *Multivariate hydrological frequency analysis and risk mapping*. Doctoral dissertation.Louisiana State Univesity.

Zhang, L., and Singh, V. P. (2006). Bivariate flood frequency analysis using the copula method. *Journal of Hydrologic Engineering*. 11(2), 150-164.

Zhang, L., and Singh, V. P. (2007). Trivarite flood frequency analysis using the GumbelHougaard Copula. *Journal of Hydrolgic Engineering*. 12(4), 431-439.