MULTI-POPULATION MORTALITY MODEL IN A STATE-SPACE FRAMEWORK

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DEDICATION

To Allah; thank you for choosing me, and blessing me much more than I deserve
"And I have chosen you for Myself" (QS Thaha, verse 41)

This thesis is dedicated to You
and to those who is always there for me
(my supervisors, my family, my friends, and my Hanania)

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ABSTRACT

Recent developments in several countries have brought forth two critical issues which are unexpected increases in life expectancy and gap in life expectancy between populations. These two issues have led to the study of mortality rates modelling. It began with the single-population regression model which comprises of the single age and time effect parameters. The single factor is limited by the fact that, firstly, the parameter is unable to capture the variances across ages and therefore tend to produce inaccurate forecasts. Secondly, the indicated model does not incorporate the inter-relationship between populations, therefore fails to produce coherent forecast in the long run. Finally, there are two-steps of separate estimations involved in the modelling procedure which are estimation procedure for Poisson regression approach and estimation procedure for forecast model. To overcome these limitations, this study proposed an extended version of multi-population mortality model. This extended model takes into account the quadratic age-effect parameter which captures the relevant information across age-groups and will help to ensure more coherent way of forecast in the future. In addition, the proposed multi-population mortality model is reformulated into the state-space framework by combining the Poisson regression and forecasting procedures into a single estimate. The mortality model in a unified estimation is proposed to avoid unreliable forecasts. This study produces a cohort mortality life table based on the assumptions of future mortality improvements. The proposed methods were applied to the Malaysian mortality data for male and female. The proposed methods are then compared with the other existing methods. The results have shown that it is vital to incorporate accurate information across ages in order to obtain accurate mortality rates forecast. Other than that, this thesis has shown that the mortality model in a state-space framework could significantly improve the independent estimation method.

ABSTRAK

Perkembangan terkini di beberapa buah negara telah membawa kepada dua isu kritikal iaitu peningkatan jangka hayat yang tidak dijangka dan jurang jangka hayat antara populasi. Kedua-dua isu ini telah membawa kepada pemodelan kadar mortaliti. Ia bermula dengan model regresi tunggal-populasi yang terdiri daripada parameter tunggal umur dan masa. Faktor tunggal adalah masih terhad, pertama, parameter ini tidak dapat mengambil varians di seluruh peringkat umur, maka ia cenderung untuk menghasilkan ramalan yang tidak tepat. Kedua, model tersebut tidak merangkumi hubungan antara populasi, oleh itu gagal menghasilkan ramalan koheren dalam jangka masa panjang. Akhir sekali, terdapat dua langkah anggaran berasingan prosedur pemodelan iaitu anggaran untuk regresi Poisson dan model ramalan. Untuk mengatasi kekangan tersebut, kajian ini mencadangkan versi model diperluas bagi mortaliti pelbagai penduduk. Model yang diperluas ini merangkumi parameter kuadratik umur yang mengambil maklumat di seluruh kumpulan umur dan juga dapat menghasilkan ramalan yang koheren pada masa hadapan. Sebagai tambahan, model mortaliti pelbagai penduduk dirumus semula kepada rangka ruang-keadaan dengan menggabungkan regresi Poisson dan ramalan ke dalam satu anggaran. Model mortaliti dalam satu anggaran dicadangkan untuk mengelak ramalan yang tidak boleh dipercayai. Kajian ini telah dapat menghasilkan jadual hayat kohort mortaliti berdasarkan kepada andaian akan berlakunya pembaikan mortaliti masa depan. Kaedah yang dicadangkan telah digunakan untuk data mortaliti lelaki dan wanita di Malaysia. Kemudian, kaedah-kaedah yang dicadangkan ini dibandingkan dengan kaedah-kaedah lain yang sedia ada. Keputusan menunjukkan bahawa memasukkan maklumat yang tepat di seluruh peringkat umur adalah penting untuk mendapatkan ramalan kadar mortaliti yang tepat. Selain itu, tesis ini telah menunjukkan bahawa model mortaliti dalam rangka ruang-keadaan boleh memperbaiki kaedah anggaran berasingan dengan lebih baik.

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LIST OF ABBREVIATIONS

AE	–	Average Measurement Error
AIC	–	Akaike Information Criterion
APC	–	Age Period Cohort
CAE	–	Common Age Effect
CBD	–	Cairns Blake Dowd
CMI	–	Continuous Mortality Investigation
DFA	–	Dynamic Factor Analysis
DOSM	–	Department of Statistics Malaysia
EM	–	Expectation Maximization
ER	–	Explanation Ratio
LC	–	Lee-Carter
LIAM	–	Life Insurance Association of Malaysia
M9903	–	Mortality Table in the year 1999 to 2003
MAPE	–	Mean Absolute Percentage Error
MLE	–	Maximum Likelihood Estimation
MVN	–	Multivariate Normal Distribution
OH	–	O'Hare and Li
PCA	–	Principal Component Analysis
RM	–	Ringgit Malaysia
RMSE	–	Root Mean Squared Error
SAINT	–	Spread Adjusted International Trend
SOA		Society of Actuaries
SVD	–	Singular Value Decomposition

LIST OF SYMBOLS

β_x	–	Additional age-specific component
x	–	Age interval
$a_{x,t}^{C}$	–	Annuity price for cohort mortality table
$a_{x,t}^{P}$	–	Annuity price for period mortality table
D	–	Coefficient matrix of a state index
Δ^d	–	Drift parameter
ϵ_t	–	Error term of mortality index
$\varepsilon_{x,t}$	–	Error term with mean zero and variance σ_ε
Λ	–	Fixed variance of the initial state vector
ϕ	–	Full set of parameters of the model considered
g	–	Iteration step
D_ξ	–	Linear constraints of ξ
f_ξ	–	Linear constraints of ξ
W	–	Maximum value of the likelihood
$\bar{x}$	–	Mean average of the sample age range
u	–	Mean level at time t
ξ	–	Mean of initial state vector
K_t	–	Measurable variable for $(\{k_t, \dots, k_{t-1}\})$
M_t	–	Measurable variable for $(\{m_t, \dots, m_{t-1}\})$
$m_{x,t}$	–	Mortality at age x and year t
$m_{x,t,i}$	–	Mortality rates for age-group x in time period t at a specific population i
k	–	Normalizing constant
$x = 1, \dots, N$	–	Number of age-group
$D_{x,t}$	–	Number of death
$E_{x,t}$	–	Number of exposure
δ	–	Number of model's parameters.
$l = 1,2,3 \dots, L$	–	Number of multiple trends

ϑ	–	Number of observed rates
c	–	Number of period effect parameter
$i = 1, \dots, M$	–	Number of population
$b = 1, \dots, B$	–	Number of samples generated for simulation purposes
l_x	–	Number of survivors at the beginning of x age interval
α_x	–	Overall mortality rate across ages
$\emptyset_p$	–	Parameter of ARIMA model for coefficient p
θ_q	–	Parameter of ARIMA model for coefficient q
γ_{t-x}	–	Parameter to capture the trend in the year of birth.
$a_{x,t}$	–	Price of life annuity immediate
$q_{x,t}$	–	Probability of an individual with age x and time t will die
$((x - \bar{x})^2 + \hat{\sigma}_x^2)$	–	Quadratic effect parameter
U_t^c	–	Stochastic drift
${}_k\hat{p}_{x,t}$	–	Survival probability of insured at age x, year k time t.
Ξ_t	–	The column matrix with the value of 1.
v_t^k	–	The discount factor with interest rate r
ψ	–	The expected joint-log likelihood
T	–	The final year for observed mortality rates
t	–	The historical years for observed mortality
ω	–	The maximum attainable age of the survivor
h	–	The number of forecasted year ahead.
k_t^c	–	The period effect parameters
$(\bar{x} - x)^+$	–	The sample age range of the equation $\max(\bar{x} - x, 0)$
k_t	–	Time-varying mortality index
λ_i	–	Unique component of a three way model
ϕ_{g+1}	–	Update equation parameter for g iteration steps
Q	–	Variance matrix for hidden state
R	–	Variance matrix for observed equation

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CHAPTER 1

INTRODUCTION

1.1 Background of the Study

The significant increase in human lifespan indicates greater advances that a country has achieved in terms of medical care, lifestyle, education, socioeconomic status and geography. As a consequence, there seems to be no limit for the life expectancy to increase in the future. Although living longer seems good, the unexpected increase in longevity may bring many new challenges to the individual, government and pension providers as well. The uncertainty surrounding the life expectancy which is also known as longevity risk, has raised several concerns such as pension providers may incur losses due to prolonged payment of insured claims (Dowd *et al*., 2010), individuals may suffer from limited retirement income (Villegas, 2015), and government may have inadequate funds to provide support in terms of healthcare cost (Roy *et al*., 2012).

There are major problems which arises as a consequence of people living longer than expected. The overestimate occurs because of the inaccurate estimate of the life expectancy obtained by using deterministic mortality table which does not allow for the forecasts of uncertainty (O'Hare, 2012). In order to avoid predicting lower mortality rates than what has been actually experienced by the policyholders, many attempts have been proposed to forecast mortality. It began with a single-population mortality model proposed by Lee and Carter (1992). However, the single-population mortality model is limited because it does not incorporate the relationship between populations, leading to a high possibility of unreasonable cross over or diverging rates in the future. Hence, Li and Lee (2005) suggested the need of joint modelling in order to ensure a coherent way of forecasting in the future. Hyndman and Booth (2013) defined a coherent forecast as a forecast that retains a biologically reasonable shape such as the forecasts for male mortality rates remain higher than that for female without diverging. The idea of joint modelling is sparked by an observation related with the mortality trends between populations that became integrated by geography, technology, education, transportation, social and behavioural conditions. Therefore, it is believed that the mortality projections between closely linked populations are implausible to diverge in the future. Inaccurate estimates due to the diverging forecasts would understate the prediction of life expectancy, thus resulting into underestimation of premiums and reserves amount of annuity (Lin *et al*., 2015).

The building of multi-population mortality model is important to produce biologically reasonable forecast as compared to single population mortality model in terms of two aspects. First, the coherent forecasts with different time horizons are generated by capturing the degree of correlation between mortality improvements of populations using central tendency measurement. Second, multi-population mortality model allows for the "borrowing strength" concept where the sharing of information is enabled between small populations, or between small and large populations (Jong *et al*., 2016). Small populations are the populations with small number sizes, populations with limited amount of calendar years and age range data, and populations with poor data record (Chen *et al*., 2015). Small populations with limited amount of data often display substantial variability or volatile patterns in the past trends. As a consequence, the forecasting measurement which is sensitive to the historical pattern could become unreasonable. Hence, to ensure reasonable forecasts between small populations, multi-population mortality model is proposed.

Despite the importance of multi-population mortality model and some researches have been carried out on this subject, there is still a lot of development that need to be done in order to improve the statistical properties of the existing multi-population mortality model. There are two main weaknesses of the existing multi-population mortality model that the study aims to address which are, the previous multi-population mortality model ignores the variances for all age group which eventually would lead to inaccurate forecast reading. Hence, to overcome this issue, this study incorporates the quadratic age effect parameter to the multi-population mortality model's design. On the other hand, previous studies did not give much attention on reformulating the multi-population mortality model into a unified estimation. Most of the existing multi-population model involve separate estimation procedure which comprises of estimation procedure for linear function model and estimation procedure for time-varying index model. The main concern associated with separate estimation procedure is that the errors in the first linear regression model is ignored while being forecast, resulting into erroneous prediction interval (Pedroza, 2006). Thus, many authors have reformulated the single-population mortality model into a state-space framework so that there would be only one-step estimation method in the mortality modelling procedure.

Realising the significant contribution that the multi-population mortality has brought into, this thesis proposes another new extension of multi-population mortality model which incorporates the quadratic-age effect in an augmented common factor approach. Then, the proposed model is reformulated into unified estimation procedure via state-space framework. The model's performances are then validated by using goodness of fit test, standardised residual analysis, coherent forecast and bootstrap technique. Finally, the mortality table is produced and the results of the annuity application are compared.

1.2 Problem Statement

Longevity risk has become a pressing concern in many developed countries, and starting from 1992, single-population mortality model with the aim to forecast has been proposed. Single-population mortality model however because it does not incorporate similar information between closely related groups of the population such as gender, states, and countries. Such limitation approach tends to allow for future forecast divergence between populations (Li and Hardy, 2011). Since then, a myriad of multi-population mortality models has been introduced so far, but little of them are appropriate to deal with across full ages and time varying variabilities. An appropriate method to deal with full ages and time varying variabilities is vital to be considered in Malaysia. This is because, as opposed to other countries, Malaysia mortality pattern is much more volatile due to limited amount of calendar year and age range data. There is, therefore, a definite need for research to build multi-population mortality model that could adequately capture relevant information related to mortality rates in order to produce accurate forecast reading in the long run.

Another source of constraint in the study of mortality model (single population and multi-population mortality models) which could have affected the overall measurement of in-sample and out-sample accuracy is that the estimation procedure composed of two types of estimations processes which are the estimation in regression model and the estimation in time series model. This type of estimation is also called independent estimation. A key issue associated with independent estimation is unreliable forecast (Fung *et al*., 2017b). The issue of independent estimation has opened another new research direction in recasting the existing single-population mortality model into a state-space framework. State-space method is suggested since it could estimate the regression and the time series equation of mortality model in a unified way. So far, there is only one study that has reformulated the multi-population mortality model in a state-space framework simultaneously. However, previous study proposed by Liu *et al*. (2016b) has not dealt with full-ages correlation. Thus, to overcome this limitation, this study proposed a model that will incorporate full ages correlation into the model's structure along with a state-space representation.

1.3 Objectives

The objectives of the study are based on the three main research problems:

(a) To develop multi-population mortality models in a separate and unified estimation.

(b) To improve the forecast accuracy of multi-population mortality models.

(c) To validate the consistency of the developed model using simulation.

(d) To analyse the impact of systematic mortality improvements on the annuity pricing.

1.4 Scope of Study

Malaysia mortality data are obtained from the Department of Statistics Malaysia (DOSM). The dataset comprises of the number of deaths and the number of exposures for male and female of Malaysia since the beginning of 1980 until 2015. There are a total of 17 age groups for both male and female, with five-year age span range from 0 up to 80 years. The mortality data encompasses of three different dimensions which are the year, age, and population dimensions. Hence, there are a total of 31 time periods, 17 age groups and 2 populations that will be accounted into the modelling framework of this study.

The projected mortality rates in this study are used to produce cohort mortality table. The produced cohort mortality table is then compared with the period mortality tables which are Abridged Malaysia life table and M9903 mortality table to calculate annuity price. Abridged Malaysia life table was obtained from the Department of Statistics Malaysia, which constitutes of five-year age gap from 0 to 80 years old. M9903 mortality table was produced from Life Insurance Association Malaysia (LIAM), and it was published in 2007 constituted of mortality experiences of insured lives data from the year 1999 until 2003. M9903 can be obtained from the Society of Actuaries (SOA, 2007). These two period life tables are presented in Appendix F.

1.5 Significance of Study

Like many other country, Malaysia is no exception of being recognized as one of the developed countries that face the rise in life expectancy. Due to the rapid increase in life expectancy, Malaysia is said to reach ageing population more than triple from the current 2.0 million to 6.0 million in 2040. Rapid increases of ageing population will give a huge impact to the whole social structure. Since the rising of life expectancy is unlikely to reduce in the long run, the study about the modelling of mortality rate is vital in order to minimize the impact of longevity risk in the future such as inadequate retirement income payment and reduced health care. Therefore, there is a vital need to solve these issues properly by mitigating the longevity risk.

The study of mortality is important to avoid underestimation of future life expectancy. Currently, there is no official projected or cohort mortality table provided in Malaysia. For public use, the Department of Statistics Malaysia (DOSM) has released a period mortality table which is called as abridged life table, generated from Coale-Demeny model. The Coale-Demeny model is deterministic in nature and is not suitable to be used for the current mortality experiences in most of the developed countries (Murray *et al*, 2000). In addition, the single parametric form of the Coale-Demeny model is inflexible for explaining complex mortality behaviours, and is outdated (Murray *et al.*, 2010). Other than DOSM, Life Insurance Association of Malaysia (LIAM) published period mortality table called M9903 in 2007. M9903 mortality table is based on the lives assured mortality data from the period 1999 to 2003 (SOA, 2007). However, M9903 is limited by the fact that the mortality table is deterministic in nature and the mortality table is outdated for a decade. Other than that, M9903 is not reflecting the whole Malaysian mortality since the standard mortality table is only based on individuals who are able to purchase the insurance (McCarthy *et al.*, 2010). Hence, being aware of the limitations of these two existing period mortality tables, this thesis aims to generate cohort mortality table and then to compare the impact of longevity risk on annuity pricing based on the period mortality table and cohort mortality table. Medved *et al*. (2011) reported that even though a standard mortality table (abridged life table and M9903 mortality table) has been provided in the country, the generation of another mortality table could still be used as long as it could generated higher provisions as compared to the standard mortality table.

The incorporation of joint movement between mortality rates is vital in order to obtain accurate forecast mortality rates reading and projected mortality table. Hahn (2014) stated that, multi-population mortality model that includes the interdependency between populations could improve the statistical properties of the model's forecasts since the dependency parameter is useful in terms of sharing information and borrowing strength from the other populations. Sharing information and borrowing strength are some of the important factors for countries with limited amount of data to improve the mortality rates forecasts. In Malaysia, the study of multi-population mortality model is still scarce and crucial to be employed since it has limited amount of data. Thus, motivated by the advantage of multi-population mortality model, this study seeks to propose a multi-population mortality model that could assimilate the experiences of mortality trend between Malaysia populations.

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