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Unbundling Generators' Reactive Power Output in a Competitive Market by Using Graph and Circuit Theory

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Abstract: This paper suggests a new method to allocate reactive power output of individual reactive power sources to system loads by tracing the current flow. Based on solved load flow and the network parameters, the method converts power injections and line flows into real and imaginary current injections and flows. These currents are then represented independently as real and imaginary current networks. Since current networks are acyclic lossless networks, proportional sharing principle and graph theory is used to trace the relationship between current sources and current sinks. The contributions from each current source are finally translated into reactive power contributions. The IEEE 14-bus system is used to illustrate the effectiveness of the method. Comparison of the results with previous methods is also given.

Keywords: Ancillary services, Directed graphs, Energy management, Load flow analysis, Power flow tracing.

1. INTRODUCTION

The traditional monopolistic nature of power systems is changing into deregulated systems in many countries. The aim is to optimize the system welfare by introducing a competitive environment among participants. The transmission open access attracts large number of market players and hence increased the volume and number of energy transactions [1]. As markets develop in managing the energy transactions, it does not take into account the network losses and system security. System operator is responsible in keeping the system normal by obtaining some of the resources from the industry participants in the form of ancillary services. The reactive support provided by generators is one of the six ancillary services specified in the FERC Order no. 888 [2].

Reactive power support can be provided by a number of means such as shunt capacitors, synchronous condensers, static var compensators (SVC) and synchronous generators. Because reactive power does not travel very far, it is usually necessary to produce reactive power close to where it is needed [3]. Thus, the opportunity for market power arises as a result of the limited number of potential suppliers. Moreover these technical characteristics can significantly increase the price of reactive power support.

Developing fair and equitable reactive power allocation method has been an active topic of research particularly in the new paradigm with many transactions in place at any time. Besides, due to non linear nature of power flow, it is difficult to evaluate reactive support allocations accurately. Therefore it required to use approximate models, tracing algorithms or circuit theory for reactive power allocation. The methods that employ circuit theory mostly use adjustments and partitioning of system Y-bus matrix [3-5]. The tracing methods are based on the actual power flows in the network and the proportional sharing principle. To date, several tracing algorithms have been proposed in the literature [6-14].

A novel tracing method is presented in [6-8]. But, even though the approach is conceptually very simple, it requires inverting a sparse matrix of the rank at least equal to the number of network nodes. Moreover it considers transmission losses by introducing fictitious nodes on every branch and therefore the calculation becomes very complex and time consuming for large systems. In [9-10] graph theory is applied to trace active or reactive power and it is limited to systems without loop flows and losses. The method reported in [11] is based on clustering the network into small groups of buses which are classified as Commons, i.e. a set of buses supplied by same set of generators. The obtained clusters are considered as new buses, connected together with tie lines. The disadvantage of this method is that the share of each generator in each Common is assumed to be the same. In general all the above mentioned tracing methods [6-11] are most appropriate for active power flow tracing rather than reactive power tracing.

Nodal generation distribution factor (NGDF) [12] for active and reactive power allocation is based on time consuming search algorithm. In order to overcome the difficulties arise in reactive power tracing due to interaction cause by real power and losses, [13] traces active and reactive power using real and imaginary currents respectively. This technique automatically becomes lossless real and imaginary current networks but lumping several devices and shunt branch capacitances together may leads to current loops and poor representation of transmission lines. Reference [14], proved that real and imaginary current networks are acyclic directed graphs.

The above mentioned disadvantages in tracing algorithms have been the reason for developing a new method to know how much, and to what extent, each reactive power generator supplies to each load and network losses. The algorithm uses the advantages of real and imaginary current networks along with the basic concept of graph theory. Starting from load flow solutions, it first decomposes line complex currents based on the proportion of generator and network injected currents. The amount of current attributed from each current source in the lines and to each current sink is then used to allocate the reactive power support provided by each generator. Shunt elements are handled by introducing additional fictitious nodes.

2. THE PROPOSED METHOD

Reference [9-10] reports a power flow tracing algorithm using graph theory which can only apply to systems without losses and loop flows. Detecting and solving the loop flows is a prerequisite to this method which is not easy especially when loops have complicated paths. To avoid these limitations, this paper suggests a new method to handle loop flows and form lossless network.

In the previous section, the paper has unveiled that real and imaginary current networks are lossless networks without loops [13-14]. Therefore these current network properties makes it [9] very suitable to trace the contribution of current sources to line flows and to current sinks or visa versa. Moreover, generators shunt elements and loads are considered independently instead of a net generator or a net load bus as in the original algorithm [9].

2.1 Handling Shunt Elements

In a power system, generator and loads are not the only sources and/or sinks of complex power. SVCs, transformers, shunt capacitors/reactors and line charging capacitances play a vital role in transferring reactive power between suppliers and consumers. In order to assess possible contributions from these shunt elements, it is necessary to consider the amount of current injected or absorbed by these equivalent shunt impedance seen at each bus. These shunt currents can be handled by introducing fictitious lines and treated as current sources or sinks at additional nodes as shown in Figure 1. To incorporate the effect of parallel capacitance of transmission lines, the most widely used π equivalent line model in load flow studies is considered. Figure 2 depicts the transmission line model with current injections at added nodes.

2.2 Current Flow Networks

Starting from AC power flow solution one can convert the complex power injections and line flows into complex current equivalents. Injected currents, line currents and currents due to shunt elements can be represented respectively as:

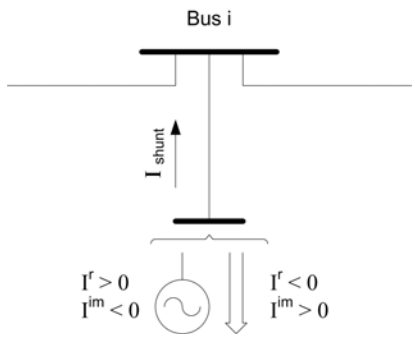

Figure 1. Representation of equivalent shunt current injection at bus i

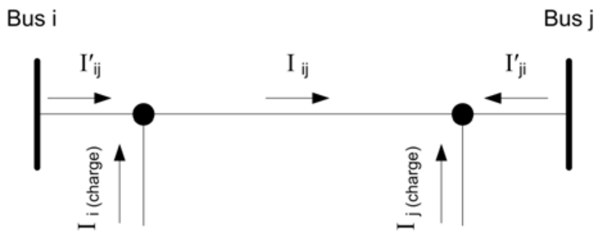

Figure 2. π equivalent of a transmission line

$$I_{inj} = \left(\frac{S_i}{V_i}\right)^* \quad (1)$$

$$I_{ij} = y_{ij}(V_i - V_j) \quad (2)$$

$$I_{i\_sh} = y_{i\_sh}(V_i) \quad (3)$$

where

I_{inj} = Injected current of bus *i*,
S_i = Injected power of bus *i*,
V_i = Voltage of bus *i*,
V_j = Voltage of bus *j*,
y_{i_sh} = Equivalent shunt admittance at bus *i*,
I_{i_sh} = Current flow through y_{i_sh},
I_{ij} = Line current from bus *i* to bus *j*,
y_{ij} = Series admittance of the line l_{ij} between buses *i* and *j*

The current $I_{i(\text{charge})}$ entered from parallel capacitors at bus i can be obtained from Equation (3) just by replacing y_{i_sh} with the admittance corresponding to half of the parallel capacitance of l_{ij}.

The complex current flow network obtained from Equations (1) to (3) can be further decoupled into real and imaginary current networks. These networks can then be used to estimate the relationship between the current sources and the current sinks using modified graph theory procedure in [9]. Details of current source and current sinks are found on [13].

2.3 The Graph Method

This method assumes that a generator has the priority to provide power to the load on the same bus and is based on the following lemmas of graph theory.

Lemma 1: A lossless, finite-nodes power system without loop flow has at least one pure source, i.e. a generator bus with all incident lines carrying outflows.

Lemma 2: A lossless, finite-nodes power system without loop flow has at least one pure sink, i.e. a load bus with all incident lines carrying inflows.

Based on these two lemmas downstream tracing sequence briefly describes the method. The downstream tracing (DSTR) is used for calculating the contribution factors of individual generators to line flows and loads. This process initially requires the formation of intermediate matrices called extraction factor matrix of lines, $\boldsymbol{A}_l$ and loads $\boldsymbol{A}_L$ from total passing power of their upstream buses respectively. The relationships involved in extraction factor matrices can be written as:

$$\boldsymbol{P}_l = \boldsymbol{A}_l.\boldsymbol{P} \tag{4}$$

$$\boldsymbol{P}_L = \boldsymbol{A}_L.\boldsymbol{P} \tag{5}$$

where

$\boldsymbol{P}_l$ = vector of line power,

$\boldsymbol{P}_L$ = vector of load power, and

$\boldsymbol{P}$ = vector of bus total passing power in the bus sequence of downstream tracing

Then the nonzero elements in $\boldsymbol{A}_l$ and $\boldsymbol{A}_L$ are calculated with the following equations:

$$\left(\boldsymbol{A}_l\right)_{\text{line j, bus i}} = \frac{\text{line j's power flow}}{\text{bus i's total pass power } P_i} \tag{6}$$

$$\boldsymbol{A}_{L_{ii}} = \begin{cases} 0 & \text{i} \notin \text{net load buses} \\ \dfrac{\text{net load power on bus i}}{\text{P}_\text{i}} & \text{i} \in \text{net load buses} \end{cases} \tag{7}$$

The next step involves the calculation of contribution factor matrix, $\boldsymbol{B}$ of generators to bus total passing power. Mathematically this can be expressed as:

$$\boldsymbol{P} = \boldsymbol{B}.\boldsymbol{P}_G \tag{8}$$

The elements of $\boldsymbol{B}$ are calculated using the equation given below:

$$\boldsymbol{B}_{i,k} = \begin{cases} 1 & (\text{k} = \text{i}, \text{k} \in \text{net gen. buses}) \\ 0 & (\text{k} = \text{i}, \text{k} \notin \text{net gen. buses}) \\ 0 & (\text{k} > \text{i}) \\ 0 & (\text{k} < \text{i}, \text{k} \notin \text{net gen. buses}) \\ \sum\limits_{l_j \in \text{i}} \left(\boldsymbol{A}_{l_{j-m}} \cdot \boldsymbol{B}_{\text{m-k}}\right) & (\text{k} < \text{i}, \text{k} \in \text{net gen. buses}) \end{cases} \tag{9}$$

where $k< i$ means k is an upstream bus of bus i, and $k> i$ means k is a downstream bus of bus i. The last expression is for the lower triangular nonzero elements. The term $l_{j\in i}$ means line j is an inflow line of bus i. $A_{l_{j-m}}$ is the unique nonzero element corresponding to line j in matrix $\boldsymbol{A}_l$ with bus m as its upstream terminal. B_{m-k} is the element in matrix $\boldsymbol{B}$ already calculated which represents the contribution of generator k to the total injection power of bus.

By substituting Equation (8) in Equations (4) and (5), contribution of each generator to line flows and loads can be calculated. The exact derivations can be found in [9].

2.4 Unbundling Generators' Reactive Power

The output of tracing procedure apportions real and reactive current sources to line currents and to each current sink within their respective real and imaginary current networks. Then the complex current contributed by each current source k to each sink i is simply:

$$I_k^i = \left(I_k^{i\_r} + jI_k^{i\_im}\right) \tag{10}$$

where I_k^i is the complex current of source k attributed to sink i. $I_k^{i\_r}$ and $I_k^{i\_im}$ are real and imaginary component of I_k^i respectively. Then the reactive power share of each current source to each current sink Q_k^i can be represented as:

$$Q_k^i = \text{Im}\left\{V_k\left(I_k^i\right)^*\right\} \tag{11}$$

Finally the total reactive power generation Q_k of each current source k can be expressed as:

$$Q_k = \text{Im}\sum_{i=1}^{inj} V_k\left(I_k^i\right)^* \tag{12}$$

where the term $\left(I_k^i\right)^*$ means the conjugate of I_k^i. Superscript *inj* represents the total number of current sinks supplied by source k.

Equation (12) shows the implicit contribution of a particular reactive power source to all current sinks including network sinks and loads. The next step consists of evaluating how much each reactive power source contributes to each system load and to network sinks. For this purpose the following derivation is used.

Splitting Equation (12) into number of load sinks, *nl* and remaining current sinks defined as network sinks, *ns*:

$$
\begin{aligned}
Q_k &= \operatorname{Im}\sum_{l=1}^{nl} V_k\left(I_k^l\right)^* + \operatorname{Im}\sum_{m=1}^{ns} V_k\left(I_k^m\right)^* \\
&= \sum_{l=1}^{nl} Q_k^l + \sum_{m=1}^{ns} Q_k^m
\end{aligned} \tag{13}
$$

where I_k^l and Q_k^l are the complex current share from source k to load l and component of Q_k due I_k^l to respectively. Similarly I_k^m and Q_k^m represents the complex current share to network sink m and component of Q_k due to I_k^m respectively.

Note that in Equation (13), source k may not necessarily be a physical reactive power source but it can be a reactive power contributor of the network. This consideration is realistic because unlike real power, the network not only absorbs reactive power but it can also supply reactive power to the system.

3. RESULTS AND ANALYSIS

A number of simulations have been carried out to demonstrate functionality of the method. However, only the result of a 4-bus test system and the IEEE 14-bus system is illustrated. A comparison with the methods suggested by Bialek [6] and Kirschen [13] is also made to show the validity and accuracy of the proposed method.

3.1 Simple Test System Analysis

Figure 3 shows the single line diagram of the 4-bus test system. There are two generators at bus 1 and bus 4 and two loads at bus 2 and bus 3 in this system. The computed load flow is also indicated in this figure. Using load flow solution and system data listed in Table 1, the equivalent real and imaginary current networks is obtained.

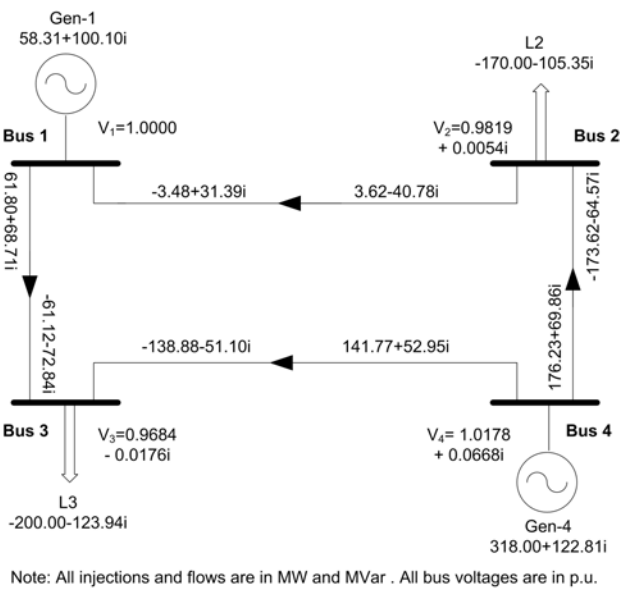

Figure 3. Four bus test system

Table 1. The Branch Data of the 4-Bus Test System

Line	Series Z		Shunt Y
bus to bus	R (p.u)	X (p.u)	Y/2 (p.u)
1--2	0.01008	0.0504	0.05125
1--3	0.00744	0.0372	0.03875
2--4	0.00744	0.0372	0.03875
3--4	0.01272	0.0636	0.06375

Figure 4 and 5 depicts the obtained real and imaginary current network respectively. All numerical values, line flow directions, current sources and sinks are also shown in Figure 4 and 5. Note that in these equivalent networks there can be current sources other than physical devices. The distribution of real and imaginary current from each current source to each line flow and sink can now be easily obtain by adopting the tracing procedure [9].

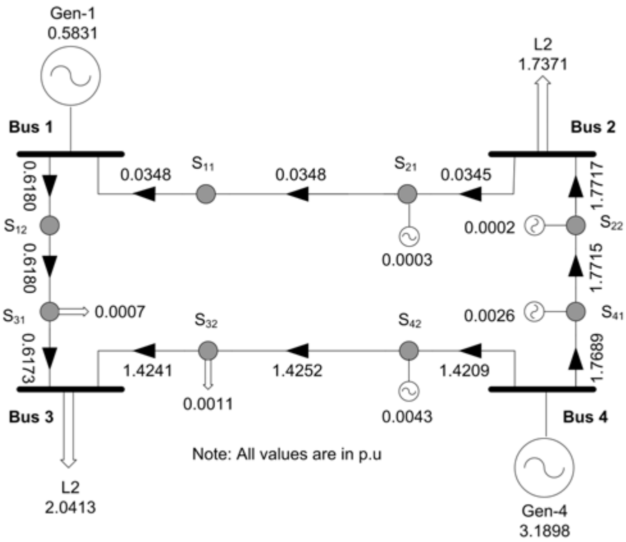

Figure 4. The equivalent real current network of the 4-bus test system

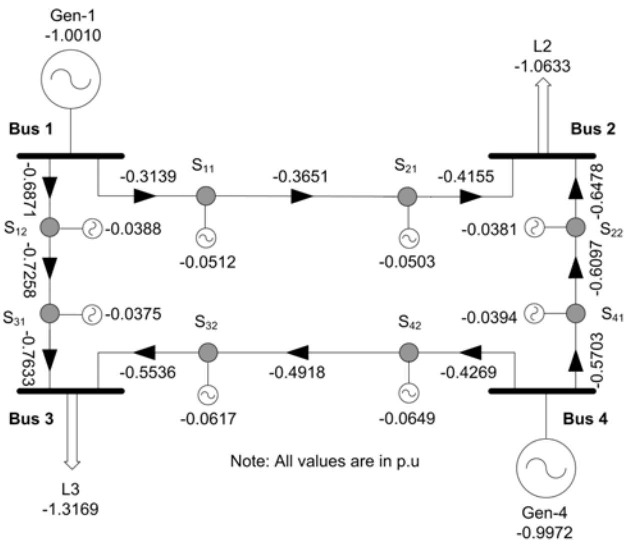

Figure 5. The equivalent imaginary current network of the 4-bus test system

The complex current contributed by the physical reactive power sources (in these case generators at buses 1 and 4) to each current sink are listed in Table 2. Finally adopting Equation (13), reactive power contribution from these two sources to system loads and network sinks are given in Table 3.

Table 2. The Contribution of Generators Current to Sinks in the 4-Bus Test System

Sink	Supplied by	
	Gen-1	Gen-4
L2	0 - 0.3139i	1.7344 - 0.5703i
L3	0.5825 - 0.6871i	1.4543 - 0.4269i
S_{31}	0.0006+0i	0.0001+0i
S_{32}	0	0.0011+0i
Total	0.5831-1.0010i	3.1899-0.9972i

Table 3. The Reactive Power Allocation Result of the 4-Bus Test

Sink	Supplied by	
	Gen-1 (MVAr)	Gen-4 (MVAr)
L2	31.388	69.631
L3	68.707	53.169
S_{31}	0	0.007
S_{32}	0	0.001
Total	100.095	122.808

3.2 IEEE 14-Bus System Analysis

Computer simulation is also conducted on IEEE 14-bus system. Load flow data of the IEEE 14-bus system is given in Table 4.

Table 4. Bus Data for the IEEE 14-Bus System

Bus	Voltage		Generation		Load		Shunt
no.	Mag	Angle	P	Q	P	Q	Suceptance
	(p.u)	(deg)	(MW)	(MVAr)	(MW)	(MVAr)	(p.u)
1	1.06	0.00	232.39	-16.89	0.00	0.00	0
2	1.05	-4.98	40.00	42.40	21.70	12.70	0
3	1.01	-12.72	0.00	23.39	94.20	19.00	0
4	1.02	-10.32	0.00	0.00	47.80	-3.90	0
5	1.02	-8.78	0.00	0.00	7.60	1.60	0
6	1.07	-14.22	0.00	12.24	11.20	7.50	0
7	1.06	-13.37	0.00	0.00	0.00	0.00	0
8	1.09	-13.37	0.00	17.36	0.00	0.00	0
9	1.06	-14.95	0.00	0.00	29.50	16.60	19.00
10	1.05	-15.10	0.00	0.00	9.00	5.80	0
11	1.06	-14.80	0.00	0.00	3.50	1.80	0
12	1.06	-15.08	0.00	0.00	6.10	1.60	0
13	1.05	-15.16	0.00	0.00	13.50	5.80	0
14	1.04	-16.04	0.00	0.00	14.90	5.00	0

The reactive power distributed from each generator to system loads and network sinks in the IEEE 14-bus system is listed in Table 5. Observe that the sum of the reactive power contributed by each generator is in conformity with the solved load flow. From Table 5, it can also be seen that some of the values attributed to reactive power sinks are negative. This implies that the method takes into consideration the interaction between real and reactive power flows. For example generator at bus 2 contributes -0.046 MVAr to reactive load at bus 6. In this case, generator at bus 2 requires more reactive power to be supplied from other sources, preferably from the reactive power sources near load at bus 6, in transferring its real power to load at bus 6. Moreover, the acquired result in Table 5 illustrates that the contribution of individual generators are mostly confined in their neighbourhood.

Table 5. The Reactive Power Allocation Result of the IEEE 14-Bus System

Load	Supplied by			
bus no.	Gen-2	SVC-3	SVC-6	SVC-8
1	0	0	0	0
2	14.143	0	0	0
3	8.504	22.260	0	0
4	0	0	0	0
5	0.919	0	0	0.055
6	-0.046	0	3.739	0
7	0	0	0	0
8	0	0	0	0
9	-0.241	0	0	8.155
10	-0.056	0	0.934	1.901
11	-0.015	0	0.995	0
12	-0.028	0	1.185	0
13	-0.060	0	3.469	0
14	-0.107	0	1.180	2.061
net-sinks	19.385	1.1338	0.739	5.184
Total	42.396	23.3936	12.240	17.357

Figure 6 shows the reactive power contribution of SVC at bus 6 obtained through alternative methods. The proposed method and the Bialek's tracing method assign the reactive power of SVC at bus 6 to only 6 loads while the Kirschen method allocates the power to a total of 7 loads. In general, Bialek's tracing method assigns only positive values for reactive power while the proposed and Kirschen's method may also allocate negative values. This difference may be due to negligence of real power flow in dealing with reactive power and the way the Bialek's method handles the system losses. From Figure 6, the main difference between the proposed method and Kirschen's method arises mainly in the number of supplied loads. This is due to the effect of lumping several elements such as parallel capacitance of transmission lines together with bus injection as suggested by Kirschen [13]. The approach used in this paper, adopts detail transmission line model and better current tracing approach for allocating reactive output of individual generator.

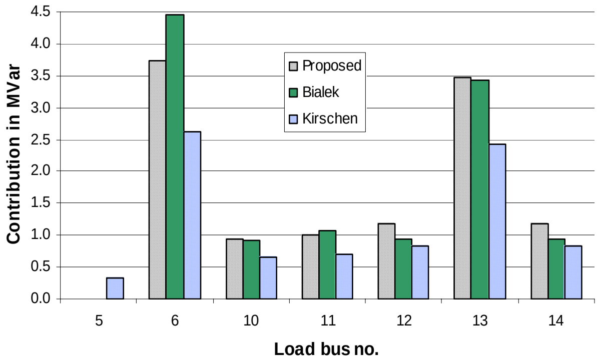

Figure 6. Distribution of reactive power from the SVC at bus 6 in the IEEE 14-bus system

4. CONCLUSIONS

In this paper, a new reactive power allocation method is proposed. Instead of power tracing, the algorithm traces real and imaginary currents to handle the problem of system losses and loop flows. The traces from current sources to current sinks are then converted to power contributions. The main advantages of converting lossy power flow networks into current flow networks is that all currents injected by the sources are completely absorbed by the sinks in the system. No current is lost in transmission lines although power loss is possible due to transmission line impedances. Moreover, by proper modelling of system elements as proposed in the paper, current flow networks are free from circulating (loop) currents. Circulating flows sometimes exits in the power flow networks.

The proposed method has been tested on a 4-bus test system and also on the IEEE 14-bus system. The test result is presented to illustrate simplicity and veracity of the method. The comparison with the other allocation methods shows that the proposed method is fair and accurate in allocating reactive power to the loads.

The method could be used to resolve some of the difficult reactive power pricing and costing issues which arise from the introduction of competition in the power industry and to ensure fairness and transparency.

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