INNOVATIVE TYPES OF FUZZY GAMMA IDEALS IN ORDERED GAMMA SEMIGROUPS

IBRAHIM GAMBO

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DEDICATION

This thesis is dedicated to my beloved father Alhaji Gambo Musa, who inspired and taught me to never give up. It is also dedicated to my beloved mother Hajiya Ummi-Kaltume, who taught me that life gives you what you believe not what you want.

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ABSTRACT

The fuzzification of algebraic structures plays an important role in handling many areas of multi-disciplinary research, such as computer science, control theory, information science, topological spaces and fuzzy automata to handle many real world problems. For instance, algebraic structures are particularly useful in detecting permanent faults on sequential machine behaviour. However, the idea of ordered $\Gamma-$semigroup as a generalization of ordered semigroup in algebraic structures has rarely been studied. In this research, a new form of fuzzy subsystem in ordered $\Gamma-$semigroup is defined. Specifically, a developmental platform of further characterizations on ordered $\Gamma-$semigroups using fuzzy subsystems properties and new fuzzified ideal structures of ordered semigroups is developed based on a detailed study of ordered $\Gamma-$semigroups in terms of the idea of *belongs to* $(\in)$ and *quasi-coincidence with* (q) relation. This idea of quasi-coincidence of a fuzzy point with a fuzzy set played a remarkable role in obtaining several types of fuzzy subgroups and subsystems based on three contributions. One, a new form of generalization of fuzzy generalized bi $\Gamma-$ideal is developed, and the notion of fuzzy bi $\Gamma-$ideal of the form $(\in, \in \vee q_k)$ in an ordered $\Gamma-$semigroup is also introduced. In addition, a necessary and sufficient condition for an ordered $\Gamma-$semigroup to be simple $\Gamma-$ideals in terms of this new form is stated. Two, the concept of $(\in, \in \vee q_k)-$fuzzy quasi $\Gamma-$ideals, fuzzy semiprime $\Gamma-$ideals, and other characterization in terms of regular (left, right, completely, intra) in ordered $\Gamma-$semigroup are developed. Three, a new fuzzified $\Gamma-$ideal in terms of interior $\Gamma-$ideal of ordered $\Gamma-$semigroups in many classes are determined. Thus, this thesis provides the characterizations of innovative types of fuzzy $\Gamma-$ideals in ordered $\Gamma-$semigroups with classifications in terms of completely regular, intra-regular, for fuzzy generalized bi $\Gamma-$ideals, fuzzy bi $\Gamma-$ideals, fuzzy quasi and fuzzy semiprime $\Gamma-$ideals, and fuzzy interior $\Gamma-$ideals. These findings constitute a platform for further advancement of ordered $\Gamma-$semigroups and their applications to other concepts and branches of algebra.

ABSTRAK

Pengaburan struktur aljabar memainkan peranan yang penting dalam banyak bidang penyelidikan multidisiplin, seperti sains komputer, teori kawalan, sains maklumat, ruang topologi dan automata kabur untuk mengendalikan pelbagai masalah dunia nyata. Sebagai contoh, struktur aljabar sangat berguna dalam mengesan kesalahan kekal pada tingkah laku mesin berturutan. Namun begitu, idea bagi $\Gamma-$semikumpulan bertertib sebagai pengitlakan semikumpulan bertertib dalam struktur aljabar jarang dikaji. Dalam penyelidikan ini, suatu bentuk subsistem kabur dalam $\Gamma-$semikumpulan bertertib yang baharu telah ditakrifkan. Khasnya, suatu platform pembangunan bagi lebih banyak pencirian $\Gamma-$semikumpulan bertertib menggunakan ciri-ciri subsistem kabur dan beberapa struktur unggulan terkabur bagi semikumpulan bertertib yang baharu telah dibangun berdasarkan kajian terperinci mengenai $\Gamma-$semikumpulan bertertib dari segi idea *kepunyaan* $(\in)$ dan *kuasi-kesekenaan* hubungan (q). Idea kuasi-kesekenaan suatu titik kabur dengan suatu set kabur ini memainkan peranan yang luar biasa dalam mendapatkan beberapa jenis subkumpulan dan subsistem kabur berdasarkan tiga hasil. Satu, suatu pengitlakan bentuk baharu bagi dwi $\Gamma-$unggulan teritlak kabur telah dikembangkan, dan suatu konsep dwi $\Gamma-$unggulan kabur berbentuk $(\in, \in \vee q_k)$ dalam $\Gamma-$semikumpulan bertertib juga telah diperkenalkan. Di samping itu, suatu syarat perlu dan cukup untuk $\Gamma-$semikumpulan bertertib menjadi $\Gamma-$unggulan mudah dari segi bentuk baharu tersebut telah dinyatakan. Dua, konsep kuasi $\Gamma-$unggulan kabur$-(\in, \in \vee q_k)$, $\Gamma-$unggulan semiperdana kabur, dan pencirian lain dalam bentuk yang biasa (kiri, kanan, selengkapnya, intra) bagi $\Gamma-$semikumpulan bertertib telah dikembangkan. Tiga, $\Gamma-$unggulan terkabur yang baharu dari segi $\Gamma-$unggulan pedalaman bagi $\Gamma-$semikumpulan bertertib dalam berbagai kelas telah ditentukan. Oleh itu, tesis ini memberi pencirian bagi jenis-jenis $\Gamma-$unggulan kabur dalam $\Gamma-$semikumpulan bertertib yang inovatif yang berklasifikasi dari segi biasa selengkapnya, intra-biasa, bagi dwi $\Gamma-$unggulan teritlak kabur, dwi $\Gamma-$unggulan kabur, $\Gamma-$unggulan kuasi kabur dan semiperdana kabur, dan $\Gamma-$unggulan pedalaman kabur. Hasil-hasil ini merupakan platform untuk perkembangan selanjutnya bagi $\Gamma-$semikumpulan bertertib beserta aplikasinya pada konsep dan cabang aljabar yang lain.

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$\in,\ \notin$	-	Belongs to and does not belongs to relation
$\in \vee q$	-	Belongs to or quasi-coincident with relation
$\in \wedge q$	-	Belongs to and quasi-coincident with relation
χ_A	-	Characteristic function/membership function
λ	-	Fuzzy subset
x_t	-	Fuzzy point with support x and value t
q_k	-	Generalized quasi-coincident with relation
$\geq$	-	Greater than or equal to
$\lambda \bigcap \mu$	-	Intersection of two fuzzy subsets of λ and μ
$\overset{-k}{\chi}_A$	-	Lower part of characteristic function of A
$U(\lambda, t)$	-	Level set
$\lambda \vee \mu$	-	Maximum of fuzzy subset λ and μ
$\lambda \wedge \mu$	-	Minimum of fuzzy subset λ and μ
$\bar{\in}$	-	Negation of belongs to relation
$\bar{q}$	-	Negation of quasi-coincident with relation
$\neq$	-	Not equal to
$\lambda \circ \mu$	-	Product of two fuzzy subsets of λ and μ
q	-	Quasi-coincident with relation
$f \bigcup g$	-	Union of two fuzzy sets of f and g
$\overset{+k}{\chi}_A$	-	Upper part of characteristic function of A

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CHAPTER 1

INTRODUCTION

1.1 Introduction

The advancements in the fascinating realm of fuzzy sets spearheaded by the work of illustrious scientist Zadeh [1] in 1965, have provided an avenue to generalize several basic notions of group theory and algebra in general. A fuzzy set is defined as a set without clear and sharp boundaries that contains the elements with only a certain degree of membership. The fuzzy sets are the extensions of classical sets, which accommodate any given elements based on the degree of membership unlike the classical sets that wholly exclude or include any given element. Semigroups play a very vital role in many areas of mathematics, for instance, mathematical analysis, coding and language theory, automata theory, and combinatorics. In 1981, Sen [2] introduced the concept of $\Gamma-$semigroups as a generalization of semigroups. In 1986, Sen and Saha changed the said definition by considering two non-empty sets [3]. Since then, $\Gamma-$semigroups have been scrutinized by many researchers [4–7]. The importance of the ideas of "belongs to" $(\in)$ and "quasi coincident with" (q) relations between a fuzzy point and fuzzy set on the other hand is an evidence from the researches conducted during the past two decades to the development fuzzy of generalization. Ordered $\Gamma-$semigroups (a generalization of ordered semigroups) play an important role in the broad study of ordered semigroups.

The idea of quasi-coincidence of a fuzzy point with a fuzzy set is pioneered by Bhakat and Das [8, 9] and Bhakat [10] hence contributed significantly for the generalization of different types of subgroup. The idea proposed by Bhakat and Das in [8] gave birth to novel vistas for future researchers in the contemporary mathematics

of the globe. Furthermore, Khan *et al.* [11, 12] in the recent time studied a novel approach toward fuzzy generalized bi ideals and filters in ordered semigroups.

Meanwhile, in this research the concept of $(\in, \in \vee q_k)-$fuzzy $\Gamma-$ideals in terms of the new types of fuzzy generalized bi $\Gamma-$ideals, fuzzy bi $\Gamma-$ideals in ordered $\Gamma-$semigroups and some characterization in ordered $\Gamma-$semigroups are introduced. Several other notions of fuzzy $\Gamma-$ideals in ordered $\Gamma-$semigroups and determination of various classes of $\Gamma-$ideals are provided.

1.2 Research Background

The concept of a fuzzy set theory was introduced by Zadeh [1] in 1965 while the ideas of fuzzy algebraic structures of groups have begun in the spearheading paper of Rosenfeld [13] in 1971. He studied the notion of fuzzy subgroups and showed that numerous outcomes in groups can be extended in an elementary form to develop the theory of fuzzy subgroups and since then, most of the contributions in this field are either to validate or generalized his work. This gives the extension of many results from groups to fuzzy groups. The notion of a $\Gamma-$semigroup was introduced by Sen [2] in 1981 which gives birth to many research areas and many applications in the context of mathematics and beyond. Furthermore, Kuroki in 1991 initiated the theory of fuzzy semigroups in [14].

Generalization plays a very useful role in mathematics, in ordered semigroups, ordered Γ-semigroups are one of their generalizations. The study of fuzzification of Γ-ideals in Γ-semigroups with their properties was studied by Williams *et al.* [15]. Likewise, Iampan [16, 17] proposed the concept of (0-) minimal and maximal ordered bi ideals in ordered Γ-semigroups and the fuzzification of ideals with filters in Γ-semigroups were also characterized. Chinram [18] studied rough prime ideals and rough fuzzy prime ideals in Γ-semigroups. In 2005, Dutta and Chanda [19] studied the

structure of fuzzy ideals $\Gamma-$rings.

Moreover, with the objective in view, Bhakat and Das [20] provided the concept of $(\in, \in \vee q)-$fuzzy subgroups by using the "belongs to" relation $(\in)$ and "quasi-coincident with" relation (q) between a fuzzy point and a fuzzy subset. In another important and pioneered contribution, Jun [21] introduced the concept of $(\in, \in \vee q_k)-$fuzzy subalgebras of BCK/BCI-algebra and also added with the basic properties of BCK-algebras. As the continuation of Jun idea, Shabir *et al.* [22] and Shabir and Mahmood [23] give the concept of more generalized form of $(\in, \in \vee q)-$fuzzy ideals and defined $(\in, \in \vee q_k)-$fuzzy ideals in semigroups, by generalizing the concept of $[x;t]\, q\lambda$ and defined $[x;t]\, q_k\lambda$ as $\lambda(x) + t + k > 1$, where $k \in [0,1)$. Khan *et al.* [24–26] investigated $(\in, \in \vee q_k)-$fuzzy ideals in ordered semigroups and discussed some important characterizations theorems in terms of this idea.

However, many forms of fuzzy algebraic structures are studeied and explored. The characterization of fuzzy subgroups of finite cyclic groups based on level subgroups of a fuzyy subgroups are studied in [27], while [28], defined an interval-valued subgroups by Rosenfeld's nature of fuzzy subgroups. The motivation of studying the finite semigroups appeared in 1950s and as a consequence, it promoted the study of linguistic models of computation and reasoning. Fuzzy interior ideals of semigroups in terms of interval valued were studied in [29], and characterizations of regular semigroups by fuzzy ideals generalization are given in [30]. Furthermore, another generalizations are given based on ordered semigroups, for instance, in [31] introduced the concept of fuzzy filters in ordered groupoids and characterize filters of ordered semigroups in terms of fuzzy ideals. Whereas, the characterization of fuzzy radicals and prime fuzzy ideals of ordered semigroups are studied in [32], in addition, [33] extended the concept of a fuzzy subgroup with threshold to the concept of a fuzzy left (right and bi)-filters with threshold of ordered semigroups. The concept fuzzy subgroups in another useful generalization of fuzzy set is introduced by Atanassov [34–36] known as intuitionistic fuzzy set. Thereafter, Davvaz *et al.* [37] used this concept to H_v-modules and introduced the concept of intuitionistic fuzzy

H_v-submodule of a H_v-module. Kim and Jun [38], introduced the concept of an intuitionistic fuzzy interior ideal of a semigroup and give some characterizations properties of semigroups. The theory of intutionistic fuzzy sets of an ordered semigroup has been recently developed [39–44]. The most recent study of complex fuzzy subgroups and complex intuitionistic fuzzy subgroups [45, 46] indicates how tremendous the advancement of fuzzy algebra is contributing to the body of knowledge and solving tedious problems.

Recently, the introduction of interval valued fuzzy sets by Zadeh [47] serves as another extension of fuzzy sets where the values of membership degrees are considered to be interval of numbers instead of numbers. Hence, the notion of interval valued fuzzy subsets gives adequate elucidation of the fuzzy subsets. This theory of fuzzy sets has shown to be a useful tool to describe an imprecise or vague data. Davvaz and Leoreanu-Foteo [48] have considered the concept of interval valued $n-$array polygroups with respect to interval-valued $t-$norms$(n - conorms)$. Ma *et al.* [49,50] studied the interval valued $(\in, \in \ \vee q)-$fuzzy ideals of pseudo-MV algebras and $BCI-$Algebras, which further studies conducted in [51, 52].

In this research, new fuzzified ideals structures of the form $(\in, \in \ \vee q_k)$ in ordered $\Gamma-$semigroups which established innovative types of fuzzy $\Gamma-$ideals are characterized. In addition, these ideals are extended to $(\in, \in \vee q_k)-$fuzzy subsystem where characterization of ordered $\Gamma-$semigroup are provided. The generalized bi $\Gamma-$ideals and bi $\Gamma-$ideals in an ordered $\Gamma-$semigroup are developed with their characterizations. In another important contribution, the quasi $\Gamma-$ideals with semiprime fuzzy characterizations are added. Furthermore, another development of fuzzy ideals, namely the fuzzy interior $\Gamma-$ideals in ordered $\Gamma-$semigroups are characterized.

1.3 Problem Statement

The use of fuzzified algebraic structure in the modern days has become a powerful tool in the field of control engineering, computer science, mechatronics and automata theory among others. Hence, the need for more studies of ordered $\Gamma-$semigroups and its characterization by the idea of fuzzy ideals. Bearing in mind the wide application of fuzzy theory in real life and the importance of the ideas of "belongs to" $(\in)$ and "quasi coincident with" (q_k) relations between a fuzzy point and fuzzy set in generalization of fuzzy systems is evident from the researches so far conducted that research on fuzzy set theory needed much more attention. Therefore, the concept of more generalized forms of $(\in, \in \vee q_k)-$ fuzzy ideals defined as $(\in, \in \vee q_k)-$fuzzy bi $\Gamma-$ideals of ordered $\Gamma-$semigroups using the idea of generalization and the concept of $[x; t]\, q\lambda$ which is defined as $[x; t]\, q_k\lambda$ by $\lambda(x) + t + k > 1$, where $k \in [0,\ 1)$ have been characterized. Thereafter, the investigation on $(\in, \in \vee q_k)$ other forms of fuzzy ideals in ordered semigroups have been characterized with their properties.

On the other hand, generalization is useful in mathematics, and serves as a great tool for discovering new results. With the idea of Rosenfeld [13] of fuzzy subgroups in view, the researches carried out come up with different forms of fuzzy algebraic structures like fuzzy subgroups [8, 20, 27, 28], on semigroups [29, 30] and fuzzy ordered semigroups [31–33]. Considering this tremendous contribution of fuzzifications in algebraic structures, a research on the ordered $\Gamma-$semigroups as the generalization of ordered semigroup is required together with the idea of "belongs to" and "quasi belongs to" to give birth to a new form of fuzzy subsystems, which a new generalized form as the innovative concept of an $(\in, \in \vee q_k)-$fuzzy $\Gamma-$ideals in ordered $\Gamma-$semigroups are presented in this research based on bi $\Gamma-$ideals, $(\in, \in \vee q_k)-$fuzzy interior $\Gamma-$ideals, $(\in, \in \vee q_k)-$fuzzy quasi $\Gamma-$ideals and $(\in, \in \vee q_k)-$fuzzy semiprime $\Gamma-$ideals of ordered $\Gamma-$semigroups. A link between the ordinary ideals with the aforementioned fuzzy $\Gamma-$ideals has been developed through level subset and characteristics function. Several characterization theorems of ordered $\Gamma-$semigroups by the properties of their $(\in, \in \vee q_k)-$fuzzy generalized bi $\Gamma-$ideals,

quasi $\Gamma-$ideals of the form $(\in, \in \vee q_k)$ and fuzzy interior $\Gamma-$ideals of the form $(\in, \in \vee q_k)-$ has been investigated. Finally, in this research a new characterization method for ordered $\Gamma-$semigroups and their different classes in terms of $\Gamma-$regular (intra regular, completely regular) ordered $\Gamma-$semigroups in terms of the new notions are also established.

1.4 Research Objectives

The main objectives of this research are stated in the following:

1. To provide $(\in, \in \vee q_k)-$fuzzy generalized bi gamma ideals in ordered $\Gamma-$semigroups and determine various classes of $\Gamma-$regular ordered $\Gamma-$semigroups by the properties of generalized bi $\Gamma-$ideals.
2. To introduce new form of fuzzy bi $\Gamma-$ideals in ordered $\Gamma-$semigroups and characterized ordered $\Gamma-$semigroups in terms of these newly introduced fuzzy bi $\Gamma-$ideals.
3. To determine fuzzy quasi $\Gamma-$ideals in ordered $\Gamma-$semigroups and investigate several properties of fuzzy quasi $\Gamma-$ideals.
4. To introduce new form of fuzzy semiprime $\Gamma-$ideals in ordered $\Gamma-$semigroups and connect the fuzzy quasi $\Gamma-$ideals with these semiprime $\Gamma-$ideals in ordered $\Gamma-$semigroups and determine various classes of $\Gamma-$regular ordered $\Gamma-$semigroups by the properties quasi $\Gamma-$ideals.
5. To initiate fuzzy interior $\Gamma-$ideals in ordered $\Gamma-$semigroups and provide classification of ordered $\Gamma-$semigroups in terms of fuzzy interior $\Gamma-$ideals.

1.5 Scope of the Research

This research focuses on the characterization of ordered $\Gamma-$semigroups in terms of the idea of belongs to $(\in)$ and quasi-coincident with (q) relation, a

new generalization of generalized bi $\Gamma-$ideals, fuzzy bi $\Gamma-$ideals of the form $(\in, \in \vee q_k)$ $\Gamma-$ideals, $(\in, \in \vee q_k)-$fuzzy generalized bi $\Gamma-$ideals, fuzzy quasi $\Gamma-$ideals, semiprime $\Gamma-$ideals, of the form $(\in, \in \vee q_k)$, the characterization of regular, left, right regular, completely regular, intra-regular ordered $\Gamma-$semigroup on the notion of $(\in, \in \vee q_k)$ are also considered. Finally, the fuzzy interior $\Gamma-$ideal of the form $(\in, \in \vee q_k)$ and other generalizations are also characterized.

1.6 Significance of the Research

Algebraic structures like ordered semigroups play a key role in information science, computer science, control theory and fuzzy automata. This research have defined new types of fuzzy subsystem which can be applied in the aforementioned fields and many others as one of the fundamental importance of this research. The fuzzification of algebraic structures has become a very powerful area of interest in handling more real world problems, hence the need for this research and more of it to tackle such life challenges and problems.

In this research, a more generalized form of fuzzy ideals would form other new bases of theories in particular and valuable contributions to mathematics in general. Furthermore, this research constitutes a platform development of ordered $\Gamma-$semigroups and their applications to other branches of algebra, as well as other science related fields are entrenched for future long term researches.

1.7 Research Methodology

In this research, new forms of fuzzy $\Gamma-$ideals in ordered $\Gamma-$semigroups are introduced. The main contribution of the present research is to define the innovative types of fuzzy $\Gamma-$ideals in ordered $\Gamma-$semigroups where several classes of ordered

$\Gamma-$semigroups in terms of these new notions are characterized.

Based on the research objectives and problem statement, the methodology employed in this research is stated in this section and is on ordered $\Gamma-$semigroups which is a generalization of ordered semigroups. This research is motivated by the pioneered research idea of quasi-coincidence of a fuzzy point with fuzzy set by many researchers. It is also significant to note that the concept of $(\in, \in \vee q)$ $-$fuzzy theory depends on belongs to $(\in)$ or quasi-coincident with (q) relation between a fuzzy point and fuzzy set (λ) i.e. $[a;t] \in \lambda$ or $a_t q \lambda$ where $t \in (0,1]$. Moreover, based on the upper/lower parts of a fuzzy subset λ of G, a characteristic function λ_A of A is introduced. Particularly, the interval $[0,1]$ is divided into sub-intervals of $\left[0, \frac{1-k}{2}\right]$ and $\left[\frac{1-k}{2}, 1\right]$ which shows the lower part lies in the interval $\left[0, \frac{1-k}{2}\right]$. Therefore, since for the fuzzy ideal theory of the form $(\in, \in \vee q_k)$ to exist, $\lambda(a) \geq \frac{1-k}{2}$ must hold, thus the fuzzy ideal of this form is related to the lower parts only and is characterized by the properties of lower parts within the corresponding interval of $\left[0, \frac{1-k}{2}\right]$. Both the method and procedures are categorized and discussed in the following paragraphs as per this research is concerned.

In [53], an ordered Γ-semigroup is defined as the following: if G and Γ are non-empty sets, then a structure $(G, \Gamma, \leq)$ is called an ordered Γ-semigroup if: $(x\alpha y)\beta z = x\alpha(y\beta z)$ for all $x, y, z \in G$ and $\alpha, \beta \in \Gamma$, and $a \leq b \rightarrow a\alpha x \leq b\alpha x$ and $x\alpha a \leq x\alpha b$ for all $a, b, x \in G$ and $\alpha \in \Gamma$. The ideas of Jun [21] in 2009 and Khan *et al.* in 2014 [11] were considered to give the new generalization in ordered $\Gamma-$semigroups using the concept between the fuzzy point a_t and a fuzzy subset F with a generalized quasi coincident q_k for $k \in [0,1)$. By Jun's idea, $a_t q_k \lambda$ is equivalent to $\lambda(a) + t + k > 1$ for $k \in [0,1)$ and $t \in (0,1]$. Taking $k = 0$ then Jun's idea become $\lambda(a) + t > 1$. Thus lead to $a_t q \lambda$ which is the existing literature of $(\in, \in \vee q)$. This research, is based on the new form of generalization in ordered $\Gamma-$semigroups. Now, considering the fuzzy point a_t, is said to belongs to A denoted as $a_t \in A$ if and only if $\lambda_A(a) \geq t$ and otherwise is denoted as $a_t \overline{\in} A$ if and only if $\lambda_A(a) < t$. Similarly, the fuzzy point a_t is said to be "generalized quasi-coincident with" A, denoted as $a_t q_k \lambda$, if and only if

$\lambda(a) + t + k > 1$ where $k \in [0, 1)$ and $t \in (0, 1]$.

This research has been conducted on the fuzzy generalized bi $\Gamma-$ideal of the form $(\in, \in \vee q_k)$ in ordered $\Gamma-$semigroups, where ordered $\Gamma-$semigroups are extensively classified in line with the new notions defined. Thereafter, some supporting examples on fuzzy generalized bi $\Gamma-$ideals in ordered $\Gamma-$semigroups are given and characterizations of fuzzy bi $\Gamma-$ideals of the form $(\in, \in \vee q_k)$ in ordered $\Gamma-$semigroups are provided.

Furthermore, the concepts of a level subset and characteristic functions are used to connect and elucidate the fuzzy generalized bi $\Gamma-$ideals of the form $(\in, \in \vee q_k)$ and fuzzy bi $\Gamma-$ideals. Some examples are constructed in support of the newly introduced definitions and other several characterization theorems of ordered $\Gamma-$semigroups of the form $(\in, \in \vee q_k)$. The notion of fuzzy bi $\Gamma-$ideals is further generalized to fuzzy bi $\Gamma-$ideals of the form $(\in, \in \vee q_k)$ in ordered $\Gamma-$semigroups with supporting examples. Moreover, for $\lambda_A(a) \geq \frac{1-k}{2}$ some theorems are determined and when $k = 0$ these theorems are reduced to the result in [54]. Further, a link is established between bi $\Gamma-$ideals and fuzzy bi $\Gamma-$ideals of the form $(\in, \in \vee q_k)$ using level subsets and characteristic functions. It is worth mentioning that every fuzzy bi $\Gamma-$ideals is also a fuzzy generalized bi $\Gamma-$ideals all of the form $(\in, \in \vee q_k)$ in ordered $\Gamma-$semigroups but the converse is not true in general. The characterization of ordered $\Gamma-$semigroup in terms of $\Gamma-$regular, left weakly $\Gamma-$regular, of bi $\Gamma-$ideals, fuzzy generalized bi $\Gamma-$ideals of the form $(\in, \in \vee q_k)$ are provided, the sufficient and necessary conditions of $\Gamma-$regular and intra $\Gamma-$regular ordered $\Gamma-$semigroups to be fuzzy bi $\Gamma-$ideals of the form $(\in, \in \vee q_k)$ are explored and characterized.

In addition, the fuzzy quasi $\Gamma-$ideals in terms of generalized quasi-coincident with relation by Jun's idea [21] is used to develop new generalization of quasi ideals called fuzzy quasi $\Gamma-$ideals of the form $(\in, \in \vee q_k)$ in ordered $\Gamma-$semigroups. Likewise, the semiprime fuzzy quasi $\Gamma-$ideals of the form $(\in, \in \vee q_k)$ in ordered $\Gamma-$semigroups are characterized and classified. Moreover, the classification of

completely $\Gamma-$regular fuzzy quasi $\Gamma-$ideals of the form $(\in, \in \vee q_k)$ in terms of the lower part of the characteristic function is provided.

Finally, the interior $\Gamma-$ideals of the form $(\in, \in \vee q_k)$ of ordered $\Gamma-$semigroups are introduced and suitable examples are provided to support the newly introduced fuzzy interior $\Gamma-$ideals based on their properties. It has been shown that this research provides an avenue to further investigations in different forms of algebraic structures especially semigroups. It is highlighted in the last chapter how future research can be conducted and extended from the current one. Figure 1.1 highlights the operational framework of this research.

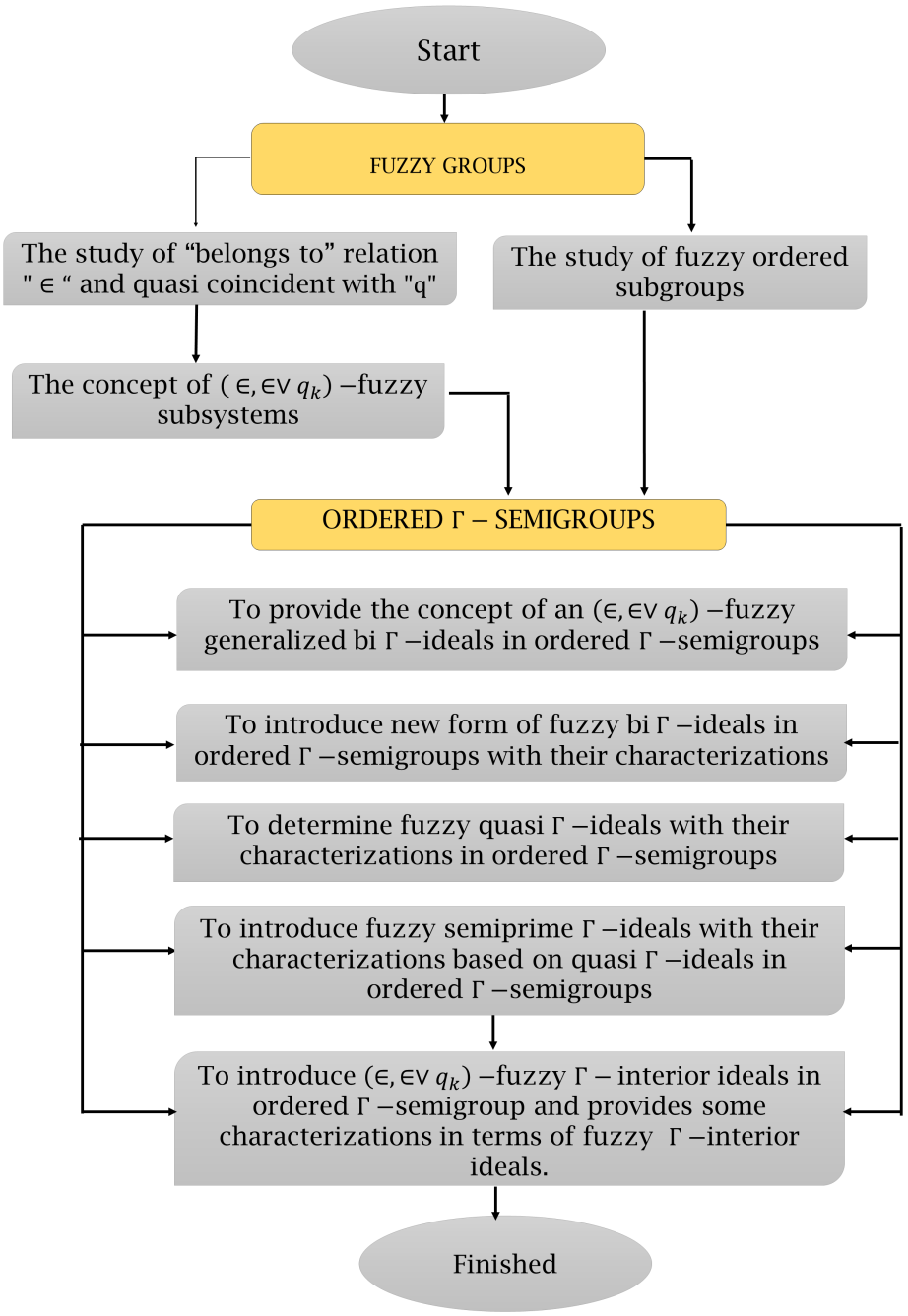

Figure 1.1 The operational framework of the research

1.8 Thesis Organization

This thesis is structured into six main chapters. Chapter 1 serves as the introduction of the thesis where the fuzzy set theory and ordered $\Gamma-$semigroups are elaborated. This chapter entails the research background that gives a general overview

of the entire research work. The statement of the problem is followed by the research objectives. In addition, the scope of the research, significance of the research, research methodology as well as thesis organization are all contained in this chapter.

The second chapter presents the detailed review of various notations, and fundamental concepts that are used in this research work. The current and most recent past literature of several types of fuzzy ideals of ordered semigroups and other related algebraic structures,various research conducted by different researchers in this area are discussed in this chapter.

Chapter 3 elaborates and provides the newly introduced concepts of generalized bi $\Gamma-$ideals of the form $(\in, \in \vee q_k)$ in ordered $\Gamma-$semigroups and fuzzy bi $\Gamma-$ideals of the form $(\in, \in \vee q_k)$ in ordered $\Gamma-$semigroups. The relation between generalized bi $\Gamma-$ideals and fuzzy generalized bi $\Gamma-$ideals of the form $(\in, \in \vee q_k)$ is presented in this chapter. Furthermore, the conditions of fuzzy generalized bi $\Gamma-$ideals of the form $(\in, \in \vee q_k)$ to be constant function using lower part are also given, and the characterization of completely regular ordered $\Gamma-$semigroups of both generalized bi $\Gamma-$ideal and fuzzy bi $\Gamma-$ideal of the form $(\in, \in \vee q_k)$ are presented in this chapter.

The fourth chapter introduces the concepts of fuzzy quasi $\Gamma-$ideal of the form $(\in, \in \vee q_k)$ in ordered $\Gamma-$semigroups and semiprime fuzzy $\Gamma-$ideals of the form $(\in, \in \vee q_k)$ in terms of quasi $\Gamma-$ideals, generalized bi $\Gamma-$ideals and bi $\Gamma-$ideals. The regular, intra-regular ordered $\Gamma-$semigroups of the form $(\in, \in \vee q_k)$ are studied and presented in this chapter. Besides, fundamental results of ordered $\Gamma-$semigroups in terms of semiprime of the form $(\in, \in \vee q_k)$ are given.

In Chapter 5, the concepts of interior $\Gamma-$ideals of the form $(\in, \in \vee q_k)$ and fuzzy $\Gamma-$ideals of the form $(\in, \in \vee q_k)$ coincide in regular, intra-regular and semisimple ordered $\Gamma-$semigroups. The upper/lower parts of fuzzy interior of the form $(\in, \in \vee q_k)$ are characterized and explained in this chapter. Numerous other types of fuzzy interior $\Gamma-$ideals and their classifications in ordered $\Gamma-$semigroups are also

established in this chapter.

The final chapter gives a recap of the whole thesis and summarizes the whole idea. The future outlook of the research, and the conclusion together with the systematic layout are also presented in Chapter 6. Figure 1.2 gives the organization of the whole thesis.

1.9 Conclusion

In this chapter, the idea about fuzzy set theory, more precisely the fuzzy subgroup and related researches are stated under the research background of this research. In addition, the objectives, statement of research problem and research methodology are included in this chapter. After the scope of this research, the motivation and importance of this research are clearly mentioned in the significance of the research.

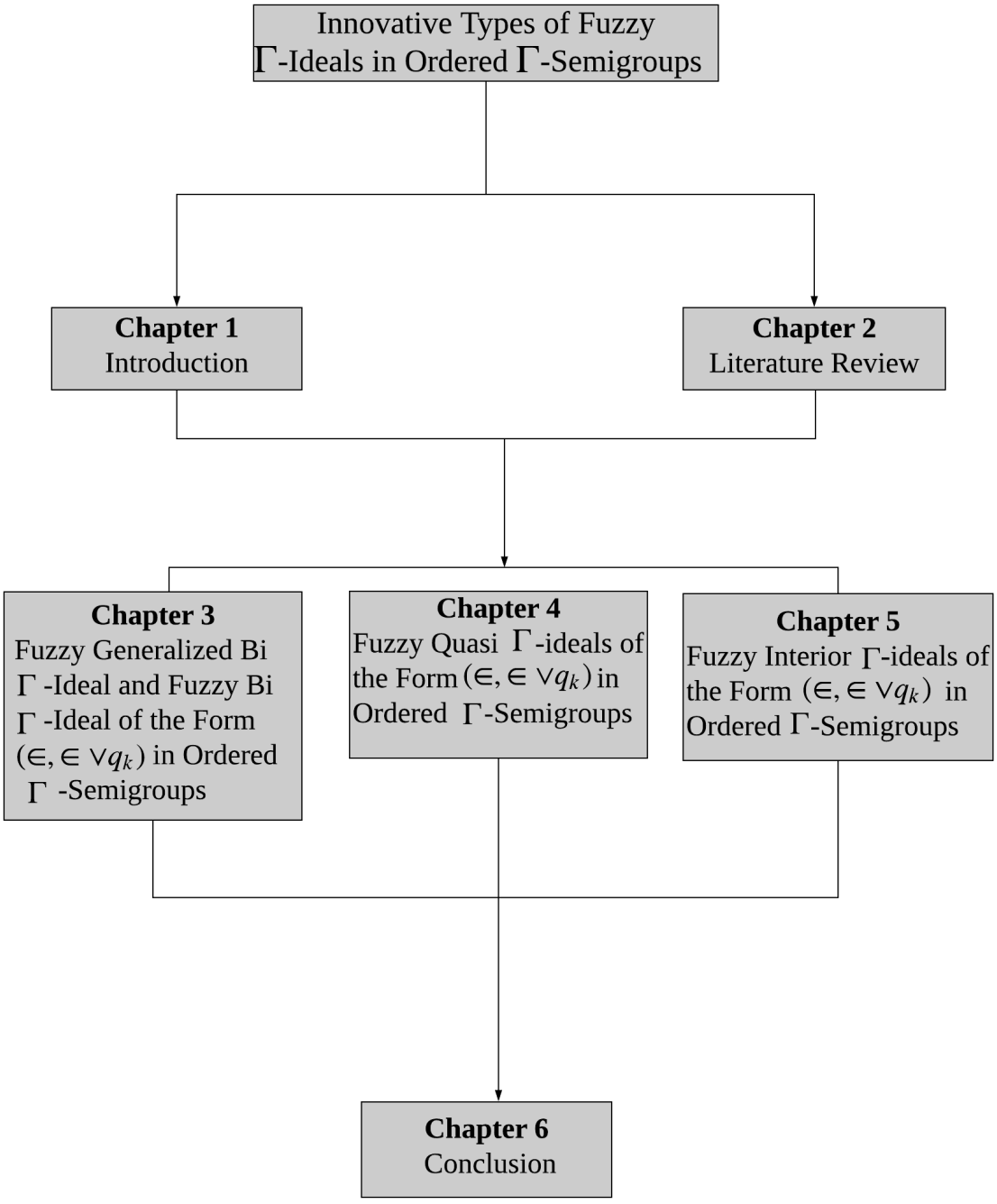

Figure 1.2 Thesis organization

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