

SYSTEMATIC STUDY OF NEUTRON EMISSION MODEL FOR NUCLEAR MUON  
CAPTURE EXPERIMENT

FAIZNUR BINTI OTHMAN

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To my beloved mother, father and brothers.

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## ABSTRACT

Neutron emission model is important for muon capture experimental verification. It provides giant resonance estimation for neutron nuclear response by muon charge exchange reaction. This study uses the neutron emission model for muon capture experiment developed in 2014 and investigate the relationship of three parameters involve in neutron emission namely percentage of the pre-equilibrium event(x), excitation energy ( $E_i^{ex}$ ), and the nuclear temperature factor (y). The influence of these parameters to the neutron energy spectrum are observed. The results are compared with the previous study on neutron emission spectrum of  $^{16}\text{O}$ ,  $^{32}\text{S}$ ,  $^{40}\text{Ca}$ ,  $^{207}\text{Pb}$  and  $^{209}\text{Bi}$  from muon capture experiment. The relationship between x,  $E_i^{ex}$  and y are deduced. The outcome of this study may provide consistency guide for neutron emission after muon capture experiment. The result can be deduced as  $x = 0.41 A^{0.48}$ ,  $E_i^{ex} = 17.31 A^{0.0020}$  and  $y = 8.76 A^{0.20}$ .

## ABSTRAK

Model pelepasan neutron adalah penting untuk mengesahkan penangkapan muon daripada eksperimen. Hal ini demikian kerana, ia akan menyediakan anggaran resonans gergasi untuk tindak balas nuklear neutron oleh caj muon hasil daripada reaksi pertukaran. Kajian ini menggunakan model pelepasan neutron untuk penangkapan muon daripada eksperimen yang dibina pada tahun 2014 dan menyiasat hubungan tiga parameter yang terlibat dalam pelepasan neutron iaitu peratusan daripada acara pra-keseimbangan ( $x$ ), tenaga pengujaan ( $E_i^{ex}$ ), dan faktor suhu nuklear ( $y$ ). Pengaruh parameter ini kepada spektrum tenaga neutron dikaji. Keputusan kajian sebelum ini dibandingkan berkaitan spektrum pancaran neutron  $^{16}\text{O}$ ,  $^{32}\text{S}$ ,  $^{40}\text{Ca}$ ,  $^{207}\text{Pb}$  dan  $^{209}\text{Bi}$  daripada eksperimen muon tangkap. Hubungan antara  $x$ ,  $E_i^{ex}$  dan  $y$  disimpulkan. Hasil kajian ini boleh memberi panduan konsisten untuk pelepasan neutron selepas penangkapan muon daripada eksperimen. Keputusan kajian dapat disimpulkan dengan  $x = 0.41 A^{0.48}$ ,  $E_i^{ex} = 17.31 A^{0.0020}$  and  $y = 8.76 A^{0.20}$ .

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**LIST OF SYMBOL AND ACRONYMS**

$\beta\beta 0\nu$	-	Neutrinoless double beta decay
$\beta\beta 2\nu$	-	Two neutrino double beta decay
$\beta^+$	-	Positive beta
$\beta^-$	-	Negative beta
$e^-$	-	Negative electron
$e^+$	-	Positive electron/Positron
$\bar{\nu}$	-	Conjugate neutrino
$\nu_\mu$	-	Muon neutrino
$\nu_e$	-	Electron neutrino
$\bar{\nu}_e$	-	Conjugate electron neutrino
$\theta$	-	Theta
$\psi$	-	Psi
$\tau$	-	Tau
$\gamma$	-	Gamma

$\mu^-$	-	Negative muon
n	-	Neutron
p	-	Proton
A	-	Atomic mass number
Z	-	Atomic number
N	-	Neutron number
PEQ	-	Pre-equilibrium
EQ	-	Equilibrium
CERN	-	"Conseil Européen pour la Recherche Nucléaire", or European Council for Nuclear Research
GMR	-	Giant magnetic resonance
RIPL	-	Reference Input Parameter Library
NME	-	Nuclear Matrix Element
MeV	-	Mega electron volt
C	-	Copper
Ca	-	Calcium
Bi	-	Bismuth
Mo	-	Molybdenum
O	-	Oxygen
Pb	-	Lead
S	-	Sulphur
Si	-	Silicon

Tl	-	Thallium
amu	-	Atomic mass unit

## CHAPTER 1

### INTRODUCTION

#### 1.1 Background

The properties of neutrinos were still unknown until in early of the twentieth century. There are three types of neutrino which are electron neutrino, muon neutrino and tau. They are in lepton family from the Standard Model of Particles Physics. Every neutrino has its anti-neutrinos where they have the same mass as neutrinos but inverse characteristics. Neutrinos are neutral, chargeless and can be their own anti-particles.

Open questions related to neutrinos that requires both theoretical and experimental explanation are still on debates. The subject of interest is the absolute mass scale, mixing, the Majorana or the Dirac nature of neutrinos, their electromagnetic properties and the possible existence of CP violation in the leptonic sector. A large enough CP violation is necessary to create the asymmetry between matter and anti-matter in the early Universe, and a large CP violation discovery in neutrino oscillations or neutrinoless double beta decay would support the evidence for the role of neutrinos in this mechanism (Itkis, Itkis, Knyazheva, & Kozulin, 2013).

The fact that neutrino have mass has firmly been established by neutrino oscillation experiments (Rodin, 2010) However, the observed oscillations cannot conclude the absolute scale of the neutrino masses.

There are two possible channels for double beta decay which are neutrinoless



double beta decay ( $\beta\beta 0\nu$ ) and two neutrino double beta decay ( $\beta\beta 2\nu$ ). The observation of a neutrinoless double beta decay would however prove that neutrinos are massive with at least one of the mass eigenvalues is larger than the corresponding effective neutrino mass. The common double  $\beta$  decay in several nuclei and their measured lifetimes have now observed by many giants experiments such as MOON, SuperNEMO, EXO and etc. They are well studied experimentally by using charge-exchange, photo-nuclear and neutrino reactions. MOON (Mo Observatory Of Neutrinos) is a high sensitivity  $0\nu\beta\beta$  experiment with the mass sensitivity of an order of 30 meV (H Ejiri, 2006).

Nuclear matrix elements associated with neutrinos and weak interactions (H. Ejiri, 2000). The transition rate of  $\beta\beta$  decay is

$$\Gamma = G|M|^2|m_{\beta\beta}|^2 \quad (1.1)$$

where  $G$  is the two-body phase-space factor,  $M$  is nuclear matrix element and  $m_{\beta\beta}$  is effective Majorana mass of the electron neutrino. The accuracy of the nuclear matrix elements calculation is affected by many numerical factors. The sensitivity of input parameter changes was estimated from the sensitivity of parameter changes in five main quantities; the single-particle energies; the interactions strengths; the single-particle wave functions' oscillator parameter in; the closure energy of the neutrino potential; and the radius of the atomic nucleus (Barea, Kotila, & Iachello, 2013).

(Izyan Hazwani Hashim, 2014) reports the experimental studies of muon capture on  $^{100}\text{Mo}$  strength distributions for the  $\beta^+$  side responses of NME, to help and confirm the theoretical evaluation for double  $\beta$  decay nuclear matrix elements. The results of muon capture strength distribution, can be used to help in deducing the nuclear responses relevant to neutrinoless double  $\beta$  decays. A neutron statistical model has been developed to support the experimental observation of nuclear muon capture.

The study was then continued by (Saroni, 2016) for the neutrino nuclear response concentrating on the nuclei with atomic mass unit between 89 amu and 109

amu. The ratio of pre-equilibrium (PEQ) to equilibrium (EQ) ratio of neutron emission in 15% to 30% was compared. The study concluded that the fraction of neutron emitted before nuclear temperature was achieved are strongly related to their mass number.

Then, in 2016 (I H Hashim, Ejiri, et al., 2016) compared the observed residual isotope (Mo-100, Nb-93 and Ta-181 ) distributions with the neutron statistical model. The study shows a giant resonance-like strength around 9-13 MeV.

## 1.2 Problem Statement

Previous study there is only model for nuclear fission such as from (Faust & Bao, 2004). Then, in (Hiroyasu Ejiri, 2010) stated that only neutron emission has cascade process.

Further of the study, the first experiment and neutrino calculator was done by (Izyan Hazwani Hashim, 2014). The study developed neutrino calculator for experiment verification. In the calculator, all relationship function does not mention clearly on which parameter should be taken. The study also provides an experiment to compare with the calculator.

(I H Hashim, Saroni, Ejiri, & Rasdi, 2016), found a problem where neutron will emit in several events which include pre-equilibrium and equilibrium. The study reported that 15% to 30% neutron will emit. Yet, the study only covers relationship of neutron emission and isotope population.

Therefore, in this thesis, we want to study relation of several parameter which are nuclear temperature factor,  $x$  constant where the fraction in pre-equilibrium and excitation level where the neutron emitted as a function of atomic mass number.

### 1.3 Objectives of the Study

The aim of this report is to provide improvement progress on the neutron emission calculator, which have been developed for neutrino nuclear response study by muon capture. This includes on:

- 1) To investigate the influence of pre-equilibrium neutron emission after muon capture process.
- 2) To investigate the effect of nuclear excitation region on neutron emission.
- 3) To study the contribution of pre-equilibrium neutron on the nuclear temperature factor.
- 4) To optimize the constant  $x$ , excitation energy, nuclear temperature coefficient by comparison on neutron emission with the previously observed neutron spectrum in muon capture experiment.

### 1.4 Scope of the Study

In order to understand the neutrino response from muon capture reaction a neutron statistical model which was developed by (I H Hashim, Ejiri, et al., 2016) was used to evaluate the nuclear excitation level during muon neutrino emission when muon was captured by the target nuclei. The model was constructed in three main focus; neutron emission, excitation energy level and isotope population. In this study three cases are investigated. In first case, the study changes the constant  $x$  for 0.01 to 0.10 in the pre-equilibrium neutron emission events. For the second case, the excitation energy between 15 MeV to 21 MeV were investigated for maximum neutron kinetic energy observation. Finally, in this study changes pre-equilibrium nuclear temperature factor from 3 to 5. The comparison of each case will be presented in  $\chi^2$  analysis and the optimized values for medium and heavy nuclei were determined.

## **1.5 Significance of Study**

This study is very useful for the development of theory in neutrino nuclear response for nuclear muon capture. The experimental data from muon capture experiment are explained in terms of neutron emission events in beta decay and delayed beta decay channel. The absolute and relative muon capture strength can give idea for the theorist to improve their calculation in order to support and reduce the NME uncertainty. This method provides new theoretical value to evaluate the experimental data from muon capture reaction. We can also have an idea of an unknown nuclear excitation range for the neutron emission after muon capture reaction and the fraction of direct and evaporation neutron emission occurs after muon capture.

## **1.6 Outline of the Thesis**

This report provides recent progress and development on the neutron emission model for the neutrino nuclear response study by nuclear muon capture. The objectives are to study the influence of pre-equilibrium neutron emission after muon capture reaction, to investigate the effect of nuclear excitation region on neutron emission, to understand the contribution of pre-equilibrium neutron on the nuclear temperature and to optimize the parameter  $x$ , excitation energy, and nuclear temperature coefficient by comparison between calculated neutron emission with the previously observed neutron spectrum in muon capture experiment. This thesis is arranged into 5 chapters. Chapter 1 includes brief introduction, background and problem statement of the study. The research objectives, scope and significance of the study also will be stated in this chapter, also thesis outline will be highlighted every topic presented in throughout the thesis. The literature review related to the double beta decay, neutrino nuclear response and nuclear muon capture are explained in detail. The overview of the Neutron Emission Model will also be included in Chapter 2. Chapter 3 provide guides to the research flow, the procedure on the ROOT compiler and the model used in the report. The results of this work and the comparison with previous findings are presented in Chapter 4. Finally, chapter 5 conclude the current finding and the plan for future work.

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