STRESS ANALYSIS OF TORISPHERICAL SHELL WITH RADIAL NOZZLE

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ABSTRACT

The high stresses at intersections are caused by discontinuity shear stresses and moments which exist to maintain compatibility at the junction. The finite element method was used to determine the stress field at the intersection of a radial nozzle attached to a torispherical crown of a cylindrical vessel. The mechanical loads acting on the structure consisted of nozzle thrust, bending moment, torsion and internal pressure. A comparison between predicted and measured readings gave acceptable results for internal pressure loading but fair for other loadings. A computer program was written to calculate interaction between any two combining loads. The results, presented graphically, could be used to predict first yield for vessels of this configuration.

Keywords : *Interaction Diagram, Pressure Vessel, Torispherical Shell*

NOTATION

D	diameter of cylindrical shell
R	radius of cylindrical shell
d	diameter of nozzle
T	thickness of vessel
t	thickness of nozzle
rk	knuckle radius
L	crown radius
H	head height
P	internal pressure
M	bending moment on nozzle
T_o	torsion on nozzle
F	axial thrust on nozzle
Z	section modulus of nozzle

Subscript

o, i, m	outer, inner, mean
vM	von Mises

1. INTRODUCTION

When pressure vessels have to be connected to a piping system, the attachment of nozzles to the crown becomes inevitable. There have been numerous detailed analyses of torispherical shells with radial nozzles, being subjected to various loadings. The nozzle has been singled out as a potential source of weakness in the sense that high stresses occur here.

The design of pressure vessels requires a careful study of many regions, the most critical of which is the knuckle region. Torispherical heads with various knuckle radii (for each head height) have been investigated by researchers to establish an ‘optimum' torispherical head. For torispherical heads of constant head height to cylinder radius ratio, Batchelor and Taylor [1] found a peak in limit pressure when plotted against knuckle radius, indicating ‘optimum' values of head height to cylinder radius ratio, H/R in the range of 0.33 to 0.5, although design rules do not permit torispherical heads with H/R<0.5. For a given value of H/R, there is an infinite number of torispherical heads having different dome and knuckle radii. Torispherical heads may achieve the optimum reduced head but they cannot act in biaxial tension in the knuckle; the knuckle must carry circumferential compression and also resist bending.

A thin-walled sharply curved knuckle region is much weaker than the spherical portion of the dome or the cylindrical vessel. On the other hand if the knuckle is less sharply curved and the wall is relatively thick compared to the knuckle radius, the knuckle region is too stiff and strong and acts as a stiffening ring between the cylindrical vessel and the spherical cap. The ASME code gives very little variation of the design pressure with D_m/T.

The radial nozzles which are welded to the vessels are designed by considering pressure loading but not the external loadings imposed by the piping system to the nozzle. The external loads on the nozzles are not generally included because the configuration of the attached piping system is not established yet, but stresses due to the external loads on the nozzle can be more critical than those due to internal pressure. The piping system which is connected to the nozzle is often redesigned several times at later stages to reduce the reactions at the nozzle, so that the resulting stresses at the nozzle are within acceptable limits.

The objective of this study is to establish the elastic stress behaviour of a torispherical vessel with nozzle. The structure, as shown in Figure 1, consists of a torispherical shell of revolution attached to a radial cylindrical nozzle at the

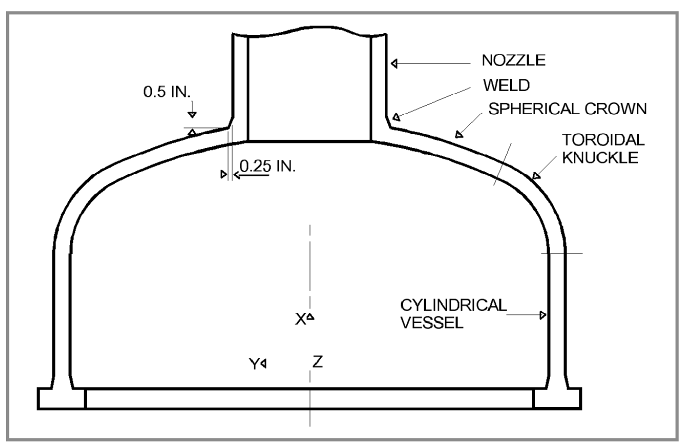

Figure 1: Torispherical vessel with nozzle

spherical crown and to a cylindrical vessel at the torus end. The nozzle is flushed with the inside surface of the vessel. Apart from internal pressure, the external loadings that were considered are axial thrust, F, bending moment, M, and torsional moment, T_o, acting at the free end of the nozzle. This study also introduces a method whereby the pressure vessel is assessed for its proximity to first yield when subjected to a combination of internal pressure and external loads on the nozzle. There are three interacting geometric locations which could influence the stress field, and the maximum stress could occur at any of these locations – they are sphere-nozzle, sphere-knuckle and cylinder-knuckle junctions.

2. LITERATURE REVIEW

Cylinders with torispherical ends have been the subject of numerous investigations over the last 80 years and satisfactory experimental and numerical correlation have been achieved. However the subject of nozzles in pressure vessels received considerable attention only in the late 1950s. The first theoretical work on sphere-cylinder intersections was initiated by Galletly [2] who utilised the theory of two intersecting thin shells. In 1959, Galletly [3] cautioned that torispherical shells designed according to the ASME code could lead to failure during testing because the use of membrane shell theory would give poor results when applied to the toroidal region of the shell.

Most of the studies were concerned with determining limit pressures, such as studies by Onat and Prager [4] in 1954, Drucker and Shield [5] in 1959, Rose and Thompson [6] in 1961, Lind [7] in 1964, Leckie and Penny [8] in 1963, Gill [9] in 1964 and Rodabaugh and Cloud [10] in 1968. Early elastic and shakedown analyses of spherical vessel with radial nozzle structures were presented by Penny and Leckie [11] and Leckie [12] respectively. Various investigators have used shell computer programs to determine the collapse behaviour of pressure vessels. The early programs used small deflection shell theory but were superseded by programs which used large deflection theories, such as the BOSOR5 [14]. A concise summary of past work on pressure vessels is given by Bushnell [13].

3. EXPERIMENTAL INVESTIGATION

The experimental result used here is part of a test programme carried out by Drabble [15] to determine the shakedown behaviour of a torispherical vessel with nozzle, under the action of internal pressure, thrust and bending moment applied to the nozzle. The vessel was made of mild steel and had a uniform thickness (see Figure 1). The weld formed a fillet, 0.5 inch high and 0.25 inch wide, at the junction between the nozzle and the main vessel. At the outer end of the cylindrical vessel, a flange was used for clamping onto the base plate of a test rig.

The model was instrumented with 39 pairs (hoop and meridional) of 0.0625-inch foil strain gauges bonded to the outer and inner surfaces of the shell. These gauges were located between S= -0.1 and S=0.5 in the meridional direction. The non-dimensional meridional distance S is defined as,

$$S = \frac{\text{actual distance (measured along shell surface) from crotch corner or weld root}}{\text{inner radius of spherical crown}}$$

The model was subjected to various combinations of internal pressure, compressive axial force and bending moment. The parameters of the vessel were as follows:-

Mean diameter of cylindrical shell, D_m	= 16.0 in.
Mean knuckle radius, r_{km}	= 2.95 in.
Mean crown radius, L_m	= 16.0 in.
Mean diameter of nozzle, d_m	= 4.5 in.
Thickness of vessel, T	= 0.5 in.
Thickness of nozzle, t	= 0.5 in.
Head height, H	= 3.967 in.
Length of cylindrical vessel	= 4.0 in.

which gives parametric ratios:-

d_m/D_m	= 0.281
D_m/T	= 32.0
dm/t	= 9.0
2L/T	= 64.0
H/D_m	= 0.248
$\lambda = D_m/2H$	= 2.017
$\rho=(r_m/L_m)\% \sqrt{(L_m/T)}$	= 0.795

4. THE FINITE ELEMENT APPROACH

A. Finite element modelling

The experimental vessel shown in Figure 1 was adopted for finite element analysis. At the base plane of the cylindrical shell, i.e. in the y-z plane, the degrees of freedom of the nodes were constrained. A computer aided engineering software package PATRAN 2.5 [16] was used for pre- and post-processing.

Since the bending moment load was non-axisymmetric, the FE modelling was done for a full 3D structure using 20-node isoparametric hexahedral elements. This is to enable a complete load interaction analysis for all regions around the intersection. At the nozzle-crown discontinuity, mesh refinement was carried out because the loadings could create high stress gradients in the meridional direction. A preliminary convergence test was carried out on the model subjected to internal pressure. The results of the mesh size convergence test led to the selection of the final FE idealisation, in the meridional direction: 8 elements in the knuckle, 8 in the crown, 2 in the weld region, 6 in the main cylinder and 7 in the nozzle. A longitudinal section of the discretised assembly is shown in Figure 2. The full model utilised 1488 elements with a total number of 10656 nodes. The

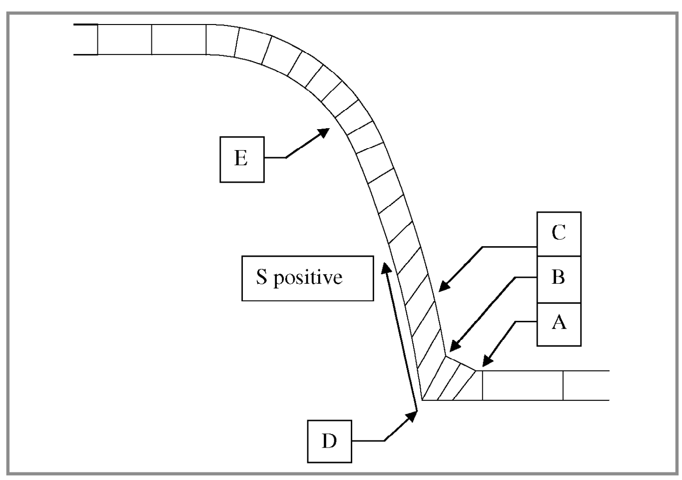

Figure 2: Finite element mesh in the meridional plane of torispherical head with nozzle

vessel material was mild steel with an elastic modulus of 30x10[6] psi and a Poisson's ratio of 0.3. The internal pressure load was applied on the cylindrical vessel, crown and nozzle. The loads of bending moment, torsional moment and thrust force were modelled to act on the free end of the nozzle. The ASAS [17] linear finite element program was used for FE analysis. The FEM output gave normal and shear stresses in the x-, y- and z-axes. These stresses were subsequently utilised in a computer program that was developed to generate a series of first yield load interaction diagrams.

5. RESULTS AND DISCUSSION

A. Elastic stress factor

The stresses used here have been normalised according to each loading condition, using the normalising factors listed below. The nominal stresses are:-

$\sigma_n = P2L_m/4T$ for internal pressure P

$\sigma_n = 32Md_o/\pi(d_o^4-d_i^4)$ for nozzle bending moment M

$\sigma_n = 4F/\pi(d_o^2 - d_i^2)$ for nozzle compressive force F

$\sigma_n = 32T_od_o/\pi(d_o^4-d_i^4)$ for nozzle torsion T_o

Hence, the stress factor, ESF, is defined as ESF= σ/σ_n where σ can be any hoop, meridional, or the von Mises effective stress. Studies made by others on the effect on maximum stress of the nozzle-crown parameters found that in all load cases, the maximum stress occurs at the junction. In the present FEM, the critical stress at the intersection occurred in the hoop direction at the root of the outside weld fillet.

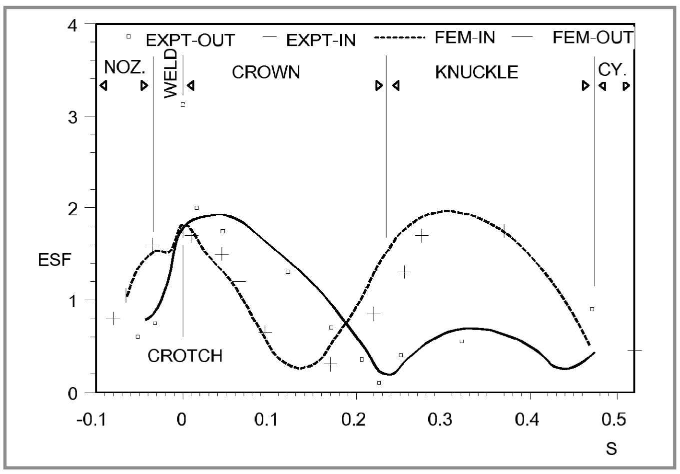

Figure 3(a): Von Mises stress distribution on longitudinal plane, due to pressure - A comparison between experimental and FEM results

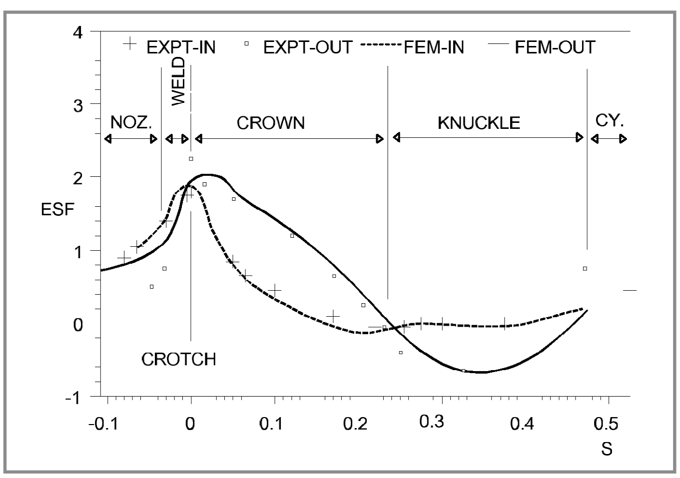

Figure 3(b): Hoop stress distribution on longitudinal plane, due to pressure - A comparison between experimental and FEM results

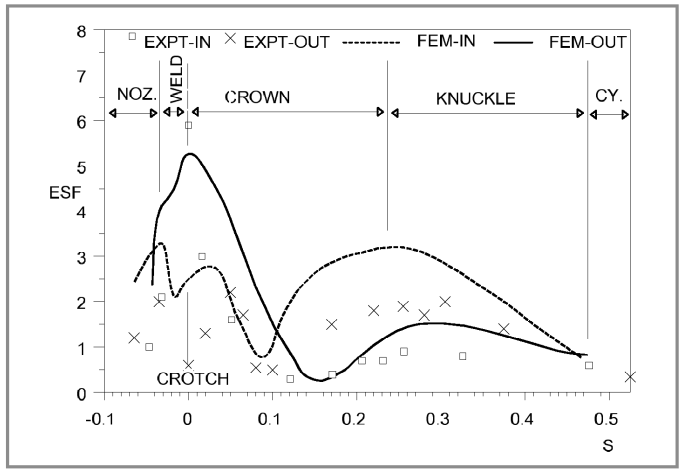

Figure 4(a): Von Mises stress distribution on longitudinal plane due to thrust on nozzle - A comparison between experimental and FEM results

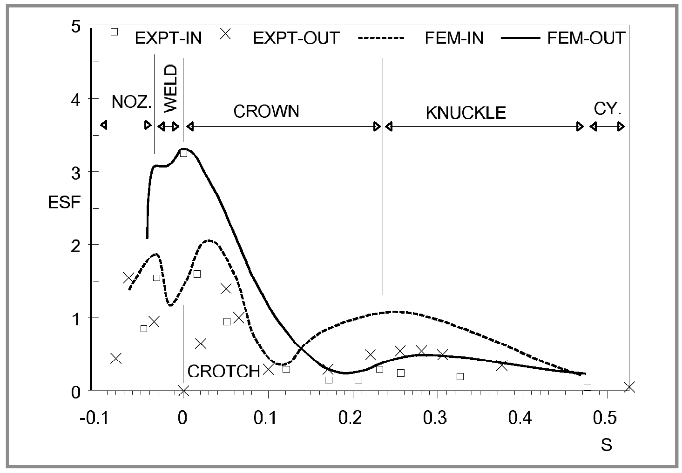

Figure 4(b): Von Mises stress distribution on longitudinal plane due to moment on nozzle - A comparison between experimental and FEM results

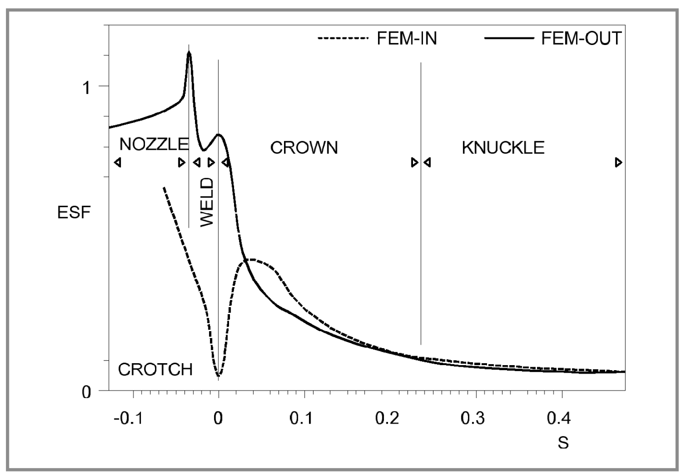

Figure 4(c): FEM von Mises stress distribution on longitudinal plane due to torsion on nozzle

Figures 3 and 4 show the ESF distribution along the meridional plane due to the four load cases. From Figures 3(a) and 3(b) pressure loading gives an overall good correlation with experiment. The crotch corner and the weld-crown regions are the highest stressed areas with $ESF \simeq 2$. The critical hoop stress distribution is often the subject of discussion. The points of interest of Figure 3(b) is the high hoop stress at the crown-weld edge and the compressive hoop stress at the knuckle outer surface which is likely to cause unsymmetric inward buckling. Galletly reported that for high D/T ratios, internal pressure can cause considerable compressive hoop stress on the inner surface of the torus region and that large diameter ratios (d/2L) can cause high meridional stress. A point worth noting is Drabble's pressure stress factor at the crotch corner. His value of 1.73 compares well with the FEM value of 1.82. Leckie et al [18] presented stress concentration factor (based on maximum shear stress) curves in spherical pressure vessels with cylindrical nozzles. Using ρ=0.795 and t=T, the maximum ESF due to pressure, is about 2.8.

Figures 4(a)-(c) present the ESF on the inner and outer surfaces of the shell for the remaining load cases. The agreement with experiment for bending moment and thrust loads is good at the crown near the intersection but fair in other regions. From thrust and moment loads, the ESFs at the weld-crown junctions are much higher than other areas but due to torque the maximum ESF occurs at the weld-nozzle junction.

Table 1: Predicted and experimental maximum von Mises effective stress factor

Load	FEM	BS 5500 [19]	Experimental [15]	Location from FEM
Torque, T_o	1.11	-	-	nozzle-weld edge
Moment, M	3.31	7.22	3.21	crown-weld edge
Pressure, P	1.95	2.2	3.13	inner surface of toroidal knuckle
Thrust, F	5.27	9.62	5.87	crown-weld edge

Table 1 shows the comparison between predicted and experimental results of maximum von Mises ESF. The table indicates that the FEM agrees reasonably with experimental tests. In Drabble's [15] experiment, the locations of first yield for the three load cases occur at the crown-weld edge.

B. Effects of weld fillet

Experimental studies by other authors, [10][20] and [21], have indicated that the presence of even a nominal amount of fillet at the nozzle-sphere junction significantly influences stress results. The agreement between thin shell theory and experimental results has always been clouded by large localised effect of fillet at the junction between the shell and the nozzle. In elastic thin shell analysis the high stress at the nozzle base has been overestimated because the presence of weld fillet in the actual vessel would actually lower the stress. This is not the case in finite element idealisations where the fillet can accurately be represented by 3D elements. To model the effects of the weld, the conventional shell theory has been modified by O'Connell and Chubb [22], where the normal forces to the shell surface at the junction were replaced by equivalent bands of pressure.

C. Effect of geometric parameters

The stress distribution at the local shell-nozzle intersection is characterised by three non-dimensional parameters, i.e. t/T, L/T and (r/L)%$\sqrt{(L/T)}$. From the parametric studies by other authors, these parameters influence the stress factors and are important in the design process of pressure vessels. One such variation of stress factor from thrust loading on the nozzle of a torispherical head was described by Chao [16] who, using thin shell theory, varied t/T while keeping the ratios L/T and $\rho=(r_m/L_m)\%\sqrt{(L_m/T)}$ constant. Chao found that the maximum ESF first decreases with increasing t/T, but then increases with further increase of t/T ratio. This means that thicker nozzles do not always lead to low stresses. Similarly for nozzle bending moment load, there are optimum values of t/T that results in minimum stresses. According to Chao [16], the reason for the optimum t/T is the changeover of maximum stress from hoop to meridional as t/T is increased. From Chao's [16] minimum stress curve of thrust and bending moment loading, the present sphere-cylinder configuration gives minimum stress when t/T is between 0.55 and 0.65. On the other hand, there is no optimum value of t/T for the case of internal pressure loading; a higher t/T ratio always results in lower stresses.

Many researchers generalise the effects at the intersection of pressure vessels by the sphere and nozzle parameters. According to Mershon [20], there is some evidence that in the reinforced condition, with 2L/T increasing, the hoop stress on the outer surface increases. This is supported by Lind [23], who discovered that the failure mechanism is restricted to certain ranges of the shape parameters. For a very thin nozzle in a vessel with very thick walls yielding occurs entirely in the nozzle. For low values of the diameter ratio (d/2L < 0.1) and for thick-walled nozzles, yielding occurs in the main vessel. In general stresses increase with increasing diameter ratio up to a d/D.0.8 after which the stresses are reduced or levelled off.

D. Load interaction

In elastic analysis, the usual procedure of determining stress is to add the simple membrane action to the effect of edge force and edge moment. In load interaction analysis separate effects from internal pressure, bending, torsional moment and thrust on the nozzle were combined to determine the first yield behaviour at the vessel-nozzle intersection when two loads interact. From the 4 load cases, 6 load interactions could be plotted. Figures 5(a)-(f) show the first yield load interaction curves at the vessel-nozzle junction. Two load pairs which exhibit the most critical linear interaction are $\overline{M}:\overline{P}$ and $\overline{M}:\overline{F}$, while three load pairs, $\overline{T}_o:\overline{M}$, $\overline{T}_o:\overline{P}$ and $\overline{T}_o:\overline{F}$, can simply and conservatively be represented by a circular relation. The symmetry of these five pairs, shown only in first quadrant in Figures. 5(a)-(e), means that the yield behaviour is not affected by the direction of the interacting loads.

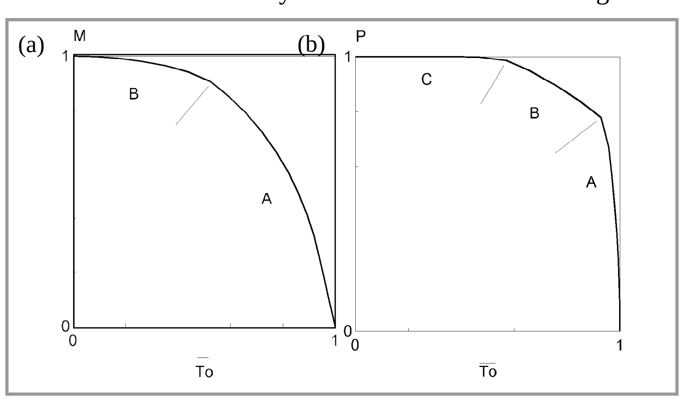

Figure 5(a) Torque: Moment interaction (b) Torque: Pressure interaction

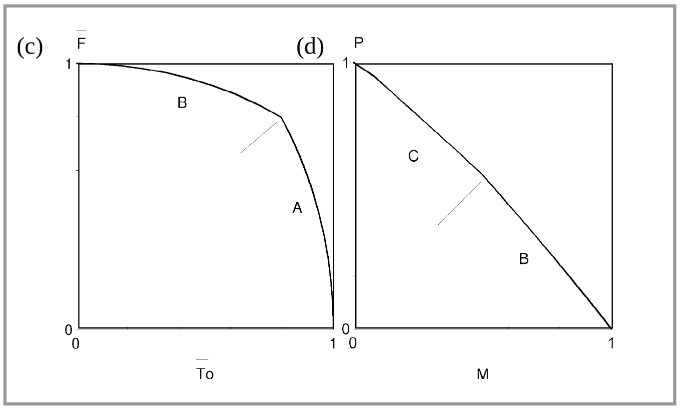

Figure 5(c) Torque:Axial Force Interaction   (d) Moment:Pressure Interaction

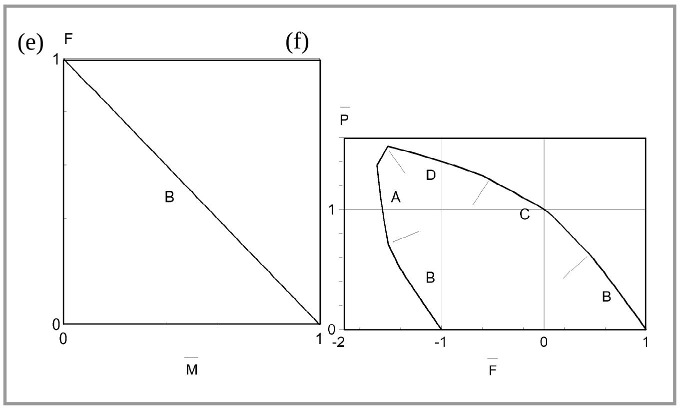

Figure 5(e) Moment:Axial force interaction (f) Axial Force:Pressure interaction

Load interaction diagrams for torispherical shell with nozzle. (Locations A-E are shown in Figure 2)

The $\overline{F}:\overline{P}$ interaction in Figure 5(f) can be simply represented by a linear relation for the first and third quadrant and a square relation in the second and fourth quadrant. However if a general relation is to be assigned to the $\overline{F}:\overline{P}$ pair, a linear relation fits well. In general, a compressive force on the nozzle plus internal pressure is a critical combination while a tensile nozzle thrust plus internal pressure results in low stress field. For this particular vessel configuration, a combination of a nozzle tensile force of $\overline{F}=1.5$ and an internal pressure of $\overline{P}=1.5$ can increase both yield loads by about 50%.

In general, the location of maximum stress from combined loadings can be predicted from the maximum location due to individual loads. From the load interaction curves, sudden changes in slope can be attributed to yielding moving from one location to another as the load combination is varied. The $\overline{F}:\overline{P}$ interaction is a case where the maximum stress can occur at a location which is not critical in either a thrust or pressure loading, for example, a tensile nozzle force and pressure combination causes maximum stress to occur at the crotch corner (see Figure 5(f)).

Drabble [15] constructed a three-load initial yield locus from the available strain gauge readings. The general trend of Drabble's load interaction agrees well with the present result. In one of the experimental tests, Witt et al [24] conducted a superposed loading on a spherical shell with protruded nozzle. The model which had geometric ratios d/2L=0.078, d/t=9.5 and 2L/T=81.33 was loaded simultaneously with axial thrust and internal pressure. From inspection of the larger stresses, they remarked that the method of stress superposition was accurate, thus giving added confidence of using the method of superposition in generating the interaction relationship in the present analysis.

E. BS 5500 code [19]

The conventional method of analysis and design of pressure vessels by the use of standard design codes, such as the ASME and BS, are based on the classical membrane theory and hence involves many inconsistencies and inaccuracies at the geometric junctions. In BS 5500 [19], the design of the opening is carried out by applying a stress concentration factor (based on the maximum principal stress) to the nominal stress (sphere pressure membrane stress) in the spherical shell rather than in the nozzle. The nominal stresses for other load categories are found in Sect. G.2.5 of BS 5500 [19]. The SCF can be read off once the values of ρ and t/T are known. For comparison the SCF values in the FEM and experimental results in Table 1 are normalized with the same factor as above.

6. CONCLUSIONS

The following main conclusions may be made:

1. For all load categories, high stresses occurred at the vessel-nozzle junction where there is a severe geometric discontinuity.
2. Nozzle thrust load gives the highest stress while torsional moment gives the lowest stress.
3. When torque is one of the combining loads, a circular interaction is proposed. For other load combinations a linear relation is proposed. ■

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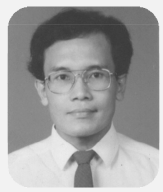

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