FREE VIBRATION OF LAMINATED COMPOSITE SHELL STRUCTURES FILLED WITH FLUID

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To my beloved Father and Mother,
Mat Daud bin Hamat and Wan Azizah binti Wan Ibrahim
for their everlasting love, and endless support, throughout my life.
Also to my wonderful siblings,
Faiz, Hazman, Irsyad, Ayuni and Izzni.

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ABSTRACT

Free vibration of layered truncated conical and circular cylindrical shells filled with fluid based on Love's first approximation theory are analysed in this research. In addition, investigation regarding the free vibration of laminated composite circular cylindrical shells filled with fluid using the first order shear deformation theory also presented. In this study, the shell is filled with quiescent fluid and analysed using the spline method. The shell equations are assumed to be in a separable form, which hence a set of coupled ordinary differential equations in the term of displacement functions is obtained for the case of the Love's first approximation theory. For the case under first order of shear deformation theory, the rotational functions are included. These functions are approximated using the spline function, bringing into a set of field equations together with boundary conditions, that reduce to a system of homogeneous simultaneous algebraic equations on the assumed spline coefficients. The resulting generalised eigenvalue problem is solved to get as many eigen frequencies as required by starting from the least. From the eigenvectors on the spline coefficients, the mode shapes can be constructed. In the first case, the effects of the relative layer thickness, cone angle, length ratio, and boundary conditions on the frequencies of truncated conical shell filled with fluid are presented. Through the application of the same theory, the effect of the relative layer thickness, length-to-radius ratio, thickness-to-radius ratio, circumferential node number, and boundary conditions on the frequencies of circular cylindrical shell filled with fluid are investigated. In the case of first order shear deformation theory; a cross-ply, anti-symmetric angle-ply, and symmetric angle-ply laminated composite circular cylindrical shell filled with fluid are analysed. Parametric studies have been conducted with respect to the length-to-radius ratio, thickness-to-radius ratio, material properties, ply orientations, number of layers, and boundary conditions on the frequencies. The contribution of this research is to provide solutions for free vibration of laminated composite conical and cylindrical shells filled with fluid using spline method. The frequency of the shell filled with fluid is found to be lower than the frequency of the shell without fluid due to the effect of fluid in the shell that acts as the added mass to it. Material properties, ply orientations, number of layers, boundary conditions, relative layer thickness, length-to-radius ratio, thickness-to-radius ratio, circumferential node number, cone angle, and length ratio significantly affect the frequencies of the shell. Furthermore, simply supported boundary conditions are found to have the lowest frequency followed by clamped-free and clamped-clamped boundary conditions.

ABSTRAK

Getaran bebas bagi lapisan cengkerang kon separuh dan lapisan cengkerang silinder bulat yang dipenuhi bendalir berdasarkan teori penghampiran pertama Love dianalisis dalam kajian ini. Tambahan pula, kajian mengenai getaran bebas bagi cengkerang silinder bulat komposit berlamina yang dipenuhi bendalir menggunakan teori ubah bentuk pemotongan tertib pertama juga dibentangkan. Dalam kajian ini, cengkerang yang dipenuhi bendalir statik dianalisis menggunakan kaedah *spline*. Persamaan cengkerang diandaikan dalam bentuk bolehpisah, yang mana, satu set persamaan perbezaan biasa berganding dalam sebutan fungsi-fungsi anjakan diperolehi untuk kes teori penghampiran pertama Love. Bagi kes dibawah teori ubah bentuk pemotongan tertib pertama, fungsi-fungsi putaran dimasukkan. Fungsi-fungsi ini dihampirkan dengan menggunakan fungsi *spline*, seterusnya membawa kepada satu set persamaan bidang bersama-sama dengan syarat-syarat sempadan, yang diturunkan kepada sistem persamaan homogen algebra serentak pada pekali *spline* yang diandaikan. Masalah nilai eigen teritlak yang terhasil diselesaikan bagi mendapatkan seberapa banyak frekuensi eigen seperti yang dikehendaki bermula dari yang paling kecil. Daripada vektor eigen bagi pekali *spline*, bentuk mod boleh dibina. Dalam kes pertama, kesan-kesan bagi ketebalan lapisan relatif, sudut kon, nisbah panjang, dan syarat-syarat sempadan terhadap frekuensi-frekuensi cengkerang kon separuh yang dipenuhi bendalir dibentangkan. Menggunakan aplikasi teori yang sama, kesan bagi ketebalan lapisan relatif, nisbah panjang-jejari, nisbah ketebalan-jejari, bilangan nod lilitan, dan syarat-syarat sempadan terhadap frequensi-frekuensi cengkerang silinder bulat yang dipenuhi bendalir dikaji. Dalam kes teori ubah bentuk pemotongan tertib pertama; lapis silang, antisimetri lapis sudut, dan simetri lapis sudut cengkerang silinder bulat komposit berlamina yang dipenuhi bendalir dianalisis. Kajian secara parameter terhadap nisbah panjang-jejari, nisbah ketebalan-jejari, sifat-sifat bahan, orientasi lapis, bilangan lapisan, dan syarat-syarat sempadan terhadap frekuensi-frekuensi telah dijalankan. Sumbangan penyelidikan ini adalah untuk menyediakan penyelesaian untuk getaran bebas bagi komposit berlamina cengkerang kon dan cengkerang silinder yang dipenuhi bendalir mengunakan kaedah *spline*. Frekuensi cengkerang yang dipenuhi bendalir didapati lebih rendah daripada frekuensi cengkerang tanpa bendalir yang disebabkan oleh kesan bendalir di dalam cengkerang yang bertindak sebagai penambah jisim kepadanya. Sifat-sifat bahan, orientasi lapis, bilangan lapisan, syarat-syarat sempadan, ketebalan lapisan relatif, nisbah panjang-jejari, nisbah ketebalan-jejari, bilangan nod lilitan, sudut kon, dan nisbah panjang memberi kesan yang bermakna kepada frekuensi-frekuensi cengkerang. Tambahan pula, syarat-syarat sempadan yang disokong mudah didapati mempunyai frekuensi yang paling rendah diikuti oleh syarat-syarat sempadan yang bebas-apit dan yang diapit-apit.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

AGE	-	AS4/3501-6 Graphite/Epoxy
Al	-	Aluminium
C-C	-	Both ends are clamped
C-F	-	One end is clamped and the other end is free
CST	-	Classical Shell Theory
FSDT	-	First order Shear Deformation Theory
HSG	-	High Strength Graphite Epoxy
KGE	-	Kevler-49 Epoxy
PRD	-	PRD-490 III Epoxy
SGE	-	S-Glass Epoxy
S-S	-	Both ends are simply supported
St	-	Steel

LIST OF SYMBOLS

A_{ij}	-	Elastic coefficients representing the extensional rigidity
B_{ij}	-	Elastic coefficients representing the bending-stretching coupling rigidity
D_{ij}	-	Elastic coefficients representing the bending rigidity
$E_x^{(k)}$	-	Young's modulus along x directions of the *k*-th layer
$E_\theta^{(k)}$	-	Young's modulus along θ directions of the *k*-th layer
$G_{xz}^{(k)}, G_{x\theta}^{(k)}, G_{\theta z}^{(k)}$	-	Shear modulus in the respective directions of the *k*-th layer
H	-	Thickness parameter
$H(X - X_j)$	-	The Heaviside step function
I_1	-	Normal inertia coefficient
I_3	-	Rotary inertia coefficients
K	-	Shear correction factor
L	-	Length parameter
L_{ij}, L_{ij}^*	-	Differential operator occurring in the equations of motion
$M_x, M_\theta, M_{x\theta}$	-	Moment resultants in the respective directions of the shell
$N_x, N_\theta, N_{x\theta}$	-	Stress resultants in the respective directions of the shell
N	-	Number of intervals of spline interpolation
$Q_{ij}^{(k)}$	-	Elements of the stiffness matrix for the material of *k*-th layer
$\bar{Q}_{ij}^{(k)}$	-	Elements of the transformed stiffness matrix for the material of *k*-th layer
$Q_{xz}, Q_{\theta z}$	-	Transverse shear resultants in the respective directions of the shell
$R(x)$	-	Spline function

R_0	-	Inertial coefficient of a layered shell
U, V, W	-	Displacement functions in x, θ, z direction
$\bar{U}, \bar{V}, \bar{W}$	-	Non-dimensionalised displacement functions in x, θ, z direction
X	-	Non-dimensionalised meridional distance coordinate
X_s	-	The equally spaced knots of spline interpolation
a, b	-	Distance of the small and large ends of the conical shell from the vertex (or centre)
$\left.\begin{matrix} a_i \\ c_i \\ e_i \\ g_i \\ l_i \end{matrix}\right\}, \left.\begin{matrix} b_j \\ d_j \\ f_j \\ p_j \\ q_j \end{matrix}\right\}$	-	Spline coefficients
$e_x, e_\theta, e_{x\theta}$	-	Strain components at an arbitrary point of the shell
h	-	Thickness of the shell
h_k	-	Thickness of the *k*-th layer of the shell
i, j, k	-	Summation or general indices
ℓ	-	Length of the shell
n	-	Circumferential node number
r	-	Radius of reference surface of shell at a general point
r_a, r_b	-	The radius at the small and the large end of the cone
S_{ij}	-	Coefficients in the equations of motion
t	-	Time coordinate
u, v, w	-	Displacements in x, θ, z direction
u_0, v_0	-	The in-plane displacements of the reference surface of the shell
$y*(x)$	-	Cubic spline function
x, θ, z	-	Axial, circumferential and normal coordinates of any point on the shell
z_k	-	Distance of the top of the *k*-th layer from the reference surface of the shell
α	-	Semivertical angle of the cone shell

β	-	length ratio a/b of the conical shell
δ_k	-	Relative layer thickness of the k -th layer
$\varepsilon_x, \varepsilon_\theta$	-	Normal strain components of the reference surface of the shell
γ	-	Ratio of the constant thickness to radius of small end of the cone
γ'	-	Ratio of the constant thickness to the distance of small end from vertex of the cone
$\gamma_{x\theta}, \gamma_{xz}, \gamma_{\theta z}$	-	Shear strain of the reference surface
$\kappa_x, \kappa_\theta, \kappa_{x\theta}$	-	Change in curvature on the reference surface of the shell
λ	-	Non-dimensional frequency parameter
$\upsilon_{x\theta}, \upsilon_{\theta x}$	-	Poison's ratio
ω	-	Angular frequency
Ψ_x, Ψ_θ	-	Shear rotational functions of the shell
ψ_x, ψ_θ	-	Shear rotations of any point on the middle surface of the shell
$\bar{\Psi}_X, \bar{\Psi}_\theta$	-	Non-dimensionalised shear rotationa functions of the shell
ρ	-	Mass density of the material of the shell
σ_x, σ_θ	-	Normal stress in the respective directions of shells
$\tau_{x\theta}, \tau_{\theta z}, \tau_{xz}$	-	Shear stress in the respective directions of shells
θ	-	Ply orientation angle

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Background of the Study

A composite structure consists of two or more constituent materials with different physical or chemical properties combined which results into a material with the characteristics that is definitely different from the individual components. Composite structures are commonly composed of reinforcing and matrix materials (Soedel, 2004).

The reinforcing materials mostly exist in the form of fibres and act as reinforcer or load-carrying agent. Fibre materials can be metals like iron, aluminum, copper, titanium, steel, and nickel, or organic materials such as graphite, glass, carbon, and boron. The functions of matrix materials are important to support and seal the fibres. The matrix can be among of organic, ceramic, or metallic materials (George, 1999).

Composite plays an important role throughout human history. The concept of composite is very ancient. Back in the ancient Egypt, it was recorded that straw was added to mud in order to strengthen bricks. It was also recorded that wood strips were glued at different angles in order to create plywood. In addition, Eskimos applied moss into the ice to build up an igloo. Besides, swords and armours were layered to add up strength as per shown by samurai swords; which was produced through repeated processes of folding and reshaping in order to form a multi-layered composite (George, 1999).

Composite structures offer high strength and stiffness as well as lightweight characteristics as major advantageous. Composite structure can be designed to be far stronger than steel as it can be engineered to be strong in a specific direction. Light in weight is also one of the factor for composite structure as to be used in many industries such as automotive and aircraft since lightweight indicates better fuel efficiency. In addition, composite structures also have the characteristics of corrosion resistance as well as better damping and shock absorbance. Swimming pool and bathtub are two other examples related to composite materials application.

In general, composite materials can be classified into three categories, namely fibre, particle, and laminated composites. If the reinforcement is made of fibre, then it is called as fibre composite. The reinforcement is in the form of particle for particle composites. Concrete is one familiar example of particle composites. Laminated composite consists of layers that is combined together to form a laminate, in which each of the layer is made from the first two types of composites. Each layer is called a ply or lamina. The lamina is the fundamental building block of laminated composite materials (Reddy, 2004; Ye, 2002). Figure 1.1 illustrates the laminated composite material.

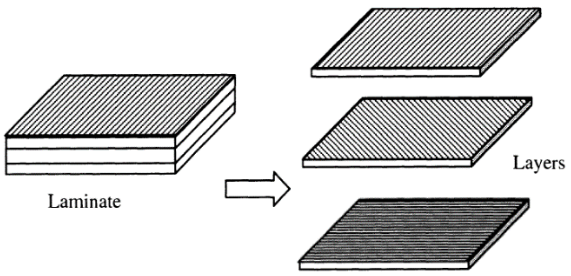

Figure 1.1 Laminated composite material (Ye, 2002)

A fibre-reinforced composite lamina consists of many fibres embedded in a matrix material. The fibre usually comes in the form of continuous and discontinuous, woven, unidirectional, bidirectional, or randomly distributed. The laminated composite has an interesting criteria which enables users to choose and design the right material combination and fibre orientation for an optimum design. Variation of fibre direction in each layer enables the different strength and stiffness in various direction to be tailored. This variation is the reason of the popular usage of laminated composite in most composites. For example, a unidirectional fibre-reinforced lamina have strong strengh in the direction of fibre but poor strength in the transverse direction of the fibre (Reddy, 2004). A unidirectional laminate has the form $\theta = 0^\circ$ for all plies (Vinson and Sierakowski, 2008).

Other types of laminates are angle-ply and cross-ply. Angle-ply laminates have lamina orientations of either $+\theta$ or $-\theta$ at $0^\circ < \theta < 90^\circ$, meanwhile cross-ply laminates use only $\theta = 0^\circ$ and $\theta = 90^\circ$ plies orientations in order to make a laminate. The examples for angle-ply and cross-ply laminates are shown in Figure 1.2.

(a)	(b)
+60°	90°
-60°	0°
+60°	90°
+60°	90°
-60°	0°
+60°	90°

Figure 1.2 Examples of (a) angle-ply and (b) cross-ply laminates

Shell structure is used tremendously in designing a modern structure because of its strength and stiffness characteristics due to its curvature, which is greatly significant in resisting the external forces (Ventsel and Krauthammer, 2001). Cylindrical and conical shell structures are noticeable in aviation, ship, building, missiles, and pressure vessel. Apart from that, the application of shell structure with the interaction of fluid can be found in many engineering applications such as containers, reservoirs, silos, nuclear power reactors, and pipe systems. In addition to that, the structures may have quiescent or flowing fluid, partially filled fluid, filled with fluid or submerged in a fluid.

Commonly, there are two types of vibration, namely force vibration and free vibration. By definition, a force vibration occur due to time-dependent external loads (Kraus, 1967). In other words, the vibration is produced when force is exerted by the external loads and the vibration will stop as the external loads is released from the system. Meanwhile, free vibration occurs in the absence of external load and it is initiated by some initial and boundary conditions (Ventsel and Krauthammer, 2001), which will give a continuous vibration with the same amplitude. The frequency of free vibration is known as natural frequency, and such frequency only depends on the geometric and material of the shell (Ventsel and Krauthammer, 2001). The natural frequencies need to be acknowledged in order to avoid the destructive effect of weather and the resonance which created by oscillating equipment or adjacent rotating (such as electrical machinery, jet and reciprocating aircraft engine, marine turbines) (Ventsel and Krauthammer, 2001; Kraus, 1967). Hence, it is essential to understand the vibrational characteristics of shell structure for industrial application.

Various studies on theoretical and experimental investigation of vibration behaviour of shell structure with fluid or without fluids have been conducted. The method used to determine the vibrational behaviour of the shell must be correctly adopted to ensure the result can meet its efficiency and accuracy. Currently, there are many methods that can be used for this purpose such as the Rayleigh-Ritz method (Zhu, 1994; Zhu, 1995), the finite element method (Carrera, 2002; Ramasamy and Ganesan, 1999), the Galerkin method (Lam and Loy, 1995a; Lee and Lu, 1995), the wave propagation approach (Zhang *et al*. 2001a; Zhang *et al*., 2001b), the general differential quadrature (GDQ) method (Tornabene *et al*., 2009; Asadi and Qatu, 2012), the multiquadric radial basis function method (Ferreira *et al*., 2007), and the spline method (Viswanathan *et al*., 2013). On the other hand, excellent review works on the composite material are accessible in Soedel (2004), Ye (2002), George (1999), Gibson (1994). Extensive studies on the development of theory and methods have been reviewed by Kraus (1967), Leissa (1973), Qatu, (2004) and Reddy (2004). Most of the studies were firstly applied onto isotropic shells, and were subsequently extended to a study related to the laminated composite ones.

In this study, free vibration of shell structures (truncated conical and circular cylindrical shell) filled with quiescent fluid using spline method is presented. The spline method applies a lower order approximation which is simple and effective in terms of its accuracy (Bickley, 1968). For the case of truncated conical shell, the equations of motion used are based on the Love's thin shell theory. The effects of relative layer thickness, cone angle, length ratio, type of materials and boundary conditions on the frequencies of two layered of shells are presented in this study.

For the case of circular cylindrical shell, the equations of motion are derived using two theories, which are Love's thin shell theory and First Order Shear Deformation Theory (FSDT). The first case study refers to the two layered of circular cylindrical shell based on Love's thin shell theory. Parametric studies are performed to analyse the frequency response of the shell with reference to the relative layer thickness, length parameter, thickness parameter, circumferential node number, type of materials, and boundary conditions.

Further, cross-ply, anti-symmetric angle-ply and symmetric angle-ply of laminated composite circular cylindrical shell which described by FSDT are investigated. The effects of shell geometries, type of materials, ply orientations, layer of materials and boundary conditions on frequencies are studied.

1.2 Problem Statement

High demands on composite structures in industry fields lead to further analysis on composite structures. In fact, popular usage of composite can be seen in automotive, building, and aircrafts industries. It is a necessity to find the natural frequency of the structures in order to avoid the destructive effect of weather and resonance due to adjacent rotating or oscillating equipment. Geometric parameters, angle orientations, and boundary conditions affect the frequencies of the composite structures (Asadi and Qatu, 2012).

In addition, Viswanathan and Navaneethakrishnan (2003) considered the free vibration of an empty cylindrical shell by using spline method. Furthermore, the method has been successfully used in solving the free vibration of an empty layered cylindrical shell. The spline method is one of the collocation methods and uses low degree polynomials in each of the interval compared to high degree polynomials, which does not suffer from Runge's phenomenon, which is a problem of oscillation using polynomial interpolation with polynomials of high degree.

Besides, Zhang *et al.* (2001b) considered the free vibration of an isotropic cylindrical shell filled with fluid. Therefore, this study, free vibration of laminated composite shell structures filled with fluid using spline method is analysed. The frequency parameter values on various fixed parameters of laminated composite shell structures filled with fluid for conical as well as cylindrical shells are obtained.

1.3 Objectives

The purpose of this research is to investigate the free vibration of laminated composite shell structures filled with fluid. This involves the mathematical formulation which included the derivation of the governing differential equations of motion and the transformation of the resulting governing equations into non-dimensional ordinary differential equations. The non-dimensional ordinary differential equations are approximated by using the spline method and resulted in an eigenvalue problem. The eigenvalue problem is solved for frequency parameters. This research embarks on the following objectives:

1. To obtain the frequency parameter values for various fixed parameters of layered truncated conical shell filled with fluid under Love's first approximation theory.

2. To determine the frequency parameter values for various fixed parameters of layered circular cylindrical shell filled with fluid under Love's first approximation theory.

3. To acquire the frequency parameter values for various fixed parameters of cross-ply laminated composite circular cylindrical shell filled with fluid under first order shear deformation theory.

4. To generate the frequency parameter values for various fixed parameters of anti-symmetric angle-ply laminated composite circular cylindrical shell filled with fluid under first order shear deformation theory.

5. To obtain the frequency parameter values for various fixed parameters of symmetric angle-ply laminated composite circular cylindrical shell filled with fluid under first order shear deformation theory.

1.4 Scope of the Study

This study aims to investigate the free vibration of laminated composite shell structure including circular cylindrical and truncated conical shell structure by using the spline approximation technique. The equations of motion of shell structure based on Love's first approximation theory and First Order Shear Deformation Theory (FSDT) are used in the problem. The shell structure is completely filled with fluid. It is assumed that the fluid is inviscid and quiescent throughout the problem. Quiescent fluid is known as the fluid with zero velocity. Hence, the effect of the fluid is introduced as added mass to the shell.

For the first problem, two layered truncated conical shell filled with quiescent fluid based on Love's first approximation theory is investigated. By applying the same theory, two layered circular cylindrical shell filled with fluid is solved. Then, cross-ply, anti-symmetric angle-ply, and symmetric angle-ply of laminated composite circular cylindrical shell filled with fluid based on FSDT is investigated.

In the case of Love's first approximation theory, the displacement components are assumed to be in a separable form in order to obtain a system of coupled differential equation consisting of the longitudinal, circumferential and transverse displacement functions, while the rotational functions are included for the case of FSDT. These functions are approximated by Bickley-type spline of suitable order. Collocation with these splines yields a set of field equations, which along with the equations supplied by the boundary conditions, and reduces to a system of homogeneous simultaneous algebraic equations on the assumed spline coefficients. The resulting generalised eigenvalue problem is solved to obtain frequency parameters and the corresponding eigenvectors. The spline coefficients are the eigenvectors from which the mode shapes are constructed. Clamped-Clamped (C-C), Clamped-Free (C-F) and Simply Supported-Simply Supported (S-S) are the considered boundary conditions. Parameter studies involving the geometries of the shell, ply orientations, material properties, layer of materials, and boundary conditions are made according to the problem.

1.5 Significance of the Study

The study of the free vibration of shell structures filled with fluid lead the researchers to a better understanding on the characteristics of the shell structure and enhanced knowledge in finding the natural frequencies of the shell.

The characteristic of the composite such as lightweight, high strength, and high stiffness provides superiority in designing any structure in engineering field as it can be engineered and designed to be strong in a specific direction. Instead of using unidirectional lamina as a structural element, each lamina is oriented at different angles or direction to strengthen the structures. This is due to the poor transverse property of the unidirectional lamina.

Shell structure which containing inviscid and quiescent fluid is significantly affecting the frequency of the structure by lowering its frequency. Hence, the effect of the fluid cannot be ignored as it influences the vibration of the structure.

1.6 Research Methodology

The research work begins with the equations of motion based on Love's first approximation theory and First Order Shear Deformation theory. After that, the equations of motion which are coupled in displacement and rotations are obtained by substituting strain-displacement relations and stress-strain relations into the equations of motion. Then, by assuming the solution in separable form, a system of ordinary differential equation in terms of the longitudinal, circumferential, transverse displacement functions for the case of Love's first approximation theory and rotational functions included in the case of FSDT problem is obtained.

Next, the equations are non-dimensionalised. Together with the boundary conditions, it is approximated by Bickley-type spline method that resulting into a generalised eigenvalue problem. Thus, it is numerically solved by using power method. The eigenvalue problem is solved for frequency parameters and the corresponding eigenvectors.

Parameter studies with respect to geometry of the shell, material properties, layer of materials, ply orientations, and boundary conditions are considered to obtain the frequency of the shell. Convergence study is carried out to check the number of interval that used in the problem. The results are compared with literature results in order to validate the present method. Extensive parameter studies are conducted with respect to each problem and only suitable and reliable results are presented in this research. The results are presented in graphs and tables.

1.7 Thesis Outline

This thesis is organised into nine chapters, in which Chapter 1 is the introduction to the research, Chapter 2 is the literature review while Chapter 3 discusses the methodology used to solve the problems. Chapter 4 to Chapter 8 represents the five research problems of this study. Lastly, Chapter 9 represents the conclusion of the thesis.

Chapter 1 introduces the background of the study, problem statement, objectives of the study, scope of the study, significance of the study and the research methodology. Chapter organisation is discussed at the end of this chapter. In Chapter 2, the definitions of shell theories, the review on the previous work of various researchers regarding to the vibrational behaviour of empty conical and cylindrical shell, as well as the shells interact with fluid are discussed. Then, the method of spline is presented.

Chapter 3 presents the mathematical formulation of shell structures under two different shell theories. Further, the spline method is implemented onto the problem. Next, the equations are reduced to the form of generalised eigenvalue problem. The power method is used to determine the frequency parameters and associated eigenvectors.

The first problem of this thesis discusses on the free vibration of two layered truncated conical shell filled with fluid under the Love's first approximation theory is given in Chapter 4. The effects of relative layer thickness, semi cone angle, and length ratio under Clamped-Clamped (C-C) and Clamped-Free (C-F) boundary conditions on the frequencies are presented. Different combination of material such as S-Glass Epoxy (SGE), High Strength Graphite Epoxy (HSG), and PRD-490 III Epoxy (PRD), aluminium (Al) and steel (St) is used.

The following Chapter 5 describes the free vibration of two layered circular cylindrical shell filled with fluid under the Love's first approximation theory. Three types of materials which are PRD, SGE and HSG are considered. The frequencies with respect to the relative layer thickness, length-to-radius ratio, length-to-thickness ratio, and the circumferential node number under Clamped-Clamped (C-C) and Simply Supported-Simply Supported (S-S) boundary conditions are analysed. Convergence study is carried out to decide the optimal number of the knots of the spline function while comparative study is made to gain conviction on the correctness of the results.

Chapter 6 discusses the free vibration of cross-ply laminated composite circular cylindrical shell filled with fluid. The formulation follows the First Order Shear Deformation theory (FSDT). The effects of shell geometry (thickness-to-radius ratio and length-to-radius ratio), material properties, boundary conditions, ply orientations and layer of the materials on frequencies are studied. Two materials which are Kevler-49 epoxy (KGE) and AS4/3501-6 Graphite/epoxy (AGE) are used respectively. The shell is constrained with C-C and S-S boundary conditions. The problem is analysed for two, three and four layered shell.

The following Chapter 7 discusses the free vibration also but for anti-symmetric angle-ply laminated composite circular cylindrical shell filled with fluid. Four and six layered shells composed of two types of material; KGE and AGE materials are used under C-C and S-S boundary conditions. Parametric studies with respect to thickness to radius ratio, length to radius ratio, material properties, ply angles and number of layers are carried out to analyse the frequencies.

Next, the free vibration of symmetric angle-ply laminated composite circular cylindrical shell filled with fluid by considering FSDT is presented in Chapter 8. Three and five layered with combination of two materials namely KGE and AGE is analysed under C-C and S-S boundary conditions. Parametric studies are made in analysing the frequencies of the shell with respect to the shell geometry, material properties, boundary conditions, ply-orientation and layer of the materials.

Chapter 9 presents the conclusion of overall analysis of this thesis. The extended problems that could be studied in the future are also discussed in the end of this chapter.

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