MIXED CONVECTION FLOW OF VISCOUS AND SECOND GRADE FLUIDS DUE TO NON–COAXIAL ROTATION

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To My Beloved:

Abah

(*Mohamad Bin Deraman*)

Ma

(*Mazni Binti Husin*)

Abang

(*Ahmad Nasafi Bin Mohamad*)

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(*Ahmad Jafni Bin Mohamad*)

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Thank you for being my supporter, listener and prayer during my PhD journey. I love you till Jannah!

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ABSTRACT

Unsteady flow of viscous and second grade fluids in non-coaxial rotation past a vertical oscillating disk have been studied by a number of researchers due to wide applications in boundary layer control, food processing, mixer machines and cooling turbine blades. Therefore, in this research, heat and mass transfer of viscous and second grade fluids were studied. The effect of magnetohydrodynamics (MHD) flow through a porous medium was considered. The main purpose of this study was to obtain the exact solutions for four problems of non-coaxial rotating flow. Two problems were studied for viscous fluid, whereas another two problems were studied for second grade fluid. All problems were considered in mixed convection flow and without magnetic and porosity effects. Appropriate non-dimensional variables were used to simplify the governing equations into non-dimensional equations along with initial and boundary conditions. Through this non-dimensional process, the non-dimensional parameters such as Grashof number, modified Grashof number, Prandtl number, Schmidt number, velocity of oscillation, magnetic, porosity and second grade fluid were obtained. The exact solutions for velocity, temperature and concentration expressions were obtained by using Laplace transform technique. From these corresponding expressions, the skin friction, Nusselt number and Sherwood number were calculated. The solutions were plotted graphically to discuss the influence of non-dimensional parameters in velocity, temperature and concentration profiles. Results show that, velocity profile with magnetic effect is lower compared to velocity without magnetic effect, whereas the velocity with heat and mass transfer phenomena is higher than just a heat transfer. It is also observed that velocity of second grade fluid solutions is always lower compared to the velocity of viscous fluid. All the obtained results are compared with published results and found to be in good agreement, validating the obtained solutions. The exact solutions obtained in this thesis provide an interesting and complete benchmark to verify numerical schemes for solving different complex flow situations.

ABSTRAK

Aliran tak mantap bagi bendalir likat and gred kedua di dalam putaran bukan sepaksi melalui cakera yang menegak telah dikaji oleh beberapa penyelidik kerana terdapat banyak aplikasi di dalam kawalan lapisan sempadan, pemprosesan makanan, mesin pencampur dan penyejuk turbin bilah. Oleh itu, dalam penyelidikan ini, aliran pemindahan haba dan jisim bagi bendalir likat dan gred kedua dikaji. Kesan aliran hidrodinamik magnet (MHD) yang melalui bahantara berliang turut dipertimbangkan. Tujuan utama kajian ini adalah untuk mendapatkan penyelesaian tepat bagi empat masalah aliran putaran bukan sepaksi. Dua masalah telah dikaji bagi bendalir likat, manakala dua lagi masalah dikaji untuk bendalir gred kedua. Semua masalah telah dipertimbangkan di dalam aliran olakan campuran dan tanpa kesan magnet dan keliangan. Pembolehubah tak bermatra bersesuaian digunakan untuk mempermudahkan persamaan menakluk ke dalam persamaaan tak bermatra bersama dengan syarat awal dan syarat sempadan. Melalui proses tak bermatra ini, parameter tak bermatra seperti nombor Grashof, nombor Grashof diubahsuai, nombor Prandtl, nombor Schmidt, halaju berayun, magnet, keliangan dan bendalir gred kedua diperoleh. Penyelesaian tepat bagi ungkapan halaju, suhu dan kepekatan diperoleh dengan menggunakan teknik penjelmaan Laplace. Dari ungkapan ini, geseran kulit, nombor Nusselt dan nombor Sherwood dikira. Penyelesaian diplotkan secara bergraf untuk membincangkan kesan parameter tak bermatra di dalam profil halaju, suhu dan kepekatan. Keputusan menunjukkan bahawa profil halaju dengan kesan magnet adalah lebih rendah berbanding dengan halaju tanpa kesan magnet, manakala halaju dengan fenomena pemindahan haba dan jisim adalah lebih tinggi berbanding dengan hanya pemindahan haba. Diperhatikan juga halaju bagi penyelesaian bendalir gred kedua adalah sentiasa lebih rendah berbanding dengan halaju bagi bendalir likat. Semua keputusan yang diperoleh dibandingkan dengan keputusan yang telah diterbitkan dan didapati penyesuaian yang sangat baik, mengesahkan penyelesaian yang diperoleh. Penyelesaian tepat yang diperoleh dalam tesis ini menyediakan suatu penanda aras yang menarik dan lengkap untuk mengesahkan skim berangka untuk menyelesaikan pelbagai situasi aliran yang sukar.

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LIST OF ABBREVIATIONS

MHD	-	Magnetohydrodynamics
Cu	-	Copper
Ag	-	Silver
CuO	-	Copper oxide
Al_2O_3	-	Alumina
TiO_2	-	Titanium oxide

LIST OF SYMBOLS

Roman Letters

$\mathbf{A_1}$	-	First Rivilin-Ericksen tensor
$\mathbf{A_2}$	-	Second Rivilin-Ericksen tensor
$\mathbf{b_1}$	-	Induced magnetic field
$\mathbf{B}$	-	Total magnetic field
$\mathbf{B_0}$	-	Applied magnetic field
c_p	-	Specific heat at constant pressure
C	-	Concentration of the fluid
C_∞	-	Concentration of fluid in free stream
C_w	-	Concentration of fluid at disk
D	-	Mass diffusivity
d/dt	-	Material time derivative
$\mathbf{E}$	-	Electric field
erfc	-	Complementary error function
exp	-	Exponential function
f	-	Dimensional non–coaxial velocity in x-direction
$\mathbf{F}$	-	Body force
F	-	Dimensional complex velocity
F^*	-	Non–dimensional complex velocity

g	-	Dimensional non–coaxial velocity in y–direction
g_x	-	Gravitational acceleration in x–direction
$\mathbf{g}$	-	Gravitational acceleration vector
Gm	-	Mass Grashof number
Gr	-	Thermal Grashof number
h	-	Dimensional non–coaxial velocity in z–direction
$H(.)$	-	Heaviside function
i	-	Imaginary number
$\mathbf{i}$	-	Unit vector in x–direction
$\mathbf{I}$	-	Identity tensor
$\mathbf{j}$	-	Unit vector in y–direction
$\mathbf{J}$	-	Current density
$\mathbf{J} \times \mathbf{B}$	-	Lorentz force
k	-	Permeability of porous medium
$\mathbf{k}$	-	Unit vector in z–direction
k_1	-	Thermal conductivity
K	-	Non–dimensional porosity parameter
M	-	Non–dimensional magnetic parameter
Nu	-	Nusselt number
p	-	Scalar pressure
p^*	-	Modified pressure gradient
p	-	Scalar pressure
p_d	-	Dynamic pressure
p_h	-	Hydrostatic pressure
$\Pr$	-	Prandtl number
q	-	Laplace transform parameter
$\mathbf{r}$	-	Vector of radius
$\mathbf{R}$	-	Darcy’s resistance

Sc	-	Schmidt number
Sh	-	Sherwood number
$\mathbf{T}$	-	Cauchy stress tensor
t	-	Dimensional time
t^*	-	Non-dimensional time
T	-	Temperature of fluid
T_∞	-	Temperature of fluid in free stream
T_w	-	Temperature of fluid at disk
u	-	Velocity in x–direction
U_0	-	Amplitude of the disk oscillation
$\mathbf{V}$	-	Velocity vector field
v	-	Velocity in y–direction
w	-	Velocity in z–direction
x	-	Coordinate axis of the plate
y	-	Coordinate axis of the plate
z	-	Coordinate axis normal to the plate
z'	-	Coordinate axis for non–coaxial rotation
z^*	-	Non-dimensional in z–direction

Greek Letters

$\mathcal{L}$	-	Laplace transform
$\mathcal{L}^{-1}$	-	Inverse Laplace transform
∇	-	Del operator
$\nabla\cdot$	-	Divergence
$I_1(.)$	-	Modified bessel function of order one
α_1, α_2	-	Material moduli or normal stress moduli
α	-	Non–dimensional second grade parameter
β_C	-	Coefficient of concentration expansion
β_T	-	Coefficient of thermal expansion

$\delta(.)$	-	Dirac delta function
μ	-	Dynamic viscosity
υ	-	Kinematic viscosity
ρ	-	Density of fluid
ρ_∞	-	Density of fluid in free stream
σ	-	Electrical conductivity
$\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$	-	Normal stress
τ	-	Skin friction
τ^*	-	Non–dimensional skin friction
τ_{xy}, τ_{xz}	-	Shear stress in x–direction
τ_{yx}, τ_{yz}	-	Shear stress in y–direction
τ_{zx}, τ_{zy}	-	Shear stress in z–direction
ω	-	Frequency of oscillation
ω^*	-	Non–dimensional frequency of oscillation
ωt	-	Non–dimensional phase angle
ϕ	-	Porosity of the medium
Ω	-	Angular velocity
$\mathbf{\Omega}$	-	Angular velocity vector

Subscripts

c	-	Cosine
d	-	Dynamic
h	-	Hydrostatic
s	-	Sine
w	-	Conditions on the wall
∞	-	Free stream condition

Superscript

T	-	transpose operation

CHAPTER 1

INTRODUCTION

1.1 Introduction

This chapter presents the main area of fluid mechanics for Newtonian fluids and non-Newtonian fluids, along with an introduction on the research background, statement of the problem, objectives of research, scope of research, and the significance of research.

1.2 Research Background

The study on convective transport of momentum, heat and mass in fluid flow has received special attention, which perhaps is mainly due to their potential applications in industries such as oil and gas, drilling, food stuffs, polymer processing, blood and cosmetic products. There are three types of convective transport, namely forced, free, and mixed convections. Forced convection occurs when the flow is caused either by external force or by imposing non-homogeneous boundary condition on velocity, such as moving or oscillating flat plate. Contrary to forced convection, in natural or free convection, the transport phenomenon occurs due to buoyancy force that arises from density differences caused by temperature and concentration variation in the fluid. A situation where the free and forced convection mechanisms simultaneously and significantly contribute to the above transport phenomena is called mixed or combined convection. Combined convection phenomenon occurs in many technical and industrial problems such as electronic devices cooled by fans, nuclear reactors cooled during an emergency shutdown, a heat exchanger placed in a low velocity environment, solar collectors, and so on. Mass transfer is the movement of mass from one to another location caused by absorption, evaporation, drying, distillation and many more. It is commonly used in engineering field for physical processes that involve diffusive and convective transport of chemical species within physical system, such as reaction engineering, separation engineering, heat transfer engineering and other chemical engineering. Over time, various publications on mixed convection with different boundary conditions and situations have appeared.

There are various types of fluid which are responsible for the motion of convection flow. Normally, the fluid is divided into two types, namely Newtonian and non-Newtonian fluids. Newtonian or viscous fluids obey the Newtons law of viscosity and are usually described by Navier Stokes equations. In general, all gases and most liquids with simpler molecular formula and low molecular weight, such as water, benzene, ethyl alcohol, hexane and most solutions of simple molecules are Newtonian fluids. Different from Newtonian fluids, non-Newtonian fluids do not obey Newtons law of viscosity, since they have variable viscosity at constant temperature, and their viscosity depends on the applied force. Examples of non-Newtonian fluids include syrupy mixture of corn-starch and water, quicksand, slurries, pastes, gels, polymer solutions etc. These non-Newtonian fluids are usually divided into three main categories, which are differential type, rate type and integral type. Differential and rate type models are used to describe the response of fluids that have slight memory such as dilute polymeric solutions, while the integral models are used to describe materials such as polymer melts that have considerable memory.

One of the most popular subclasses of differential type of fluids is called the second grade fluid; also known as a viscoelastic fluid. This fluid model was first proposed by Coleman and Noll in (1960). It is found in polymer fluids, where these fluids exhibit both the viscous and elastic characteristics. Viscous materials, like honey, resist shear flow and strain linearly with time when stress is applied. Meanwhile, elastic materials strain instantaneously when stretched, and quickly return to their original state once the stress is removed. In mathematical analysis, the problems of Newtonian fluids are simpler compared to non-Newtonian fluids problems for simple flow geometry. It is due to the fact that the mathematical systems of Newtonian fluids are not as much complicated and their solutions are convenient. Even the Newtonian fluid problems for complicated flow geometries are more difficult to solve due to the complex form of Navier Stokes equations. The problems of non-Newtonian fluids, on the other hand, are very complicated due to additional non-Newtonian terms in the constitutive equations. Therefore, the present study aims to investigate unsteady mixed convection flow of incompressible viscous and second grade fluids in oscillating infinite vertical disk. Difficulty to cater non-Newtonian fluid further increases when these non-Newtonian fluids are incorporated in other physical phenomena such as heat transfer, mass transfer or heat and mass transfer together or by changing the physical configuration of the problem. One of the complicated physical configurations is when the fluid and disk are in rotating motion.

Historically, the rotating fluid theory was developed during the process of understanding and predicting the flow phenomena on the earth surface, especially at large scale atmospheric and oceanic flows. Significance of rotating fluid can also be observed by study of the mathematical modeling of rotating flow. Modeling of rotating flow is critically important across wide range of scientific, engineering, and product design applications, providing design capability for products such as jet engines, pumps, and vacuum cleaners, as well as modeling capability for geophysical flows. Even for applications where rotation is not essentially evident, the subject is often fundamental to understand, and modeling the details of the flow physics is important. Examples include the vortices produced in flow along a channel, the secondary flow produced for flow around a bend, and wing-tip vortices produced downstream of a wing. Nevertheless, rotating flows over a flat plate are of great importance in terms of their relevance to a wide variety of technical applications such as meteorology, cosmical and geophysical fluid dynamics. The Coriolis force in fundamental rotating flow equations is more significant in comparison with inertial and viscous forces. Coriolis force in a fluid is responsible for the differences between the dynamics of non-rotating and rotating fluids. In many geophysical and industrial energy system flows, Coriolis force has a significant influence on the fluid dynamic of the system. In physics, the Coriolis force is defined as a deflection of moving objects in a frame rotating in the opposite direction. For example, when a frame rotates in clockwise direction, the moving object will deflect to the left. If the frame rotates counterclockwise, the deflection of object will move to the right. This effect is very important for earth rotation, which is evident by observing free-moving objects to veer toward the right in the Northern Hemisphere and to the left in the Southern Hemisphere. From the literature survey, rotation can be divided into two types, which are coaxial and non-coaxial rotation. Coaxial rotation is defined as fluid having a common axis or coincident axes on a straight line, whereas non-coaxial rotation is rotation that involves two rotating flows between axes separated by a distance known as length (Erdogan (1997)). Based on the above discussion, it is interesting to study the behavior of the fluid motion influenced by non-coaxial rotation in heat and mass transfers.

The rotating flow of an electrically conducted fluid under the influence of a magnetic field or commonly known as magnethohydrodynamic (MHD) flow in heat and mass transfer has been conducted extensively in recent years. The study on MHD flow has attracted the attention of researchers due to its wide range of useful applications in several areas of science and technology, such as in medical science of magnetic drug targeting for transporting drugs to the whole human body (Mustapha *et al.,* (2009; 2010)) and MHD flow as a controller of boundary layer transition (Poggie and Gaitonde (2002), Nishihara *et al.* (2005)). Further, the effect of magnetic field on flows through a porous medium has some specific applications in ground water flow, irrigation problems, recovery of crude petroleum, heat-storage beds, thermal and insulating engineering, chromatography and chemical catalytic reactors (Hayat *et al.* (2008), Salah *et al.* (2013)). This study on porous medium circulates about the permeability, tensile strength and electrical conductivity, which can sometimes can be derived from the respective properties of its constituents (solid matrix and fluid) and the media porosity and pores structure, but such derivation is usually complex. Even the concept of porosity is only straight-forward for a poroelastic medium.

The present study focuses on magnetic and porosity effects. Specifically, the problem of non-coaxial MHD mixed convection flow of fluid in a porous medium is the main focus since it is still not available in the literature. All problems considered in the present study are tackled by using Laplace transform technique, since exact solutions are considerably important for comparison with the numerical scheme. Laplace transform has been used in this study because of most engineering problems involve functions with respect to time, such as piecewise continuous functions, periodic functions, steps functions, and delta functions. Therefore, there is a need of a method to solve differential equations involving such functions, thus Laplace Transform, which was introduced by a French mathematician Pierre Simon de Laplace (1749 - 1827 M), has been chosen. This transform provides a systematic alternative approach for solving differential equations where mathematical transformations are used to simplify the solution of problems. The purpose of using a transformation is to create a new domain to ease the handling of the problem being considered. Once results have been obtained in the new domain, they can be converted back into the original domain by taking Inverse Laplace transform. The Laplace transform takes an ordinary differential equation in the time, t domain into an algebraic equation in the q domain, after the solution. This is then rearranged using algebraic rules to obtain an expression for a function, with respect to transform variable. Then, the solution of the differential equation as a function of t is found by taking inverse transform. While in this problem, the Laplace transform has distinct advantages because initial and boundary conditions are involved at early stage and automatically incorporated into the solution. Further discussions on these topics are provided in Chapter 2 by looking at related literatures done by previous researchers relevant to this study.

1.3 Statement of Problem

This research focuses on the investigation of the behavior of unsteady mixed convection flow in a rotating disk executing non-coaxial rotation. Focus is given to the flow motion induced by oscillating infinite vertical disk. Two types of fluids, which are viscous and second grade fluids, are considered. This research explores the following questions:

(i) How does the mathematical model behave in the problem of unsteady mixed convection flow of viscous and second grade fluids in non-coaxial rotation?

(ii) How does the mathematical model behave for this problem involving concentration, magnetic field and porosity effects?

(iii) How can the exact solutions for complicated mixed convection flow for the proposed fluid models be obtained?

(iv) How do the physical parameters embedded in the fluid flow models affect the behaviors of velocity, temperature and concentration profiles?

1.4 Objectives of Research

The objective of this research is to investigate theoretically the unsteady mixed convection flow of non-coaxial rotation for viscous and second grade fluids. This investigation includes:

(i) to derive and extend the appropriate governing equations, together with initial and oscillating boundary conditions based on a suitable physical model,

(ii) to obtain exact solutions for the velocity, temperature and concentration profiles by using the Laplace transform method for

(a) MHD and porosity effects,

(b) heat and mass transfer phenomenon, and

(c) MHD and porosity effects as well as heat and mass transfer phenomenon,

(iii) to compute the skin-friction, Nusselt and Sherwood numbers, and

(iv) to analyze graphically and in tabulated form of the obtained exact solutions of velocity, temperature and concentration together with skin friction, Nusselt and Sherwood numbers.

1.5 Scope of Research

This research focuses on unsteady mixed convection flow of incompressible viscous and second grade fluids in non-coaxial rotation, specifically the oscillating infinite vertical disk. In both viscous and second grade fluid problems, the first two problems focus on the fluid motion induced by heat transfer, or heat and mass transfers (double diffusion) together, without magnetic and porosity effects. Consequently, the last two problems of viscous and second grade fluids focus on the fluid motion induced by heat flow and double diffusion in the presence of magnetic and porosity effects. The proposed problems are solved analytically by using the Laplace transform technique. Accordingly, the skin friction, Nusselt number and Sherwood number are calculated. MATHEMATICA software is utilized to find the complicated inverse Laplace transformation. The obtained results will then be plotted graphically using MATHCAD. In order to check the accuracy, the results will be compared with the published work in the literature.

1.6 Significance of Research

The results obtained from this project are significant because of the following reasons.

(i) to build a better understanding on the rheological behavior of non-coaxial rotation of fluid flows in oscillating infinite vertical disk,

(ii) to enhance the knowledge on the magnetic, porosity, heat and mass transfers characteristics in rotating viscous and second grade fluids,

(iii) to give insight on the physical behavior of non-coaxial rotation of fluid flows affected by mixed convection phenomenon,

(iv) to introduce new knowledge of theoretical study that can be a good reference to researchers, engineering applications and education, and

(v) these exact solutions can be used as a check of correctness for the solutions of more complex mathematical models obtained through numerical schemes.

1.7 Research Methodology

The unsteady dimensional momentum, energy and mass equations of the incompressible viscous and second grade fluids shall be modeled in the form of partial differential equations with initial and oscillating boundary conditions. These governing equations, together with conditions, will then be transformed into non-dimensional equations by using the corresponding non-dimensional variables. After that, the Laplace transform technique, subjected to non-dimensional initial and boundary conditions, will be applied into non-dimensional equations in order to obtain the analytical solutions of velocity, temperature and concentration profiles. This technique is chosen because it is applicable to the geometry of infinite vertical disk for the proposed problems, and shall be applied in each case. Then, the results for skin friction, Nusselt number and Sherwood number of the fluid flow are computed.

For the sake of physical understanding, analytical results for velocity, temperature and concentration profiles will be plotted graphically for the emerging flow parameters such as Grashof number, modified Grashof number, Prandtl number, Schmidt number, amplitude parameter, second grade parameter, phase angle parameter and time parameter. MATHCAD and MATHEMATICA are the main tools in this research to plot the solutions and find the inversion of Laplace transform. As the velocity is a complex function, the graphs for both primary (real part) and secondary (imaginary part) velocities shall be shown separately. The results will be used to ensure the correctness of the solutions by satisfying all imposed initial and boundary conditions. The limiting cases will then be compared with those of previous publications to confirm the correctness of the obtained solutions. The operational framework for research methodology is shown in Figure 1.1.

1.8 Thesis Organization

This thesis contains eight chapters. Chapter 1 discusses the research background, which describes all definitions of problem, followed by statement of problem, objectives of research, scope of research, significance of research, research methodology and thesis organization. The following Chapter 2 reviews some published researches related to proposed problems, as acknowledged in the objectives.

Chapter 3 presents the derivation of governing momentum equations for non-coaxial rotation of viscous and second grade fluids with simultaneous effects of heat and mass transfers (double diffusion). An oscillating disk for sine and cosine cases with the effect of MHD and porous medium is also derived.

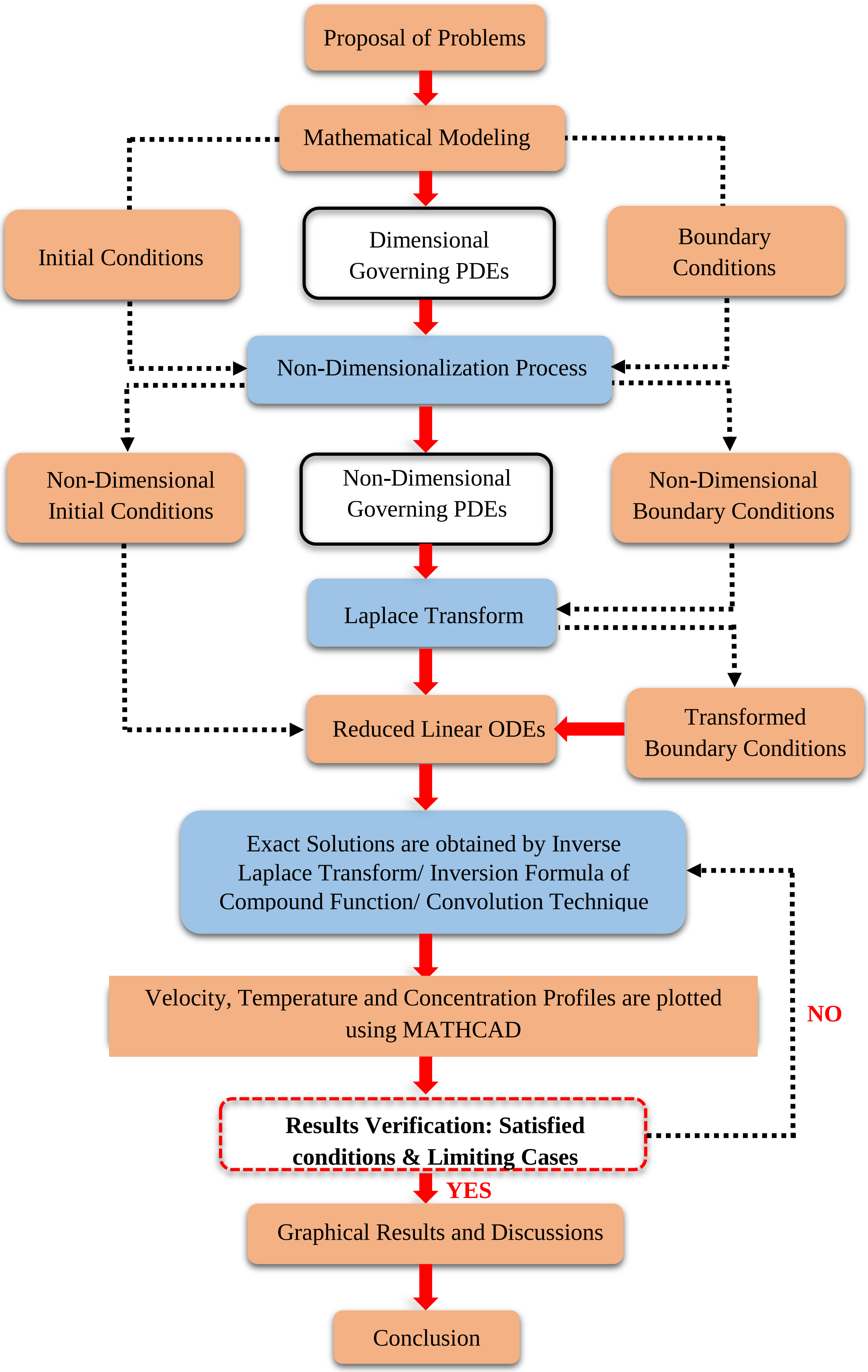

Figure 1.1: Operational Framework

Chapter 4 presents the exact solution for unsteady MHD viscous fluid due to non-coaxial rotation over an isothermal oscillating vertical disk through a porous medium. The dimensional governing equations are reduced to non-dimensional form by using some suitable non-dimensional variables. Then, the expressions of velocity and temperature profiles are obtained by using the Laplace transform method. These profiles are plotted by using the MATHCAD software in order to investigate the behavior of various parameters involved. The comparison of solutions between MHD and without MHD is displayed graphically and discussed in detail in this chapter. The results validation are obtained in two ways, which are by comparing the present solution with published result by Guria *et al.* (2010) and comparing the present exact solution with numerical solution by using Stehfest–Algorithms. The numerical results for skin friction and the Nusselt number are calculated, then tabulated in tables.

Chapter 5 is an extension of work in Chapter 4 by considering mass transfer. In this chapter, the new velocity profiles and concentration profile are obtained by using the same procedure explained in Chapter 4 and discussed via figures. Chapter 6 focuses on the similar problem as in Chapter 4 but considering the second grade fluid. The new expression is used to obtain velocity for comparison with viscous fluid solution in Chapter 4. The exact solutions are also obtained by using the Laplace transform, and the research methodology of this chapter can be referred from Chapter 4.

Chapter 7 presents the extension of the problem reviewed in Chapter 6 by considering mass transfer. Finally, Chapter 8 summarizes this research, inclusive of suggestions for future researches. References and appendixes are listed at the end of this thesis.

REFERENCES

Agrawal, H., Ram, P. and Singh, V. (1983). Combined influence of dissipation and Hall effects on free convective flow in a rotating fluid. *Indian Journal of Pure and Applied Mathematics*. 14(3): 314–321.

Ahmad, I. (2012). Flow induced by non-coaxial rotations of porous disk and a fluid in a porous medium. *African Journal of Mathematics and Computer Science Research*. 5(2): 23–27.

Alfvén, H. (1942). Existence of electromagnetic-hydrodynamic waves. *Nature*. 150(3805): 405.

Ali, F. (2014). *Exact Solutions for Unsteady Flows of Newtonian and Non-newtonian Fluids Using Laplace Transform*. Universiti Teknologi Malaysia: Ph. D. Thesis.

Ali, F., Khan, I., Ul Haq, S. and Shafie, S. (2013). Influence of thermal radiation on unsteady free convection MHD flow of Brinkman type fluid in a porous medium with Newtonian heating. *Mathematical Problems in Engineering*. 2013: 1–13.

Anderson, J. D. and Wendt, J. (1995). *Computational fluid dynamics*. Vol. 206. Springer.

Asghar, S., Hanif, K., Hayat, T. and Khalique, C. (2007). MHD non-Newtonian flow due to non-coaxial rotations of an accelerated disk and a fluid at infinity. *Communications in Nonlinear Science and Numerical Simulation*. 12(4): 465–485.

Baghel, R., Kumar, G. and Sharma, R. (2013). Two dimensional unsteady free convective flow of a viscous incompressible fluid through a rotating porous medium. *Defence Science Journal*. 42(1): 59–62.

Choi, S. U. and Eastman, J. A. (1995). *Enhancing thermal conductivity of fluids with nanoparticles*. Technical Report. Argonne National Lab., IL (United States).

Coleman, B. D. and Noll, W. (1960). An approximation theorem for functionals, with applications in continuum mechanics. *Archive for Rational Mechanics and Analysis*. 6(1): 355–370.

Das, S., Guchhait, S. and Jana, R. (2015). Hall effects on unsteady hydromagnetic flow past an accelerated porous plate in a rotating system. *Journal of Applied Fluid Mechanics*. 8(3): 409–417.

Das, S., Jana, M., Guria, M. and Jana, R. (2008). Unsteady viscous incompressible flow due to an oscillating plate in a rotating fluid. *Journal of Physical Sciences*. 12: 51–64.

Das, S., Jana, M. and Jana, R. (2013). Unsteady hydromagnetic flow due to concentric rotation of eccentric disks. *Journal of Mechanics*. 29(01): 169–176.

Das, S., Jana, M. and Jana, R. (2014). Oscillatory flow due to eccentrically rotating porous disk and a fluid at infinity embedded in a porous medium. *Meccanica*. 49(1): 147–153.

Das, S. and Jana, R. (2014). Hall effects on unsteady hydromagnetic flow induced by an eccentric–concentric rotation of a disk and a fluid at infinity. *Ain Shams Engineering Journal*. 5(4): 1325–1335.

Das, S., Maji, S., Ghara, N. and Jana, R. (2012). Combined effects of Hall currents and slip condition on steady flow of a viscous fluid due to non-coaxial rotation of a porous disk and a fluid at infinity. *Journal of Mechanical Engineering Research*. 4(5): 175–184.

Das, S., Maji, S., Guria, M. and Jana, R. (2009). Unsteady MHD Couette flow in a rotating system. *Mathematical and Computer Modelling*. 50(7): 1211–1217.

Erdogan, M. E. (1997). Unsteady flow of a viscous fluid due to non-coaxial rotations of a disk and a fluid at infinity. *International Journal of Non-Linear Mechanics*. 32(2): 285–290.

Erdogan, M. E. (1999). Flow due to parallel disks rotating about non-coincident axis with one of them oscillating in its plane. *International Journal of Non-Linear Mechanics*. 34(6): 1019–1030.

Ersoy, H. V. (2003). Unsteady viscous flow induced by eccentric-concentric rotation of a disk and the fluid at infinity. *Turkish Journal of Engineering and Environmental Sciences*. 27(2): 115–124.

Ersoy, H. V. (2010). MHD flow of a second order/grade fluid due to noncoaxial rotation of a porous disk and the fluid at infinity. *Math. Comp. Appl*. 15: 354–363.

Ersoy, H. V. (2012). Unsteady flow produced by oscillations of eccentric rotating disks. *Mathematical Problems in Engineering*. 2012: 1–14.

Ersoy, H. V. (2015). Periodic flow due to oscillations of eccentric rotating porous disks. *Advances in Mechanical Engineering*. 7(8): 1–8.

Farhad, A., Norzieha, M., Sharidan, S. and Khan, I. (2012). Hydromagnetic rotating flow in a porous medium with slip condition and Hall current. *International Journal of Physical Sciences*. 7(10): 1540–1548.

Fosdick, R. and Rajagopal, K. (1979). Anomalous features in the model of second order fluids. *Archive for Rational Mechanics and Analysis*. 70(2): 145–152.

Guria, M., Das, S. and Jana, R. (2007). Hall effects on unsteady flow of a viscous fluid due to non-coaxial rotation of a porous disk and a fluid at infinity. *International Journal of Non-Linear Mechanics*. 42(10): 1204–1209.

Guria, M., Das, S., Jana, R. and Ghosh, S. (2009). Oscillatory Couette flow in the presence of an inclined magnetic field. *Meccanica*. 44(5): 555.

Guria, M., Kanch, A., Das, S. and Jana, R. (2010). Effects of Hall current and slip condition on unsteady flow of a viscous fluid due to non-coaxial rotation of a porous disk and a fluid at infinity. *Meccanica*. 45(1): 23–32.

Hayat, T., Asghar, S., Siddiqui, A. and Haroon, T. (2001). Unsteady MHD flow due to non-coaxial rotations of a porous disk and a fluid at infinity. *Acta Mechanica*. 151(1-2): 127–134.

Hayat, T., Ellahi, R. and Asghar, S. (2004a). Unsteady periodic flows of a magnetohydrodynamic fluid due to noncoxial rotations of a porous disk and a fluid at infinity. *Mathematical and Computer Modelling*. 40(1): 173–179.

Hayat, T., Ellahi, R. and Asghar, S. (2007). Unsteady magnetohydrodynamic non-Newtonian flow due to non-coaxial rotations of disk and a fluid at infinity. *Chemical Engineering Communications*. 194(1): 37–49.

Hayat, T., Ellahi, R., Asghar, S. and Siddiqui, A. (2004b). Flow induced by non-coaxial rotation of a porous disk executing non-torsional oscillations and a second grade fluid rotating at infinity. *Applied Mathematical Modelling*. 28(6): 591–605.

Hayat, T., Fetecau, C. and Sajid, M. (2008). Analytic solution for MHD transient rotating flow of a second grade fluid in a porous space. *Nonlinear Analysis: Real World Applications*. 9(4): 1619–1627.

Hayat, T. and Hutter, K. (2004). Rotating flow of a second-order fluid on a porous plate. *International Journal of Non-Linear Mechanics*. 39(5): 767–777.

Hayat, T., Nadeem, S., Asghar, S. and Siddiqui, A. (2005). Effects of Hall current on unsteady flow of a second grade fluid in a rotating system. *Chemical Engineering Communications*. 192(10): 1272–1284.

Hayat, T., Nadeem, S. and Siddiqui, A. (2004c). An oscillating hydromagnetic non-Newtonian flow in a rotating system. *Applied Mathematics Letters*. 17: 609–614.

Hayat, T., Zamurad, M., Asghar, S. and Siddiqui, A. (2003). Magnetohydrodynamic flow due to non-coaxial rotations of a porous oscillating disk and a fluid at infinity. *International Journal of Engineering Science*. 41(11): 1177–1196.

Hussain, S., Jain, J., Seth, G. and Rashidi, M. (2017). Free convective heat transfer with hall effects, heat absorption and chemical reaction over an accelerated moving plate in a rotating system. *Journal of Magnetism and Magnetic Materials*. 422: 112–123.

Hussanan, A., Khan, I. and Shafie, S. (2013). An exact analysis of heat and mass transfer past a vertical plate with Newtonian heating. *Journal of Applied Mathematics*. 2013: 1–9.

Imran, M., Imran, M. and Fetecau, C. (2014). MHD oscillating flows of rotating second grade fluids in a porous medium. *Communications in Numerical Analysis*. 2014: 1–12.

Imran, M., Sarwar, S., Vieru, D. and Nazar, M. (2015). General solution for free convection of viscous fluid near an infinite isothermal vertical plate that applies a shear stress to the rotating fluid. *American Journal of Applied Mathematics*. 3(3-1): 6–13.

Ismail, Z. (2016). *Unsteady MHD Flow of Viscous and Second Grade Fluids in a Porous Medium*. Universiti Teknologi Malaysia: Ph. D. Thesis.

Ismail, Z., Khan, I., Awang, R., Nasir, N. M., Salleh, M. Z. and Shafie, S. (2015a). Rotation effects on unsteady magnetohydrodynamic second grade fluid flow in a porous medium past an infinite inclined plate. *American Institute of Physics Conference Series*. 1643: 555–562.

Ismail, Z., Khan, I., Mohamad, A. and Shafie, S. (2015b). Second grade fluid for rotating MHD of an unsteady free convection flow in a porous medium. *Defect and Diffusion Forum*. 362: 100–107.

Ismail, Z., Khan, I. and Shafie, S. (2013). Rotation and heat absorption effects on unsteady MHD free convection flow in a porous medium past an infinite inclined plate with ramped wall temperature. *Recent Advances in Mathematics*. 161–167.

Jaluria, Y. (1980). *Natural convection: heat and mass transfer*. Vol. 5. Pergamon.

Jana, M., Das, S. and Jana, R. (2012a). Effects of rotation and radiation on the hydrodynamic flow past an impulsively started vertical plate with ramped plate temperature. *International Journal of Applied Information Systems*. 3(4): 39–51.

Jana, M., Das, S. and Jana, R. N. (2012b). Unsteady couette flow through a porous medium in a rotating system. *Open Journal of Fluid Dynamics*. 2(4): 149.

Jat, R. and Jhankal, A. K. (2003). MHD viscous flow through a porous medium past an oscillating plate in a rotating system. *Indian Journal of Engineering and Materials Sciences*. 10(1): 37–40.

Jha, B. and Ajibade, A. (2010). Free convection heat and mass transfer flow in a vertical channel with the Dufour effect. *Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering*. 224(2): 91–101.

Jha, B. K. (1998). Effects of applied magnetic field on transient free-convective flow in a vertical channel. *Indian Journal of Pure and Applied Mathematics*. 29: 441–446.

Jha, B. K. and Ajibade, A. O. (2011). Diffusion-thermo effects on free convective heat and mass transfer flow in a vertical channel with symmetric boundary conditions. *Journal of Heat Transfer*. 133(5): 052502.

Jha, B. K. and Apere, C. A. (2011). Time-dependent MHD Couette flow in a rotating system with suction/injection. *ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik*. 91(10): 832–842.

Jha, B. K. and Apere, C. A. (2012). Time-dependent MHD Couette flow of rotating fluid with Hall and ion-slip currents. *Applied Mathematics and Mechanics*. 33(4): 399–410.

Jiann, L. Y., Ismail, Z., Khan, I. and Shafie, S. (2015). Unsteady magnetohydrodynamics mixed convection flow in a rotating medium with double diffusion. *American Institute of Physics Conference Series*. 1660: 050082.

Khalid, A., Khan, I. and Shafie, S. (2015). Exact solutions for unsteady free convection flow of Casson fluid over an oscillating vertical plate with constant wall temperature. *Abstract and Applied Analysis*. 2015: 1–8.

Khan, A., Khan, I., Ali, F., Shafie, S. et al. (2014a). Effects of wall shear stress on unsteady MHD conjugate flow in a porous medium with ramped wall temperature. *PloS one*. 9(3): e90280.

Khan, I., Ali, F., Norzieha, M. and Sharidan, S. (2010). Exact solutions for accelerated flows of a rotating second grade fluid in a porous medium. *World Applied Sciences Journal (Special Issue of Applied Math)*. 9: 55–68.

Khan, I., Khan, A., Farhad, A., Qasim, M. and Sharidan, S. (2013). Unsteady hydromagnetic rotating flow through an oscillating porous plate embedded in a porous medium. *Mathematical Problems in Engineering*. 2013.

Khan, M., Saeed, S., Fetecau, C. and Azram, M. (2014b). Influence of Hall Current on Oscillatory Couette Flow In The Presence of an Inclined Magnetic Field through Porous Medium. *Journal of Porous Media*. 17(1): 81–92.

Lahurikar, R. (2010). On flow past an impulsively started infinite vertical isothermal plate in a rotating fluid-solution. *Bulletin of the Marathwada Mathematical Society*. 11(1): 41–49.

Maji, S., Ghara, N., Jana, R. and Das, S. (2009). Unsteady MHD flow between two eccentric rotating disks. *Journal of Physical Sciences*. 13: 87–96.

Manna, G., Maji, S., Guria, M. and Jana, R. (2007). Unsteady viscous flow past a flat plate in a rotating system. *Journal of Physical Sciences*. 11: 29–42.

Marneni, N. (2008). Transient free convection flow between tow long vertical parallel plates with constant temperature and mass diffusion. In *Proceedings of the World Congress on Engineering (WCE), London, UK,(11)*.

Marneni, N. and Raghavan, V. R. (2009). Natural Convection Flow in Vertical Channel Due to Ramped Wall Temperature at One Boundary. In *ASME 2009 Heat Transfer Summer Conference collocated with the InterPACK09 and 3rd Energy Sustainability Conferences*. American Society of Mechanical Engineers. 73–80.

Maxwell, J. C. (1873). 'A treatise on electricity and magnetism, Clarendon Press'.

Mohamad, A. Q., Khan, I., Ismail, Z. and Shafie, S. (2014). The unsteady free convection flow of second grade fluid in rotating frame with ramped wall temperature. *American Institute of Physics Conference Series*. 1605: 398–403.

Mohamad, A. Q., Khan, I., Ismail, Z. and Shafie, S. (2016). The unsteady free convection flow of rotating second grade fluid over an oscillating vertical plate. *Jurnal Teknologi*. 78(3-2): 57–63.

Mustapha, N., Amin, N., Chakravarty, S. and Mandal, P. K. (2009). Unsteady magnetohydrodynamic blood flow through irregular multi-stenosed arteries. *Computers in Biology and Medicine*. 39(10): 896–906.

Mustapha, N., Mandal, P. K., Johnston, P. R. and Amin, N. (2010). A numerical simulation of unsteady blood flow through multi-irregular arterial stenoses. *Applied Mathematical Modelling*. 34(6): 1559–1573.

Narahari, M. (2009). Natural convection in unsteady Couette flow between two vertical parallel plates in the presence of constant heat flux and radiation. In *Proceedings of the 11th WSEAS International Conference on Mathematics and Computational Methods in Science and Engineering*. 73–78.

Nishihara, M., Jiang, N., Rich, J. W., Lempert, W. R., Adamovich, I. V. and Gogineni, S. (2005). Low-temperature supersonic boundary layer control using repetitively pulsed magnetohydrodynamic forcing. *Physics of Fluids*. 17(10): 106102.

Ostrach, S. (1952). Laminar natural-convection flow and heat transfer of fluids with and without heat sources in channels with constant wall temperatures. *NASA Report*. 2863.

Ostrach, S. (1954). *Combined natural-and forced-convection laminar flow and heat transfer of fluids with and without heat sources in channels with linearly varying wall temperatures*. Technical Report. National Aeronautics and Space Administration Cleveland OH Glenn Research Center.

Ozerinc S., K. S. and A.G., Y. (2010). Enhanced thermal conductivity of nanofluids: a state-of-the-art review. *Microfluidics and Nanofluidics*. 8(2): 145–170.

Paul, T., Jha, B. and Singh, A. (1996). Transient free convective flow in a vertical channel with constant temperature and constant heat flux on walls. *Heat and Mass Transfer*. 32(1-2): 61–63.

Poggie, J. and Gaitonde, D. V. (2002). Magnetic control of flow past a blunt body: Numerical validation and exploration. *Physics of Fluids*. 14(5): 1720–1731.

Raji, A. W. M. (2010). *Differential Equations for Engineering Students*. Universiti Tun Hussein Onn Malaysia.

Ram, P. and Takhar, H. (1993). MHD free convection from an infinite vertical plate in a rotating fluid with Hall and ionslip currents. *Fluid Dynamics Research*. 11(3): 99–105.

Reddy, G. R. and Reddy, N. B. (2012). MHD oscillatory flow past a vertical porous plate embedded in a rotating porous medium. *Annals of the Faculty of Engineering Hunedoara*. 10(3): 253–258.

R.Lakshmi, M. (2014). Investigation of viscous fluid in a rotating disk. *IOSR Journal of Mathematics*. 10(5): 42–47.

Rossow, V. J. (1958). *On flow of electrically conducting fluids over a flat plate in the presence of a transverse magnetic field*. Technical Report. National Advisory Committee for Aeronautics.

Sahoo, S., Panda, J. and Dash, G. (2010). Hydromagnetic oscillatory flow and heat transfer of a viscous liquid past a vertical porous plate in a rotating medium. *Indian Journal of Science and Technology*. 3(7): 818–822.

Sajid, M., Javed, T. and Hayat, T. (2008). MHD rotating flow of a viscous fluid over a shrinking surface. *Nonlinear Dynamics*. 1(51): 259–265.

Salah, F., Aziz, Z. A. and Ching, D. L. C. (2013). On accelerated MHD flows of second grade fluid in a porous medium and rotating frame. *International Journal of Applied Mathematics*. 43(3): 1–8.

Seth, G., Ansari, M. S. and Nandkeolyar, R. (2011a). Effects of rotation and magnetic field on unsteady Couette flow in a porous channel. *Journal of Applied Fluid Mechanics*. 4: 95–103.

Seth, G., Ansari, M. S. and Nandkeolyar, R. (2011b). Unsteady hydromagnetic couette flow within a porous channel. *Tamkang Journal of Science and Engineering*. 14(1): 7–14.

Seth, G., Bhattacharyya, A. and Tripathi, R. (2017). Effect of Hall Current on MHD Natural Convection Heat and Mass Transfer Flow of Rotating Fluid Past a Vertical Plate with Ramped Wall Temperature. *Frontiers in Heat and Mass Transfer (FHMT)*. 9(1): 1–12.

Seth, G., Kumbhakar, B. and Sharma, R. (2015). Unsteady Hydromagnetic Natural Convection Flow of a Heat Absorbing Fluid within a Rotating Vertical Channel in Porous Medium with Hall Effects.. *Journal of Applied Fluid Mechanics*. 8(4): 767–779.

Seth, G., Nandkeolyar, R. and Ansari, M. (2011c). Effect of rotation on unsteady hydromagnetic natural convection flow past an impulsively moving vertical plate with ramped temperature in a porous medium with thermal diffusion and heat absorption. *International Journal of Applied Mathematics and Mechanics*. 7(21): 52–69.

Seth, G., Nandkeolyar, R. and Ansari, M. S. (2012a). Effects of Hall current and rotation on unsteady MHD Couette flow in the presence of an inclined magnetic field. *Journal of Applied Fluid Mechanics*. 5: 67–74.

Seth, G., Nandkeolyar, R. and Ansari, M. S. (2013). Effects of thermal radiation and rotation on unsteady hydromagnetic free convection flow past an impulsively moving vertical plate with ramped temperature in a porous medium. *Journal of Applied Fluid Mechanics*. 6(1): 27–38.

Seth, G. and Sarkar, S. (2015). MHD natural convection heat and mass transfer flow past a time dependent moving vertical plate with ramped temperature in a rotating medium with Hall effects, radiation and chemical reaction. *Journal of Mechanics*. 31(1): 91–104.

Seth, G., Singh, J. and Mahato, G. (2012b). Effects of Hall current and rotation on unsteady hydromagnetic Couette flow within a porous channel. *International Journal of Applied Mechanics*. 4(02): 1250015.

Seth, G., Tripathi, R. and Sharma, R. (2016). An analysis of MHD natural convection heat and mass transfer flow with Hall effects of a heat absorbing, radiating and rotating fluid over an exponentially accelerated moving vertical plate with ramped temperature. *Bulgarian Chemical Communications*. 48(4): 770–778.

Singh, A. K., Singh, N., Singh, U. and Singh, H. (2009). Convective flow past an accelerated porous plate in rotating system in presence of magnetic field. *International Journal of Heat and Mass Transfer*. 52(13): 3390–3395.

Singh, J. K., Rohidas, P., Joshi, N. and Begum, S. G. (2017). Influence of Hall and ion-slip currents on unsteady MHD free convective flow of a rotating fluid past an oscillating vertical plate. *International Journal of Heat and Technology*. 35(1): 37–52.

Stehfest, H. (1970). Algorithm 368: Numerical inversion of Laplace transforms [D5]. *Communications of the ACM*. 13(1): 47–49.

Vijayalakshmi, A. (2010). Radiation effects on free-convection flow past an impulsively started vertical plate in a rotating fluid. *Theoretical and Applied Mechanics*. 37(2): 79–95.

Villinger, H. (1985). Solving cylindrical geothermal problems using the Gaver-Stehfest inverse Laplace transform. *Geophysics*. 50(10): 1581–1587.

Whitaker, S. (1986). Flow in porous media I: A theoretical derivation of Darcy's law. *Transport in Porous Media*. 1(1): 3–25.

Zafar, A. A., Vieru, D. and Akhtar, S. (2015). Magnetohydrodynamics of rotating fractional second grade fluid in porous medium. *Journal of Prime Research in Mathematics*. 10: 45–58.

Zin, N. A. M., Khan, I. and Shafie, S. (2017). Exact and numerical solutions for unsteady heat and mass transfer problem of Jeffrey fluid with MHD and Newtonian heating effects. *Neural Computing and Applications*. 1–17.