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Speed Effect to a Quarter Car ARX Model Based on System Identification

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***Abstract*—This paper presents the effect of car speeds on a quarter car passive suspension system model dynamics. The model is identified using system identification technique, in which the input-output data are collected by running a test car on an artificial road surface with two different speeds i.e., 10 km/h and 20 km/h. The quarter car passive suspension system dynamics is assumed to have an ARX model structure and identified using linear least-square estimation algorithm. The car vertical body acceleration, which is the output variable, is measured by installing an accelerometer sensor on the car body, above the suspension. On the other hand, the car shaft acceleration, which is the input variable, is measured by installing an accelerometer sensor at the lower arm of the car suspension. The best model for the 10 km/h car speed gives the output order (n_a) = 4, the input order (n_b) = 2, delay (*d*) = 1, the best fit = 90.65%, and the Akaike's Final Prediction Error (FPE) = 5.315e-06. In contrast, the 20 km/h speed results in 4th output order (n_a), 1st the input order (n_b), 1st delay (*d*), the best fit of 91.05%, and 7.503e-05 Akaike's FPE. These results show that the higher speed reduces the effect of the road surface to car dynamics, which is indicated by the order of the model.**

***Keywords*—a quarter car passive suspension system dynamics; speed; running test; artificial road surface; ARX model**

I. INTRODUCTION

The suspension system is the main part of a ground vehicle, which consists of a coil spring and a damper (shock absorber). When the vehicle drives over a bumpy road, the combination of a damper and coil spring that are mounted on each wheel will be absorbed the shocks. The spring part has a function to store energy and will supply it later on. The spring makes the vehicle wheel to move up and down with respect to the frame of the vehicle, while the damper softens the suspension moves entailed by the spring by absorbing the shocks. The damper is built from a steel or aluminum hydraulic cylinder filled with oil and pressurized with nitrogen. When the suspension moves, a piston is forced to move through the oil-filled cylinder. The energy produced by the motion of the piston is dissipated as heat which in turn is absorbed by the oil.

Suspension systems affect the handling and riding quality of cars, so they have been of great interest to both academia and industry [1]. The riding problem arises mainly from vehicle vibration. The major source of the vehicle vibration is road surface irregularities or roughness [2], [3]. The vehicle suspension systems receive the vibration from tire, transfer it into the vehicle body and finally to the passenger.

The purposes of the suspension system are to support the vehicle weight, keep the wheels on the ground, minimize the transient force to the body, maintain good ride comfort, and enhance handling performance [4]. Thus, the most vital factor to improve handling and comfort of driver and passengers is by improving the vehicle suspension system [5]. One way to improve the suspension system is by designing its controller [1], [5]-[7].

In order to design the controller for a vehicle suspension system, a mathematical model of the system is needed. The best way to determine the mathematical model is by using system identification. System identification is an art of modeling. Its basic philosophy is to form a good mathematical model to fit input-output data of a system instead of building a model from physical laws, although the physical structures do provide some guidance in choosing model structures and experiment designs [8]. One of the well-known system identification algorithms is using the least-squares estimation method, and it has been applied in this works.

A quarter-car model is an easy way to understand and represent the vehicle suspension system dynamic. A quarter car model describes the vibration transmission from the road surface to the vehicle body, and then to the passenger. Therefore, a quarter car model offers a quite reasonable representation of the actual suspension system dynamic [9]. In this paper, a quarter car is assumed to have an Autoregressive Exogenous (ARX) model structure and used to be a candidate model of the suspension system. The ARX model is assumed as a single input and single output (SISO) model.

The car input-output data used to identify the ARX model are measured by running a test car on an artificial road with two different speeds, at 10 km/h and 20km/h.

II. MATERIAL AND METHOD

A model represents essential aspects of a system with respect to certain purposes and takes on several different forms such as:

- Cognitive models are the conceptual aspect of models underlying human reasoning and perception, inductive learning, decision-making, and planning.
- Normative models which define the specified or desired function, goal, or purpose of system or process. Such models are often found in engineering design and government regulation.
- Descriptive and functional models are used for scientific and technology purpose. Such models are often subdivided into quantitative models (described by number or parameters) and qualitative models (described by categorical data).

Based on the above definition, the system dynamic can be described as two models. First, the physical model is represented by the kinetic motion and secondly, the mathematical model to represent the equation of the system kinetic. The system mathematical model is usually obtained from the system physical model. The system physical and mathematical model is a part of descriptive and functional model categories and is used in control engineering.

The vehicle suspension system can divide to be three types, they are:

1) Dependent Type: For dependent suspension type, there is a rigid linkage between the two wheels of the same axle. Therefore a force acting on one wheel side will affect the opposite wheel. For each wheel motion that caused by road, irregularities will affect the coupled wheel as well. This suspension type can bear shocks with a great capacity than independent suspension.

2) Semi-independent Type: This kind of suspension type has both the characteristics of a dependent as well as independent suspension. For this type, the wheel move relative to one another as in independent suspension but the position of one wheel has some effect on the other wheel. This is done with the help of twisting suspension parts.

3) Independent Type: The independent suspension means that the suspension is set-up in such a way that allows the wheel on the left and right side of the vehicle to move vertically independent up and down while driving on an uneven surface. A force acting on the single wheel does not affect the other as there is no mechanical linkage present between the two hubs of the same vehicle. This types of suspension usually offer better ride quality and handling due to less unsprung weight. The main advantages of independent suspension are that they require less space and provide easier steerability, low weight, etc.

Based on category, the car suspension system can be divided into three; they are: passive, semi-active and active. The preliminary category is a passive suspension system. Most of the cars now still use this category of the suspension system. Passive suspension system contains springs or coil (restoring) element and damper or absorber (dissipative) element. In a passive suspension system, there is no external power added in order to control the system dynamics. The main feature of the suspension system is the cyclic interchange of kinetic and potential energy with some energy dissipation in damping devices.

Nowadays, the independent suspension type widely used in a single car. Why a quarter model reasonable uses to represent the vibration transmission mechanism from road surface irregularities to the vehicle body and the end to the passenger. Fig. 1 shows a quarter passive suspension system dynamic model.

The ARX model of the quarter car passive suspension dynamic system is derived based on the dynamic model shown in Fig. 1, where M_s is the sprung mass, Z_s is sprung mass position, K_s is the suspension spring element, C_s is the suspension damping element and Z_o is the unsprung mass position. Applying Newton's Second Law, the equation of the quarter car is,

$$M_s\ddot{Z}_s = C_s(\dot{Z}_o - \dot{Z}_s) + K_s(Z_o - Z_s) \qquad (1)$$

The input and output variables are sprung mass vertical acceleration and unsprung mass vertical acceleration, respectively. By differentiating equation (1) two times and by assuming that the output variable is $y = \frac{d^2 z_s}{dt^2} = \ddot{Z}_s$ and the input variable is $u = \frac{d^2 z_o}{dt^2} = \ddot{Z}_o$, the following equation is obtained.

$$M_s\ddot{y} + C_s\dot{y} + K_s y = C_s\dot{u} + K_s u \qquad (2)$$

Converting equation (2) into discrete form and assumed that the current value of input variable is avoided, the following equation is obtained.

$$y(kT) = -a_1 y(k-1)T) - a_2 y(k-2)T) + b_1 u(k-1)T) \qquad (3)$$

Note that the structure of equation (3) represents the ARX model.

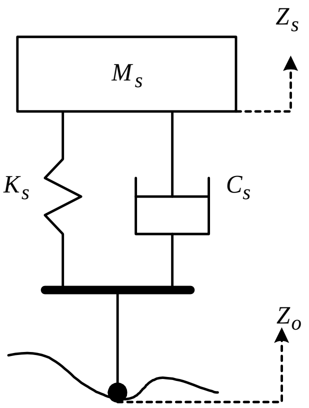


Fig. 1 A quarter car dynamic model

III. RESULT AND DISCUSSION

A. *Result*

Normally, the mathematical model of a system can be derived by applying some physical law. The method used a tailored model description that yields a system of ordinary differential equation. Since not all the coefficients of the differential equations is readily available, especially when the system is complex. Therefore, a procedure for calibrating the parameters of the virtual prototype is needed. For use on a test bench, the desired model parameters need to be adjusted as the testing proceeds. The system identification technique can be achieved efficiently by means of a mathematical optimization algorithm.

System identification is the process of the deriving mathematical system model of a system from observed data in accordance with some predetermined criterion. The term system identification was coined by Lotfi Zadeh in 1962 [10] and decried system identification as:

"*Identification is the determination on the basis of input and output of a system within a specified class of systems to which the system under test is equivalent*."

This definition is of course highly systems oriented, the term caught on and soon become standard terminology in the control community. The system identification block diagram is as bellow:

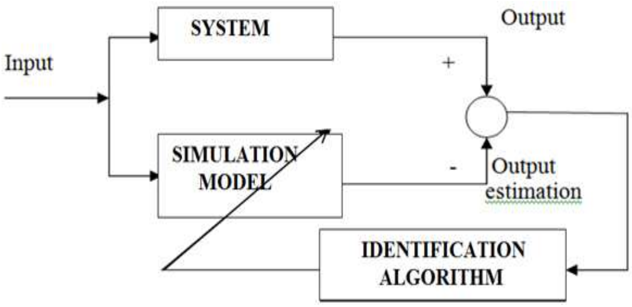


Fig. 2 System identification block diagram

The determination of the model of a dynamical system from observed input-output data using system identification procedure entails the following three steps:

1) *Experiment Design:* Its purpose is to obtain a good experimental data, and it includes the choice of the measured variable and of the character of the input signal.

2) *Selection of Model Structure:* A suitable model structure is chosen using prior knowledge and trial and error.

3) *Choice of the Criterion to Fit:* A suitable cost function is chosen, which reflects how well the model fits the experimental data.

4) *Parameter Estimation:* An optimization problem is solved to obtain the numerical values of the model parameters.

5) *Model Validation:* The model is tested in order to reveal any inadequacies.

System identification is the field of the modeling dynamic system from experimental data, and it is a fundamental problem in system theory. The area of system identification has greatly matured over the last two decades [8]. However, a majority of effort has been directed towards technique of linear system.

As mention previous, a quarter passive suspension system is assumed has an ARX model structure. An ARX model has been widely used in engineering fields to model dynamic response of a system. General form of ARX model is written as,

$$y(kT) = -a_1 y(k-1)T) - a_{n_a} y(k-n_a)T) + \\ b_1 u(k-1)T) + \cdots + b_{n_b} u(k-d-n_b)T) \tag{4}$$

where, n_a and n_b are output variable and input variable orders respectively and, d is delay time.

The best parameter estimation using least square estimation is given by,

$$\hat{\Theta}_N = \left[\frac{1}{N}\sum_{t=1}^{N} \varphi(t)\varphi^T(t)\right]^{-1} \left[\frac{1}{N}\sum_{t=1}^{N} \varphi(t)\varphi^T(t)\right] \tag{5}$$

where,

$\varphi(t) = [-y(k-1)T) \ldots - y(k-n_a)T)\ u(k-1)T) + \cdots + u(k-d$

$\Theta^T = [a_1 \ldots a_{n_a}\ \ b_1 \ldots\ b_{n_b}]$

B. *Input-Output Data Collection*

In system identification, the system model is identified based on the input-output data of the system collected through experiments or real test system. Then the data pre-processing is performed to determine whether the data is informative or not [11], [12]. To get informative data, in this research the data collection process is done experimentally, the high resolution and accuracy data acquisition hardware and sensors are used.

Data acquisition serves the purposes are an analysis of the logged data and the improvement of the object of measurements. A data acquisition system is an instrument that has the capability measuring some parameters [13]. This system generally electronics based, and it is made of hardware and software. For the hardware, part consists of sensors, cables, and electronic components. Meanwhile, the software part is made of data acquisition logic or the analysis software and other utilities that can be used to configure the logic or to transfer data from data acquisition to laptop or a data recording device. In this research, data logging, carried out by the data acquisition, used to measure the sprung mass and unsprung mass vertical acceleration.

The car sprung mass vertical acceleration and unsprung mass vertical acceleration are the output and input variables, respectively, which are measured using accelerometers installed on a test car, which runs on an artificial road surface. The sprung mass vertical acceleration data are measured by installing an accelerometer sensor at the top of suspension on the vehicle body as shown in Fig. 3. While the unsprung mass vertical acceleration data are recorded using an accelerometer sensor that is installed at vehicle axle as represented in Fig. 4. Fig. 5 illustrates the experimental data collection process.

The test car used in this research is a light passenger car. The following table elaborates the test car specification.

TABLE I
SPECIFICATION OF TEST CAR

Component	**Specification**
Year	2010
Front suspension type	MacPherson Strut & coil spring
Rear suspension type	Multilink & coil spring
Front tire pressure	22 psi
Rear tire pressure	20 psi

Artificial road surfaces are made of plywood and the beam woods. The dimension of the plywood used to build the artificial road surface and the bump cover is represented by Fig. 5. Fig. 6 shows the three types of beam wood dimension, while the full assembly of the artificial road surface and the test process are depicted in Fig. 7. A different dimension of beam woods is placed on the plywood with random distance. In order to the right and left sides of car suspensions sense the same vibrations, two artificial road surfaces are made identical.

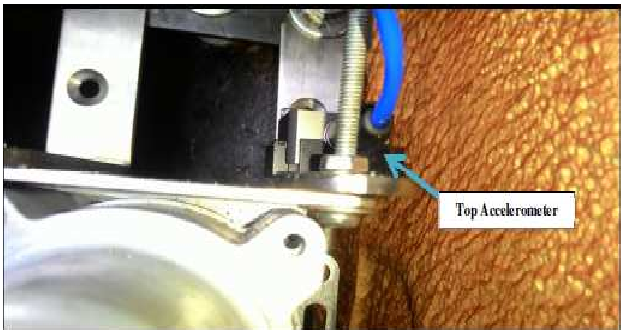


Fig. 3 Accelerometer sensor installation for vertical body Acceleration

Fig. 4 Accelerometer sensor installation for vertical shaft acceleration

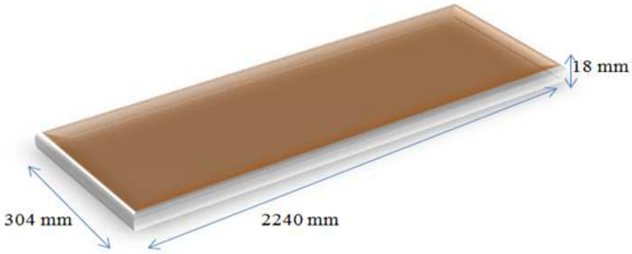


Fig. 5. The plywood dimension

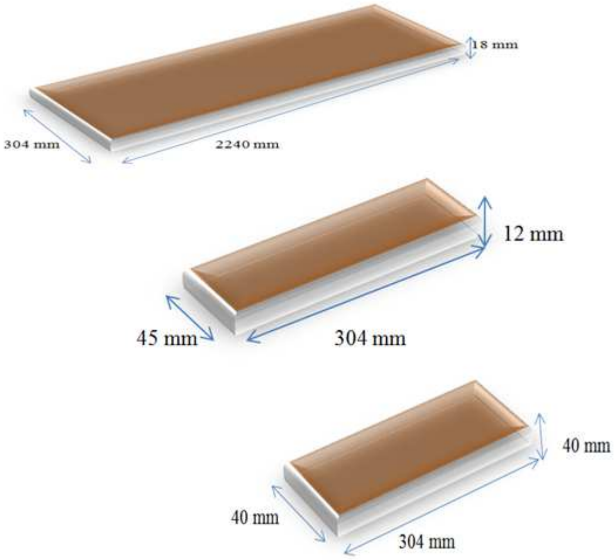


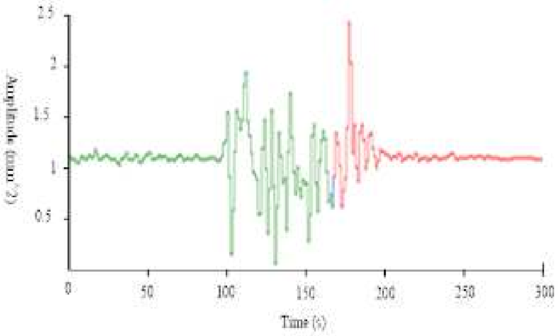
Fig. 6 Dimension of the three types of beam

Fig. 7 Running test car

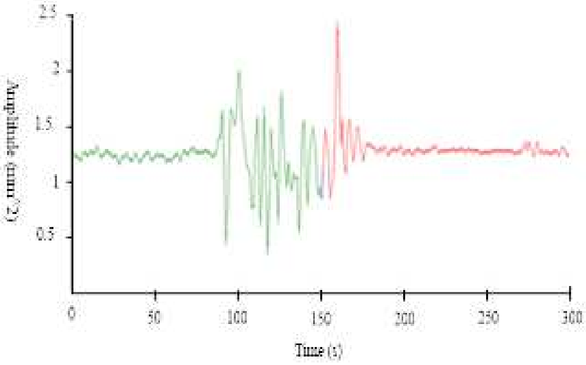
B. System Identification and Validation

The input-output data of the quarter car suspension system are collected from two different speeds as mentioned previously. From each set of data, the best ARX model of a quarter car passive suspension system is identified using the least square estimation algorithm.

1) 10 km/h Speed: The data of sprung mass vertical acceleration as the output variable and the unsprung mass vertical acceleration as input variable at 10km/h speed are shown in Fig. 8(a) and 8(b), respectively. The data will be divided into two parts in which the first half is used for estimating the model parameter and the second half is for model validation purpose.



(a) Sprung mass vertical acceleration signal as output variable



(b) Unsprung mass vertical acceleration as input signal
Fig. 8 Input-output data (10 km/h)

The best ARX model of a quarter car passive suspension system dynamic produced from system identification technique for the data shown in Fig. 8 is represented as in equation (6). The model validation for best fit and Akaike's Final Prediction Error (FPE) are 90.65% and 5.315e-06, respectively. On the other hand, the comparison between measured signal and the model response is presented in Fig. 9. The autocorrelation of residual and cross-correlation between residual and input are described as shown in Fig. 8 and 9, respectively.

$$\begin{aligned} y(kT) = {} & 0.9669y\big((k-1)T\big) + 0.0134y\big((k-2)T\big) \\ & -0.0004y\big((k-3)T\big) + 0.0057y\big((k-4)T\big) \\ & +0.9973u\big((k-1)T\big) - 0.9728u\big((k-2)T\big) \\ & +0.0170u\big((k-3)T \end{aligned} \tag{6}$$

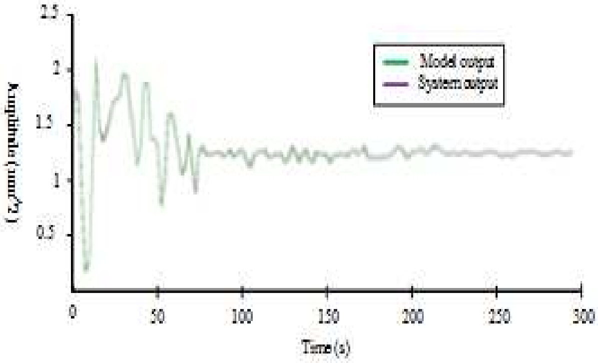


Fig. 9 Comparison between measured signal and model response from 10 km/h speed of run test

Fig. 9 shows the response of the estimated model and the response of the system from 10 km/h speed of run test car data. The graph of the identified model response to the system to the artificial road surface has a similar trend. It means that the estimated model has the same dynamic response with the system.

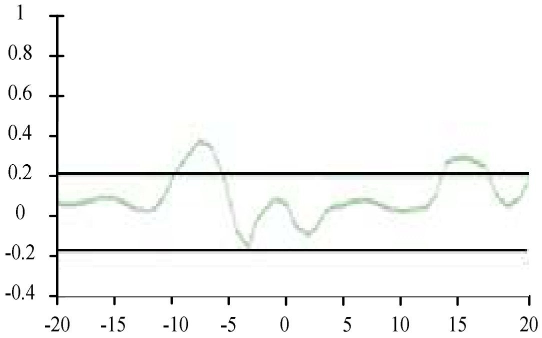


Fig. 10 Auto correlation of residual from 10 km/h run test speed

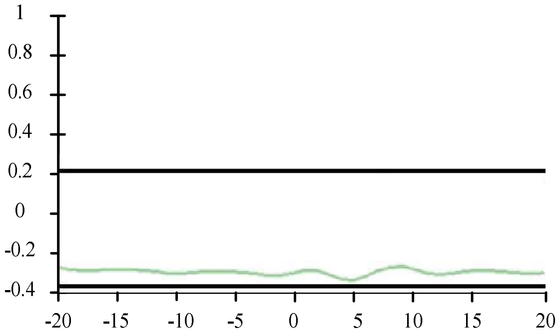
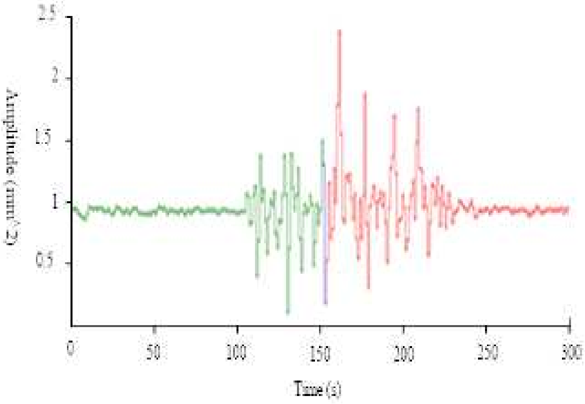


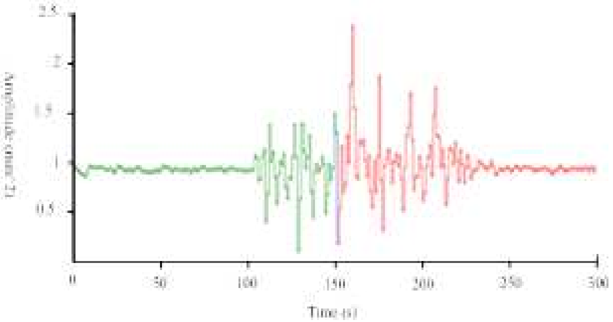
Fig. 11 Cross-correlation of residual and input from 10 km/h run test speed

The residual autocorrelation graph and cross-correlation with input are laid in the confidence interval limit as shown in Fig. 10 and 11. This means that the residual does not correlate. The autocorrelation and cross-correlation between the residual and the input are very small.

2) 20 km/h Speed: The input-output data at 20km/h speed are illustrated in Fig. 12. Input and output variables are sprung mass vertical acceleration signal and the unsprung mass vertical acceleration signal, respectively. The data is divided into two parts; one-half is to estimate the model parameter and the other part is for model validation.



(a) Sprung mass vertical acceleration signal as the output variable



(b) Unsprung mass vertical acceleration as the input signal
Fig. 12 Input-output data (20 km/h)

The best ARX model of a quarter car passive suspension system dynamic from this set of data is shown in equation (7). The best fit and Akaike's Final Prediction Error (FPE) are 91.05% and 7.503e-05, respectively. Fig. 13 shows the comparison of measured signal and the best ARX model response. Fig. 14 and 15 present the autocorrelation of residual and cross-correlation between the residual and input.

$$\begin{aligned} y(kT) = {} & 0.9673y\big((k-1)T\big) + 0.0002y\big((k-2)T\big) \\ & -0.0002y\big((k-3)T\big) + 0.0047y\big((k-4)T\big) \\ & +1.002u\big((k-1)T\big) - 0.9671u\big((k-2)T\big) \end{aligned} \quad (7)$$

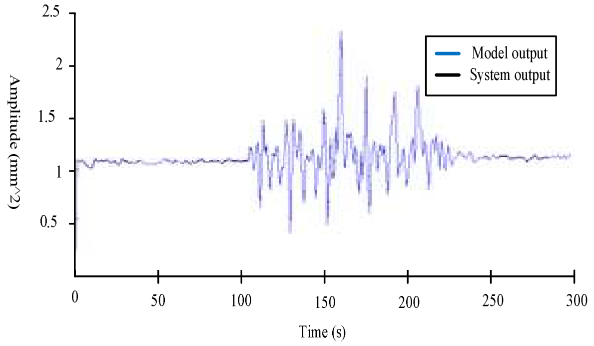


Fig. 13 Comparison between measured signal and model response from 20 km/h speed of run test

Fig. 11 shows the graph which compared the system output and the estimated model output for 20 km/h speed of run test. The model output is superimposed on the system output. It shows that the estimated model has the same dynamic response to the system.

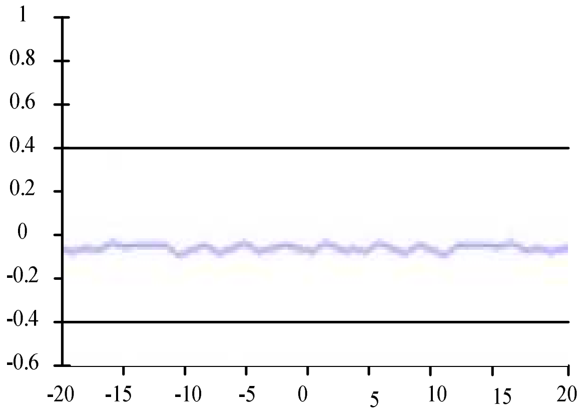


Fig. 14. Autocorrelation of residual from 20 km/h run test speed

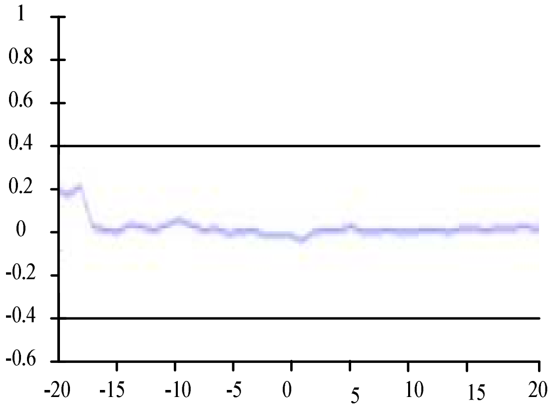


Fig. 15 Cross-correlation of residual and input from 20 km/h run test speed

The graph of the residual autocorrelation and cross-correlation with input are laid in the confidence interval limit as shown in Fig. 14 and 15. This means that the residual does not correlate. The autocorrelation and cross-correlation between the residual fulfill the criteria.

B. Discussion

System identification for identifying the best ARX models of a quarter car passive suspension system with two different speeds has been performed. It uses the input-output data collected by running a test car on an artificial road surface with 10km/h and 20km/h speeds. An artificial road surface is made from plywood and different dimensions of beam wood. It has a function to imitate the road surface. In this project, the system identification criteria have been fulfilled, and the responses of the identified models have similar trends with that of the measured data. Besides that, the residual autocorrelation and residual cross-correlation with input were also laid in limit areas. On the other hand, the obtained best fit and Akaike's FPE values showed that the identified models are acceptable.

IV. CONCLUSIONS

The ARX models of a quarter car passive suspension system with body mass (sprung mass) acceleration as the output variable and tire mass (unsprung mass) acceleration as the input variable have been obtained through system identification technique. The best model for 10 km/h car speed has the following results: output order (n_a) = 4, the input order (n_b) = 2, delay (*d*) = 1, best fit = 90.65%, and the Akaike's Final Prediction Error (FPE) = 5.315e-06. On the other hand, the 20 km/h speed produced 4th output order (n_a), 1st input order (n_b), 1st delay (*d*), 91.05% best fit and 7.503e-05 Akaike's FPE. The results showed that higher car speed reduced the effect of the road surface to car dynamics, as indicated by the value of n_b.

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