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RESEARCH ARTICLE

Separation time analysis of transient magnetohydrodynamic mixed convection flow of nanofluid at lower stagnation point past a sphere

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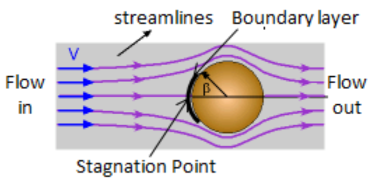
Graphical abstract



Abstract

In this paper, the unsteady magnetohydrodynamics (MHD) mixed convection flow of nanofluid at lower stagnation point past a sphere is studied. Nanoparticles Cu and TiO_2 with water as a base fluid are considered. The separation times of the flow as the boundary layer start to separate at the surface of the sphere are given attention. The governing boundary layer equations in the form of partial differential equations are transformed into nonlinear coupled ordinary differential equations and solved numerically using an implicit finite-difference scheme known as Keller-box method. Results of the separation times of boundary layer flow for viscous and nanofluid influenced by magnetic parameter and volume fraction are shown in tabular form and analysed. This study concluded that the separation times can be delayed by added more magnetic particles and small amount the volume fraction.

Keywords: Unsteady flow, mixed convection, nanofluid, magnetohydrodynamic

INTRODUCTION

Nanofluid can be described as fluid which has nanometer-sized particles suspended in a based fluid. The conventional fluid such as water, oil and ethylene glycol have poor heat transfer characteristic because of the low thermal conductivity of the fluid. Since thermal conductivity play an important role in heat transfer, many research have been done to increase the thermal conductivity for this fluid. One of the method is suspended a nanoparticles in these fluid which is conducted as an agent to enhanced thermal conductivity [1]. Term of nanofluid that refers to the suspended nanoparticles was introduced by Choi [2]. Choi et *al.* [3] showed that the small amount nanoparticles that is added in a base fluid will increased the thermal conductivity of the fluid itself. Studies of using nanofluid to enhanced heat transfer was started in the book by Das et *al.* [4], review paper by Maiga et *al.* [5], Buongiorno [6], Daungthongsuk and Wongwises [7], Trisaksri and Wongwises [8], and Kakac and Pramuanjaroenkij [9].

Mixed convection flow from a solid sphere are important in heat transfer problem because of the tremendous application such as cooling problems in turbine blades, electronic system and manufacturing processes [10]. Yuge [11] and Klaycko [12] started on mixed convection study for viscous incompressible fluid by using experimental work. Then, Hieber and Gebhart [13] solve by using analytical method with certain consideration. After that, Chen and Mucoglu [14, 15] extended using boundary layer approximation with very large Reynolds and Grashof number. Dennis and Walker [16] studied the steady forced convection flow past a sphere at low and moderate Reynolds numbers. El-Shaarawi et *al.* [17] studied mixed convection about a sphere between moderates and high Reynolds numbers with wide range of viscosity ratio. Nazar et *al.* [18] investigates the problem for whole sphere by considering constant surface temperature from lower stagnation point up to separation point. Then, Tham et *al.* [19] extended this problem by taking nanofluids with constant wall temperature as boundary condition. They observed that the volume fraction affected the skin friction as well as heat-transfer coefficient.

Further, Kasim et *al.* [20] have studied the steady problem on MHD effect on convective boundary layer flow of a viscoelastic fluid embedded in porous medium with Newtonian heating. This studied extend the problem that proposed by Anisah et *al.* [21] by considering MHD effect. Furthermore, Mohammad et *al.* [22] studied the effect of MHD on unsteady boundary layer flow past a sphere in viscous fluid. Then, Mohammad et *al.* [23] extends their problem by considering mixed convection flow and the separation times of the flow has been given a full attention.

Thus, following Mohammad et *al.* [23], separation times for unsteady mixed convection boundary layer flow past a sphere focuses on nanofluid with MHD effect is studied in this paper. Nanoparticles Cu and TiO_2 diluted into water-based fluid are considered and Tiwari and Das [24] model of nanofluid is referred. In this study, only separation times of boundary layer flow at lower stagnation point is investigated.

The present problem will focus on MHD effect on unsteady mixed convection boundary layer flow but focussing on nanofluid with constant wall temperature. The problem of flow separation also will be considered in this present study.

MATHEMATICAL MODEL

In this study, we consider unsteady two-dimensional mixed convection boundary layer flow of a nanofluid past a sphere of a radius *a*, which is placed in an incoming stream of nanofluid with a constant free-stream velocity of U_∞ and constant temperature T_∞ . It is assume that the force convection is moving upward, while the gravity sector is acts downward. It also assume that the sphere and based fluid (water) is maintained at constant temperature, T_w with $T_w > T_\infty$ for heated sphere (assisting flow) and $T_w < T_\infty$ for cooled sphere (opposing flow) and ensuring no slip velocity between them. We considered the nanoparticles shape to be spherical and the volume fraction is taken into account.

The basic equations of the problem consist of continuity, momentum, and energy equation in Cartesian coordinates $\bar{x}$ and $\bar{y}$ are [24]

$$\frac{\partial(\bar{r}\,\bar{u})}{\partial\bar{x}} + \frac{\partial(\bar{r}\,\bar{v})}{\partial\bar{y}} = 0 \tag{1}$$

$$\begin{aligned}\left(\frac{\partial\bar{u}}{\partial\bar{t}} + \bar{u}\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right) &= -\frac{1}{\rho_{nf}}\frac{\partial\bar{p}}{\partial\bar{x}} + \frac{\mu_{nf}}{\rho_{nf}}\left[\frac{\partial^2\bar{u}}{\partial\bar{x}^2} + \frac{\partial^2\bar{u}}{\partial\bar{y}^2}\right] \\ &+ \frac{\varphi\rho_s\beta_s + (1-\varphi)\rho_f\beta_f}{\rho_{nf}} g\left(\bar{T} - T_\infty\right)\sin\left(\frac{\bar{x}}{a}\right) - \frac{\sigma B\bar{u}}{\rho_{nf}}\end{aligned} \tag{2}$$

$$\begin{aligned}\left(\frac{\partial\bar{v}}{\partial\bar{t}} + \bar{u}\frac{\partial\bar{v}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{v}}{\partial\bar{y}}\right) &= -\frac{1}{\rho_{nf}}\frac{\partial\bar{p}}{\partial\bar{y}} + \frac{\mu_{nf}}{\rho_{nf}}\left[\frac{\partial^2\bar{v}}{\partial\bar{x}^2} + \frac{\partial^2\bar{v}}{\partial\bar{y}^2}\right] \\ &- \frac{\varphi\rho_s\beta_s + (1-\varphi)\rho_f\beta_f}{\rho_{nf}} g\left(\bar{T} - T_\infty\right)\cos\left(\frac{\bar{x}}{a}\right) - \frac{\sigma B_0^2\bar{v}}{\rho_{nf}}\end{aligned} \tag{3}$$

$$\frac{\partial\bar{T}}{\partial\bar{t}} + \bar{u}\frac{\partial\bar{T}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{T}}{\partial\bar{y}} = \alpha_{nf}\left[\frac{\partial^2\bar{T}}{\partial\bar{x}^2} + \frac{\partial^2\bar{T}}{\partial\bar{y}^2}\right] \tag{4}$$

subject to boundary conditions

$$\begin{aligned}\bar{t} < 0: &\quad \bar{u} = \bar{v} = 0,\ \ \bar{T} = T_\infty \ \text{ for any } \bar{x}, \bar{y}, \\ \bar{t} \ge 0: &\quad \bar{u} = \bar{v} = 0,\ \ \bar{T} = T_w \ \text{ at } \bar{y} = 0 \\ &\quad \bar{u} = \bar{u}_e(\bar{x}),\ \ \bar{T} = T_\infty \ \text{ as } \bar{y} \to \infty\end{aligned} \tag{5}$$

Where in this study $\bar{u}_e = 3/2U_\infty \sin(\bar{x}/a)$ and $\bar{r}(\bar{x}) = a\sin(\bar{x}/a)$. Here $\bar{u}$ and $\bar{v}$ are the velocity components along the $\bar{x}$ and $\bar{y}$ axes, respectively, $\bar{r}(\bar{x})$ is the radial distance from the symmetrical axis to the surface of the sphere, $\bar{u}_e(\bar{x})$ is the local free-stream velocity, T is the fluid temperature, $\bar{p}$ is the fluid pressure, φ is the nanoparticles volume fraction, β_f is the thermal expansion coefficient of the fluid fraction, β_s is the thermal expansion coefficient of solid fraction, α_{nf} is the thermal diffusivity of the nanofluid, ρ_{nf} is the density of the nanofluid, and μ_{nf} is the viscosity of the nanofluid, which are given by Oztop and Abu Nada [25]

$$\begin{aligned}&\alpha_{nf} = \frac{k_{nf}}{\left(\rho c_p\right)_{nf}}, \quad \rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s, \quad \mu_{nf} = \frac{\mu_{nf}}{(1-\varphi)^{2.5}} \\ &\left(\rho c_p\right)_{nf} = (1-\varphi)\left(\rho c_p\right)_f + \varphi\left(\rho c_p\right)_s, \\ &\frac{k_{nf}}{k_f} = \frac{\left(k_s + 2k_f\right) - 2\varphi\left(k_f - k_s\right)}{\left(k_s + 2k_f\right) + \varphi\left(k_f - k_s\right)}\end{aligned} \tag{6}$$

where k_{nf} is the effective thermal conductivity of the nanofluid, k_f is the thermal conductivity of the fluid, k_s is the thermal conductivity of solid, $\rho(c_p)_{nf}$ is the heat capacity of the nanofluid, ρ_f is the density of fluid fraction, ρ_s is the density of the solid fraction and μ_f is the viscosity of the fluid fraction.

To dimensionless the base equations, the following variables are introduced,

$$\begin{aligned}&x = \bar{x}/a, \quad y = \mathrm{Re}^{1/2}(\bar{y}/a), \quad r(x) = \bar{r}(\bar{x})/a, \quad u = \bar{u}/U_\infty, \\ &v = \mathrm{Re}^{1/2}(\bar{v}/U_\infty), \quad T = (T - T_\infty)/(\bar{T} - T_\infty), \quad t = \frac{U_\infty\bar{t}}{a} \\ &u_e(x) = \bar{u}_e(\bar{x})/U_\infty, \quad p = (\bar{p} - p_\infty)/(\rho_{nf}U_\infty^2).\end{aligned} \tag{7}$$

where $\mathrm{Re} = U_\infty a/\nu_f$ is Reynold number and ν_f is the kinematic viscosity of the fluid. Substituting these variables into equations (1)-(4) and applied the boundary layer approximation, those equations become

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) \tag{8}$$

$$\begin{aligned}&\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}\nu_f}\frac{\partial^2 u}{\partial y^2} + \\ &\lambda\alpha T\sin x - \gamma M u\end{aligned} \tag{9}$$

$$-\frac{\partial p}{\partial y} = 0 \tag{10}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\mathrm{Pr}}\frac{\alpha_{nf}}{\alpha_f}\frac{\partial^2 T}{\partial y^2} \tag{11}$$

where Pr is the Prandtl number, α is mixed convection parameter, M is magnetic parameter, λ and γ which are defined as

$$\begin{aligned}&\mathrm{Pr} = \frac{\nu_f\left(\rho C_p\right)_f}{k_f}, \quad \alpha = \frac{Gr}{\mathrm{Re}^2}, \quad M = \frac{\sigma B_0^2 a}{\rho_f U_\infty}, \\ &\lambda = \frac{\varphi\rho_s\left(\beta_s/\beta_f\right) + (1-\varphi)\rho_f}{\rho_{nf}}, \quad \gamma = \frac{1}{(1-\varphi) + \varphi\left(\rho_s/\rho_f\right)}\end{aligned} \tag{12}$$

with $Gr = \frac{g\beta(T_w - T_\infty)a^3}{\nu_f^2}$, being a Grashof number; $\alpha > 0$ is for heated sphere and $\alpha < 0$ for cooled sphere. Then the boundary condition (5) become

$$\begin{aligned}t < 0: &\quad u = v = 0,\ \ T = 0 \ \text{ for any } x, y \\ t \ge 0: &\quad u = v = 0,\ \ T = 1 \ \text{ at } y = 0 \\ &\quad u = u_e(x),\ \ T = 0 \ \text{ as } y \to \infty\end{aligned} \tag{13}$$

Evaluating equation (9) at outside boundary layer region where

$$-\frac{\partial p}{\partial x} = u_e\frac{\partial u_e}{\partial x} + Mu_e \tag{14}$$

Substitute (14) into equation (9) and get

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) \tag{15}$$

$$\begin{aligned}&\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{\partial u_e}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}\nu_f}\frac{\partial^2 u}{\partial y^2} + \\ &\lambda\alpha T\sin x - \gamma M(u - u_e)\end{aligned} \tag{16}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\mathrm{Pr}}\frac{\alpha_{nf}}{\alpha_f}\frac{\partial^2 T}{\partial y^2} \tag{17}$$

Now we use similarity transformation to solve equation (15) to (17), subject to boundary conditions (13). At lower stagnation point of the sphere, where $x = 180^o$ and $\frac{du_e}{dx} = -\frac{3}{2}$. Therefore the similarity variables are defined as

$$u = \frac{1}{r}\frac{\partial \psi}{\partial y} \qquad \text{and} \qquad v = -\frac{1}{r}\frac{\partial \psi}{\partial x} \tag{18}$$

For small time case, consider

$$\psi = t^{\frac{1}{2}} u_e(x) r(x) f(x,\eta,t), \qquad T = s(x,\eta,t), \qquad \eta = y / t^{\frac{1}{2}} \tag{19}$$

Then equation (16)-(17) becomes

$$\begin{aligned} &\frac{\mu_{nf}}{\rho_{nf}\nu_f}\frac{\partial^3 f}{\partial \eta^3} + \frac{\eta}{2}\frac{\partial^2 f}{\partial \eta^2} - \frac{3}{2}t\left[1 - \left(\frac{\partial f}{\partial \eta}\right)^2 + f\frac{\partial^2 f}{\partial \eta^2}\right] \\ &+ t\gamma M\left(1 - \frac{\partial f}{\partial \eta}\right) + \frac{2}{3}t\lambda\alpha s = t\frac{\partial^2 f}{\partial \eta \partial t} \end{aligned} \tag{20}$$

$$\begin{aligned} &\frac{\alpha_{nf}}{\alpha_f}\frac{\partial^2 s}{\partial \eta^2} + \frac{\Pr}{2}\eta\frac{\partial s}{\partial \eta} - \frac{3}{2}\Pr t f\frac{\partial s}{\partial \eta} \\ &= \Pr t\frac{\partial s}{\partial t} \end{aligned} \tag{21}$$

Subject to boundary conditions

$$\begin{aligned} t < 0: &\quad f = 0, \ \frac{\partial f}{\partial \eta} = 0, \ s = 0 \ \text{ for any } x, y, \\ t \geq 0: &\quad f = \frac{\partial f}{\partial \eta} = 0, \quad s = 1 \ \text{ at } y = 0 \\ &\quad \frac{\partial f}{\partial \eta} = 1, \quad s = 0 \ \text{ as } y \to \infty \end{aligned} \tag{22}$$

For large time case, consider

$$\psi = u_e(x) r(x) F(x,Y,t), \qquad T = S(x,Y,t), \qquad Y = y \tag{23}$$

Therefore equation (16)-(17) becomes

$$\begin{aligned} &\frac{\mu_{nf}}{\rho_{nf}\nu_f}\frac{\partial^3 F}{\partial Y^3} - \frac{3}{2}\left[1 - \left(\frac{\partial F}{\partial Y}\right)^2 + F\frac{\partial^2 F}{\partial Y^2}\right] + \gamma M\left(1 - \frac{\partial F}{\partial Y}\right) \\ &+ \frac{2}{3}\lambda\alpha S = \frac{\partial^2 F}{\partial Y \partial t} \end{aligned} \tag{24}$$

$$\frac{\alpha_{nf}}{\alpha_f}\frac{\partial^2 S}{\partial Y^2} - \frac{3}{2}\Pr F\frac{\partial S}{\partial Y} = \Pr\frac{\partial S}{\partial t} \tag{25}$$

Subject to boundary condition

$$\begin{aligned} &F = 0, \ \frac{\partial F}{\partial Y} = 0, \ S = 1 \quad at \ \ Y = 0 \\ &\frac{\partial F}{\partial Y} = 1, \quad S = 0 \quad as \quad Y \to \infty \end{aligned} \tag{26}$$

RESULT AND DISCUSSION

Equations (20)-(21) for small time and (24) to (25) is for large time cases are solved by using numerical scheme named Keller-Box method in MATLAB environment. From this output obtained, the separation times and other physical quantities are presented in tabular form. Two different types of nanoparticles, namely Cu and TiO_2 (water as base fluid), have been considered in this study and the range of volume fraction is $0 \leq \varphi \leq 0.2$. The value of mixed convection parameter is choose as $\alpha = 1$ (heated sphere) and $\alpha = -1$ (cooled sphere). Table 1 shows the thermophysical properties water and nanoparticles.

Table 1 The thermophysical properties of fluid and nanoparticles.

Physical Properties	Base Fluid (water)	Cu	TiO_2
C_p(J kg^{-1} K^{-1})	4179	385	686.2
ρ (kg m^{-3})	997.1	8933	4250
k (W m K^{-1})	0.613	400	8.9538
β x 10^{-5} (K^{-1})	21	1.67	0.9

In order to verify the solution, the present result are compared with Mohammad et al [23] for $\varphi = 0$ at Pr = 0.7 and presented in Table 2. The results show an absolute agreement.

Table 2 Comparison of separation time at the surface of a sphere when Pr=0.7 and mixed convection, $\alpha = 1$.

Magnetic parameter (M)	Prandtl number (Pr)	Separation times (t_s), $\alpha = 1$ Mohammad et al. [23]	Present
0.1	0.7	0.6130	0.6130
0.5	0.7	0.9103	0.9105
1.0	7	1.2724	1.2724

Table 3 The separation times, t_s for Pr=6.2, $\alpha = 1$ and $\varphi = 0.1$.

M	0.0	0.1	1.0	1.5
Viscous	0.4665	0.4960	1.2760	-
Cu	0.4113	0.4557	0.6407	0.8446
TiO_2	0.4488	0.4701	1.1063	1.8524

Table 4 The separation times, t_s for Pr=6.2, $\alpha = -1$ and $\varphi = 0.1$.

M	0.0	0.1	1.0	1.5
Viscous	0.3254	0.3593	0.5842	0.9875
Cu	0.3556	0.3687	0.4759	0.5700
TiO_2	0.3587	0.3671	0.7028	1.5630

Table 5 the separation times, t_s for Pr=6.2, $\alpha = 1$ and $\varphi = 0.2$.

M	0.0	0.1	1.0	1.5
Cu	0.4110	0.4390	0.5418	0.6254
TiO_2	0.4415	0.4508	0.7028	0.8969

Table 6 The separation times, t_s for Pr=6.2, $\alpha = -1$ and $\varphi = 0.2$.

M	0.0	0.1	1.0	1.5
Cu	0.3721	0.3744	0.4449	0.4977
TiO_2	0.3675	0.3735	0.4970	0.6128

Tables 3 and 4 show separation times along the sphere for the various value of magnetic parameter, M for Pr =6.2, $\alpha = 1$ (assisting flow) and $\alpha = -1$ (opposing flow) for two type of nanoparticles Cu and TiO_2. The separation times of two particles are increase with increasing value of M. The separation times of TiO_2 is slightly higher compared to Cu. The same behaviour can be seen for both cases. From these cases it shows that the separation times of TiO_2 smaller than Cu at $M < 1$, for $\alpha < 1$ (opposing flow) but higher at $M > 1$.

Tables 5 and 6 show the separation times when volume fraction $\varphi = 0.2$ at Pr=6.2 and $\alpha = 1, -1$. For this case, it is also observed that separation times are increased with the increased with magnetic parameter, M. From these tables, it shows that the separation times decreased when volume fraction is increased. . This has been proving by Selvakumar and Dhinakaran [27] by doing it experimentally and numerically. However, in this case the TiO_2 shown the effectiveness to delay the separation times compared to Cu for $\alpha = 1$. Furthermore, the separation times can be delayed by added more magnetic particles and small amount the volume fraction.

CONCLUSION

In this paper, we analysed separation times for unsteady magnetohydrodynamic mixed convection boundary layer flow of nanofluid at lower stagnation point past a sphere. We look the effect of volume fraction, φ and magnetic, M towards separation times of the flow. There are two types of nanoparticles used in this paper which are Cu and TiO_2. The governing equations are reduce into non-dimensionless equations and then transform using appropriate similarity transformation and the solved by using Keller-box method. This study concludes that:

1. The separation times decreased with the increasing of volume fraction φ.
2. The separation time increased with the increasing of magnetic parameter, M.
3. The separation times of TiO_2 are higher compare to the Cu at any magnetic Parameter, M for the both α .
4. The separation times of TiO_2 is decreased at $\alpha = -1$ when $M < 1$.

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