

EMBEDMENT OF SYMMETRY GROUPS IN THE TRIAXIAL PATTERNS OF
FOOD COVERS IN MALAYSIA

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To my beloved mother and father. Thank you for your love and support all these years.

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ABSTRACT

Triaxial weaving technique is a fading historical heritage in Malaysia. Besides Malaysia, this technique can be found in countries across the world such as Africa and South America. In Malaysia, this technique is used to produce the Malay traditional food covers through an intricate process in the form of a conic shape. Along the process, the triaxial weaving technique used by the food cover weavers produces beautiful symmetrical patterns. The symmetrical properties of the triaxial patterns including the food cover patterns have intrigued many mathematicians especially ethnomathematicians. In this research, the symmetrical properties possessed by the food cover patterns are found to be similar to the symmetrical properties found in crystals. Therefore, it justifies the implementation of the crystallographic symmetric types, also known as the wallpaper groups, on these patterns. The patterns were then analysed based on their groups and therefore, the generalised properties of these patterns were listed according to the concepts in group theory. Additionally, a new approach was developed to imitate the process of framework making which is an essential part of the food cover weaving. Through this method, more patterns were able to be introduced in this research. Finally, some of those patterns were also analysed and categorised into wallpaper groups.

ABSTRAK

Teknik anyaman tiga paksi boleh dianggap sebagai warisan sejarah yang semakin pudar di Malaysia. Selain Malaysia, teknik ini juga boleh ditemui di seluruh dunia seperti Afrika dan Amerika Selatan. Di Malaysia, teknik ini digunakan untuk menghasilkan tudung saji tradisional masyarakat Melayu melalui satu proses yang rumit untuk menghasilkan tudung saji berbentuk kon. Disamping itu, produk yang terhasil daripada penggunaan teknik anyaman tiga paksi ini mempunyai corak simetri yang cantik. Sifat simetri yang terdapat pada corak tiga paksi termasuklah corak tudung saji telah menarik minat banyak ahli matematik terutamanya ahli etnomatematik. Dalam penyelidikan ini, sifat simetri yang dimiliki oleh corak tudung saji ini didapati mempunyai persamaan dengan sifat simetri yang terdapat dalam kristal. Hal ini mewajarkan penggunaan jenis simetri kristalografi, yang juga dikenali sebagai kumpulan kertas dinding, pada corak ini. Corak tersebut kemudiannya dianalisis berdasarkan kumpulan mereka dan ciri umum corak ini telah disenaraikan mengikut konsep teori kumpulan. Selain itu, pendekatan baharu telah dibangunkan untuk meniru proses pembuatan rangka atau mata punai yang merupakan bahagian yang penting dalam penghasilan tudung saji. Melalui kaedah ini, lebih banyak corak dapat diperkenalkan dalam penyelidikan ini. Akhirnya, beberapa corak baharu juga telah dianalisis dan dikategorikan ke dalam kumpulan kertas dinding.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

The existence of mathematical elements in the weaving culture all over the world has been discussed by researchers especially in the field of ethnomathematics. Ethnomathematics is defined by D' Ambrosio as below:

‘The mathematics which is practiced among identifiable cultural groups, such as national-tribal societies, labour groups, children of a certain age bracket, professional classes and so on. Its identity depends largely on focuses of interest, on motivation and on certain codes and jargons which do not belong to the realm of academic mathematics [1].’

The Malay weaving culture was also a subject to these researches. For example, the *rombong* (basket) weaving culture has been the interest of some ethnomathematicians such as Bland [2], who discussed about the significance of the sequence of numbers in *anyaman gila* (mad weaving technique), associated with *rombong* weaving. However, in this research, *rombong* weaving is not in our scope, instead the patterns considered are the ones observed by Adam, who stated that the techniques involved are known as triaxial weaving techniques [3]. Apart from that, Adam also developed some template patterns using graphical software. These patterns are then analysed and classified into certain groups in this research.

1.2 Research Background

Triaxial weaving techniques have been known to be used in traditional Malay weaving especially in *rombong* or basket weaving and also in food cover weaving. Adam in [3] has highlighted the usage of this technique in the Malay food cover weaving particularly in Malacca and Terengganu. Some examples of the patterns established from the application of triaxial weaving technique in traditional food cover weaving are the *Corak Berdiri* and the *Lima Buah Negeri* pattern as shown in Figure 1.1.

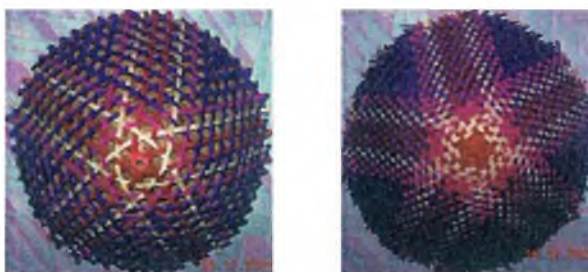


Figure 1.1 (a): *Corak Berdiri*

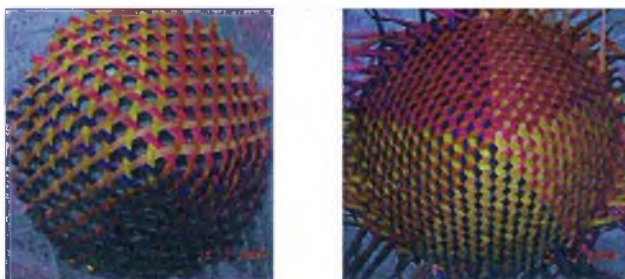


Figure 1.1 (b): *Lima Buah Negeri* pattern

The names of the food cover weavings are given based on the visualised patterns. For example, the *Corak Berdiri* is given its name due to the five ‘pillars’ that appears to be standing on the food cover. Meanwhile, the *Lima Buah Negeri* pattern is named based on the five sections that have different colour combinations. As an addition, Adam [3] had also developed some template patterns based on the triaxial weaving techniques using two colours and up to six strands. Therefore, this research was based on the template patterns developed by Adam along with the traditional weaving patterns. These patterns were analysed and categorised into

certain groups with the number of colours used as one of the defining properties. An analysis has been conducted on the relationship between the number of colours used and the patterns produced.

1.3 Statement of Problem

The research done by Adam in [3] suggested that the triaxial weaving patterns incorporated by the Malay food cover weavers can be analysed in a mathematical way. Thus, this research is conducted to classify these patterns into certain groups with consideration for the number of colours used and whether an algorithm can be developed based on the analysis.

1.4 Research Objectives

The objectives of the study are:

1. to study the concept and method used to produce triaxial weaving patterns and whether they can be defined using wallpaper groups.
2. to develop three-colour triaxial patterns by mimicking the food cover weaving process.
3. to classify the three-colour triaxial patterns into wallpaper groups.

1.5 Scope of the Study

This study was conducted based on the template patterns developed by Adam in [3] which include some patterns that are similar to the actual food cover patterns. The patterns are then analysed based on the ideas from group theory to allow the patterns to be categorised into groups.

1.6 Significance of the Study

From this research, the two-colour triaxial patterns developed in a previous study have been classified into wallpaper groups. Furthermore, three-colour triaxial patterns have been developed using the food cover weaving process as a reference. These patterns have also been classified into wallpaper groups.

1.7 Research Methodology

The research is started by studying the basic techniques of triaxial food cover weaving. Also, the planar triaxial patterns that were produced in a previous research are also studied. Next, the properties of the crystal symmetry types or also known as the wallpaper groups are discussed. Following that, an analysis on the planar patterns is carried out based on the wallpaper groups. These patterns are then categorised into different wallpaper groups. Then, more patterns are generated in the same template by using three colours. The same process of analysis and categorising is carried out on these new patterns. The research framework is illustrated in Figure 1.2.

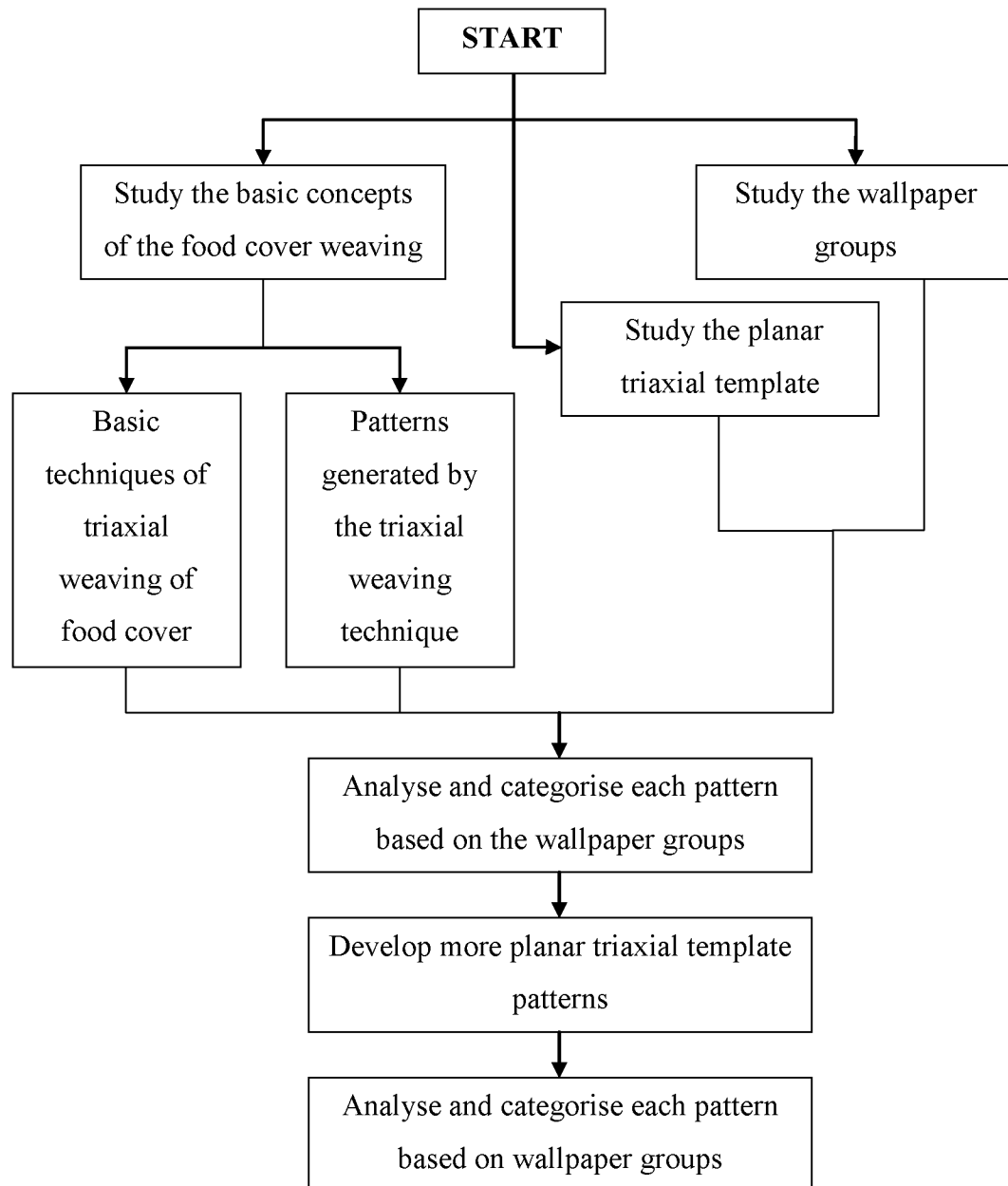


Figure 1.2: Research Framework

1.8 Thesis Organisation

In this chapter, some introduction was given on triaxial weaving technique. Apart from that, the research background, statement of problem, research objectives, scope of the study and the significance of the study were also given.

In the next chapter, some literature reviews is given on the food cover weaving techniques and the patterns produced with its properties. Some basic knowledge from an earlier research was also included as the foundation of this research.

In Chapter 3, some mathematical preliminaries are given. This includes an introduction to triaxial patterns. Other than that, the chapter also discussed about the wallpaper groups elaborately. Some examples of the patterns that can be classified into different wallpaper groups were also provided.

Next, an analysis was done on the thee-coloured triaxial patterns. The categorisation of these patterns into wallpaper groups were given in Chapter 4. Some explanations were also given to justify the result obtained.

Proceeding in Chapter 5, the process of obtaining the three-colour triaxial patterns was explained in detail. The classification of these patterns into wallpaper groups was also given. Justifications of some of the result were also discussed.

Finally, in Chapter 6, a conclusion was made and some recommendations for future researches were given.

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APPENDIX

PUBLICATION AND SEMINARS

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