SOLUTION OF DIFFUSION EQUATION OF TOTAL CONCENTRATION IN SPHERICAL SHAPE DROPS USING LAPLACE TRANSFORM

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Special dedicated to

My beloved father, Abu Bakar Md Zain and my beloved mother, Raha Ahmad.

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ABSTRACT

Liquid-liquid extraction is a separation technology that is based on the distribution of one or more components between two immiscible or almost immiscible liquids. The equipment that is used for the extraction is Rotating Disc Contactor (RDC) column. This research is concerned with solving analytically the diffusion equation of total concentration in spherical shape drops by using Laplace transform. Currently the drops move the counter in a RDC column. The total concentrations of all spherical shape drops were simulated by using Matlab software and Microsoft Office Excel. From the results, it shows that total concentration of spherical shape drops increases as time increases and the total concentration also increases as radius of drops increases. Whilst, the value of total concentration decreases when different values of diffusion constant were used.

ABSTRAK

Pengekstrakan cecair-cecair adalah teknologi pemisahan yang berasaskan pembahagian satu atau lebih komponen antara dua cecair yang tidak boleh bercampur atau hampir tidak boleh bercampur. Peralatan yang digunakan untuk pengekstrakan adalah Turus Pengekstarakan Cakera Berputar (RDC). Kajian ini berkenaan dengan mendapatkan penyelesaian analisis bagi persamaan resapan dari jumlah kepekatan dalam titisan berbentuk bulat dengan menggunakan jelmaan Laplace. Pada masa ini, titisan bergerak dalam lajur RDC. Jumlah kepekatan bagi semua titisan berbentuk bulat diperolehi dengan menggunakan perisian Matlab dan Microsoft Office Excel. Keputusan menunjukkan jumlah kepekatan bagi titisan berbentuk bulat bertambah apabila masa bertambah, dan jumlah kepekatan juga bertambah apabila jejari bagi titisan bertambah. Sementara itu, jumlah kepekatan akan menurun apabila nilai resapan tetap yang digunakan bertambah.

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LIST OF SYMBOLS/NOTATIONS

a	-	Radius of a sphere
RDC	-	Rotating disc column
C_0	-	Concentration of one drop on surface
C_1	-	Initial uniform concentration of drop
$C(r,t)$	-	Concentration at distance r from the center at time t
C_s	-	Total concentration of one drop
D	-	Diffusion equation constant
L	-	Laplace transform
IBVP	-	Initial Boundary Value Problem

CHAPTER 1

INTRODUCTION

1.1 Background of the study

Liquid-liquid extraction has become very important issues to be discussed amongst chemical engineers as well as mathematicians [1]. Liquid- liquid extraction is a separation technology that is based on the distribution of one or more components between two immiscible liquids. Normally, liquid-liquid extraction is used when separation by distillation is ineffective or very difficult because certain liquid cannot withstand the high temperature of distillation. Thus, liquid-liquid extraction is the main alternatives to be considered.

There are wide ranges of industries that use this application extraction such as petroleum industries, biotechnology, food processing, polymer processing, separation and purification of pharmaceutical and nuclear industry. The main principle of liquid-liquid extraction is separation of components of homogeneous solution by using another solution which is known as solvent [2, 3]. The extractor types that use for liquid-liquid

extraction can be categorized into broad categories. Some of these are static extraction column, the agitated column, the centrifugal column and mixers settlers.

The static extraction column is an extractor that is not involved with the mechanism of stirring to separate the extraction from solvent. Example of static extraction columns are spray column, packed column and sieve plate column. While agitated column was developed in order to control the efficiency of liquid-liquid extraction. Examples of agitated column are Rotating Disc Contactor (RDC) column, SCHEIBEL column and reciprocating plate column. The other type of extractor is centrifugal extractor where it is an advanced design to separate extract from solvent using a rapid centrifuge while mixers settlers is a series of static or agitated mixers interspersed with settling stages. The main concern of this research is only to the extractor of Rotating Disc Contactor (RDC) column.

There were models that have been developed by researches for modeling of extraction process in RDC column. The results of the modeling show that drop size and mass transfer process are important factors for the column performances. Mass transfer is the movement of molecules from high concentration to low concentration until equilibrium concentration is achieved. The models of mass transfer are based on the radial diffusion equation with a constant boundary condition [1].

Based on Crank [6], diffusion is the process by which matter is transported from one part of a system to another as a result of random molecular motions. From the diffusion equation and together with boundary value and initial value, it can solve the problem of concentration in spherical shape drops. Crank [6] solved the boundary value problem by using the method of Separation of Variable. An other method that can be used to solve the problem analytically is the Laplace transform and the aim of this study is to find the solution of diffusion equation of total concentration in spherical shape drops.

1.2 Statement of problem

Several diffusion models have been developed in order to find concentration and total concentration of drops in Rotating Disc Contactor (RDC) column. Hidayu [14], developed a new diffusion model for total concentration of tear shape drop that consists of hemisphere and cone. She solved the problem using the method of partial differential equation, separation of variable and method of variation parameter. Then, Syarafina [15], developed a diffusion model for ellipsoidal shape drop and she also solved the total concentration of spherical shape drops using forward difference explicit method.

Besides that, Crank [6] solved diffusion equation of concentration in spherical shape drops using the method of Separation of Variable. This study concerned with solving analytically the diffusion equation of total concentration in spherical shape drops by using Laplace transforms. Laplace Transform is an integral transform that can be used to solve differential equation. It is also a tool to convert a difficult problem into simpler one.

Ten different sizes of radius and three different values of diffusion constant were used to solve this problem. Therefore, this study can help us to identify the effects to the total concentration when use different sizes of radius and different values diffusion constant.

1.3 Objectives of study

i) To solve diffusion equation of total concentration in spherical shape drops by using Laplace transform method based on the initial boundary value problem (IBVP)

ii) To simulate the diffusion equation of total concentration in spherical shape drops when different values of diffusion constant were used.

1.4 Scope of study

This study will focus on solving the initial boundary value problem of diffusion equation that has its own boundary and initial conditions. By solving it using Laplace transform, the diffusion equation of total concentration in spherical shape drops can be obtained. The drops concentrations are simulated for ten classes of Rotating Disc Contactor (RDC) column and the range of radius is between 0.0003525*mm* to 0.003525*mm*.

1.5 Significance of study

This study will develop a model of diffusion equation of total concentration in spherical shape drops by using Laplace transform. The model that will be developed can be used to find other method of total concentration of spherical shape drops for various drops in RDC column.

1.6 Summary

In this introductory chapter, a short introduction of liquid-liquid extraction and type of extractor for liquid-liquid extraction has been presented. The current chapter gives direction and purpose to the research and the discussions presented here are the basis for the work done in the subsequent chapter.

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