SUPM

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ABSTRACT: This study examined Year Five children's problem posing abilities based on three different stimuli. The type of problems posed were also examined. The first stimulus was a specific-goal situation whereby each child has to create a story based on any of the addition sentences of mixed operational structures. The second stimulus was similar to the first, but involved multiplication. The third required the subjects to pose 2 new problems rather freely based on non-specific-goal situation. A sample of 35 Year 5 pupils were asked to carry out the tasks in a worksheet within an hour. The responses were coded based on a rubric and later analysed quantitavely using descriptive statistics. Six children were selected for further clinical interviews. The children showed preference for the basic Missing End problems for both addition and multiplication sentences. Eight children failed to create the addition problems correctly whilst 23 did not manage to pose a multiplication story. Twenty five children successfully completed the third task. However, the problem posing products generated were limited to the uni level or multi level types comprising of addition or/and subtraction only.

INTRODUCTION

Background

Problem posing which involves the construction of a new problems from a given situation or mathematical stories based on certain computations has begun to be recognized as an important component in the teaching and learning mathematics (English 1997; Silver 1994; Brown and Walter, 1993). Problem posing activities in mathematics not only could enhance pupils' thinking but also could enrich and consolidate their understanding related to mathematical concepts and processes (English, 1997; Brown and Walter, 1993). The role of mathematics teachers should no longer focused on demonstrating a set of procedures to be remembered one by one by pupils, but more importantly, is to create opportunities for pupils to think mathematically and construct their

own mathematical knowledge via problem posing activities (National Council for Teachers of Mathematics (NCTM, 2000). Teachers are recommended to nurture and develop the innate tendency for individuals to pose problems by giving them the opportunities to pose challenging problems (NCTM, 2000). This approach could potentially generate individuals who can think, organize and monitor ones' own thinking (Blum & Niss, 1991). NCTM (1991) explicitly recommended that pupils be given the experience to identify and pose their own problems and this should be the heart when doing mathematical activities.

This recent recommendation for school mathematics have drawn interest of many researchers throughout the world such as from America (Dickerson, 1999; Craig, 1999; Silver1996; Brown and Walter,1990), Australia (Lowrie & Whitland, 2000; English, 1999), Japan (Hashimoto, 1997, 1987), Taiwan (Leung, 1997,1993) and Singapore (Yeap and Berindeer Kaur, 2000) to study children's' abilities in posing mathematical problems. Systematic investigations relating problem posing as a cognitive process involving generation of problems from situations and experience, however, are still lacking (Lowrie and Whitland, 2000; English, 1997; Silver, 1994). This study seeks to explore the cognitive behaviors of Year 5 children when engaged in problem posing tasks as well as to analyze the quality of the problems posed.

The recent mathematics curriculum reform in Malaysia emphasized two new components in problem solving. First, children need to be aware of the processes involved while solving problems. Secondly, the curriculum explicitly stated that children should be given the opportunity to pose problems to their friends (Kementerian Pendidikan, 1998). In line with this recent development, the research is undertaken with the belief that problem posing should be emphasized from year one to six. In addition, it is also in line with the current attempts or view that problem posing abilities amongst learners should be enhanced (NCTM, 2000, 1991, 1989; Lowrie and Whitland, 2000; English 1997; Silver, 1994; Brown and Walter, 1993; Kilpatrick, 1987, Writs and Kahn,1982).

English (1998) in his study to investigate the problem-posing abilities of third-grade children within formal and informal contexts, studied the response patterns used by the children in creating problems based on the structural complexity of the mathematics sentences given. He referred these as Missing End, Missing Change and Missing Start in order of difficulty. This study that also investigated the types of structural complexities the children favour when constructing a story based on addition and multiplication perceived English's categorization to be appropriate.

OBJECTIVES OF THE STUDY

This study has two main objectives. Firstly, it aimed to examine the abilities of children to problem-pose in three different format. That is creating a story based on addition and multiplication sentences as well as generating problems in a non-goal-specific situation. Secondly, the study looked into the response patterns in the problems posed by the children. This addresses the following questions:

- 1. What problem type do the children favour when constructing a story based on addition sentences?
- 2. What problem type do the children favour when constructing a story based on multiplication sentences?
- 3. Can the children create a story based on addition sentences they have chosen?
- 4. Can the children create a story based on the chosen multiplication sentences?
- 5. Can the children posed problems based in a non-goal-specific given situation?
- 6. What are the response patterns in the problems posed by the children?

METHODOLOGY

Sample

A sample comprising of 35 Year 5 children from the high achieving class of a public school participated in this study. The children had been exposed to problem-posing tasks that involved the creation of a story from a given computation during their first three years of primary schooling but they had never been exposed to the other type of the task which would require them to generate mathematical problems based a non-goal-specific situation.

Tasks and Administration

The problem-posing tasks were designed based on school mathematics curriculum. The aim was to provide insights into children's abilities to pose mathematical problems. Each subject completed three problem-posing tasks in a set of worksheets in approximately, one hour. The problem-posing tasks were piloted to ensure that the tasks assessed the cognitive processes and content area they were designed to assess.

Task 1 was designed to investigate what sort of structural complexity do the children favour when constructing a story based on addition sentences I also used to assess the children ability to construct a story in line with the addition sentences they have chosen. The structural complexities designed were of the Missing End, Missing Change and Missing Start problem types. An Example of the task is

Construct a story which is challenging and creative based on the computation that you have chosen.

$$= 6 + 2$$
 or $8 = 6 +$ or $8 =$ $+ 2$

Task 2 was designed to investigate what sort of structural complexity do the children favour when constructing a story based on multiplication sentences. It is also used to assess the children's' abilities to construct a story in line with the multiplication sentences they have chosen. The structural complexities designed were of the Missing End, Missing Change and Missing Start problem types. The task was:

Create a story which is challenging and creative based on the computation that you have chosen.

$$= 5 \times 2 \text{ or } 10 = \times 2 \text{ or } 10 = 5 \times 10^{-5}$$

Task 3 was designed to provide novel yet meaningful situations for the children to generate creative ideas. It was also designed to investigate whether the children could pose problems based on a situation that is given. It also is used to assess the response patterns in the problems posed by the children. The patterns inferred in this task, however, were limited to the operational complexities incorporated in the children's responses. The task was:

Construct 2 problems or questions based on the situation given below.

You are encouraged to pose problems that are the most challenging and creative.

The situation is:

Baba has 7 marbles.

Mutu has 3 marbles.

Data Coding

Data Coding for Task 1 and Task 2

Data was analysed by focusing on the descriptive statistics involving frequencies and percentages. Initially the mode for the problem type was found and then the percentages of the number of responses that were compatible with the computation attempted were determined.

Problem posing responses for Task 1 and 2 were compiled according to the rubric

Table 1: Rubric For Problem Posing Task 1 and Task 2

Score	Response
0	Posed a story or situation that is not compatible with the computation chosen. Eg. "Dad has bought 8 oranges to be eaten. The next day, he bought another 6 oranges. How many oranges did Dad buy altogether?" is not compatible with '8 = 6 +"
1	Posed a story or situation that is compatible with the computation chosen. Eg; Ali has to answer altogether 8 questions in an examination. Ali has answered 6 questions. How many more questions does he need to answer?" is compatible with " $8 = 6 + $ "

Data Coding for Task 3

The quality of the responses was scored based on the rubric in Table 2 followed by descriptive statistics involving frequencies and percentages. Initially the mode for the quality was determined followed by the percentages for the range of qualities of the responses posed by the subjects.

Table 2: Rubric For Problem Posing Task 3

Score	Response
0	Did not posed any question.
1	Posed a problem where the solution could be derived directly from the situation
	given.
	Eg. " How many marbles did Mutu have?
2	Posed a solvable problem comprising addition or subtraction as one step problem
	Eg. "If Baba gave away 2 of his marbles. How many marbles has he left?"
3	Posed a solvable problem comprising multiplication or division or multistep
	problem
	Eg: "Baba gave away 3 of his marbles to Mutu and bought 4 more marbles for
	himself. How many marbles has he now?"
99	Posed a non mathematical problem.
	Eg: "Baba went to Mutu's house to play marbles. What should they do?"

Analysis of Data for Task 1

Half of the subjects chose to create a story based on the Missing End type which implies that majority of the subjects seemed to favour the least complex form of the operation. However 9 of the children resorted to complete the task associated with the most complex operational form, that is, the Missing Start but none was successful. This lack of problem diversity may have been due to the children's unfamiliarity with the other problem types namely, the Missing Change and Missing Start. Surprisingly, more than a quarter of the children (26%) failed to pose a story or situation compatible with the addition computation of their choice meaning that they failed to demathematise the situation.

Analysis of Data for Task 2

Nine of the subjects did not complete the task whilst expectedly, majority (65.7%) showed preference for the basic Missing End problem. Interestingly, 3 subjects took the challenge completing the hardest option, namely the Missing Start type but none of the responses were compatible with the chosen problem type. It was a surprise to note that majority of the subjects failed to pose a story or situation compatible with the multiplication computation chosen. Only 9 managed to do the task correctly whilst 10 did not successfully pose any mathematical problem.

Analysis of Data for Task 3

Task 3 required the children to generate 2 problems from the given non-goal-specific situation. For the first problem generation, 9 of the children were not able to create any mathematical situation whilst another 9 elicited a non-mathematical question. No one generated a straight forward question. This is perhaps due to the researcher's reminder discouraging them not to do so. However, It is interesting to note that majority of the subjects (19) managed to obtain score 2 meaning that they managed to pose a solvable problem comprising of either addition or

subtraction. The problems produced however lacked operational complexity, that is, comprised of only one operation, mainly addition. Nevertheless, 15 managed to attain the maximum score of 3 but none exhibit their abilities to produce problems involving multiplication or division operations. All of them produced multistep problems involving mainly addition and occasionally subtraction.

The subjects were required to pose another problem related to the given situation. For this second task, bigger number of children (34%) ended up not producing any product. However, the same number of subjects (34%) was able to generate problems of a greater complexity whereby incorporating at least the addition or subtraction operation as a one-step problem. Despite the children's unfamiliarity with this non-goal-specific situation, several children (31%) successfully generated multistep problems but again limited to both addition and subtraction operations only.

Interviews

6 children wee selected for further in-depth interview. A clinical approach was used in the interviews. The objectives of the interviews were mainly related to eliciting the reasons why certain objects or examples were used in their problems. The interviews revealed, for example, that examples and objects were chosen based on their personal experience and these were culturally biased.

RESULTS AND DISCUSSION

In summary, although the number of subjects involved in this study is small, nevertheless the results did give insights into the Year 5 children's ability to pose problems in 3 different stimuli, namely to create a story compatible with the addition and multiplication sentences as well as to generate problems from a given non-goal-specific situation. The children's responses for Task 1 and Task 2 revealed an overall limited range of the children's interpretation and understanding for the standard number sentences. Many showed preference for the basic Missing End problem type which presented the least level of structural complexity. This may be due to the lack of opportunities given to children to consider multiple meanings for the "+" and "x" symbols (Fuson, 1992). It is rather alarming to find out there were a substantial number of children who still could not mathematise the addition sentence of their choice. Another unexpected finding is that 27 of the subjects could not generate a story in line with the multiplication sentence. This demands urgent investigation into the development of multiplicative thinking amongst the children. However, it is enlightening, to note that although the children have not been exposed to the nongoal-specific situation activity, about two thirds of them managed to complete this task successfully. In fact, about a third of the children managed to mathematise the situation, some displayed operational complexity although limited to incorporating addition and subtraction only. One possible explanation for this inclination is that as English & Halford (1995) pointed out, is that the creation of elementary multistep problems only involves the iteration of familiar change relations whereas the more complex multistep problems involves the creation of several relational ideas which is structurally complex for the child.

The present study has raised a number of questions pertaining to the children's abilities to pose problem, particularly when posing a story in line with the basic number computation given. These questions warrant attention if we were to progress towards a more complete account of children's numerical understanding. These include the following:

The results also suggest that an alternative approach that provide children with opportunities to pose problems should be included in the teaching and learning of mathematics. It would seem reasonable to include activities where children could consider both to pose and solve problems that extend beyond the basic Missing End types. Kilpatrick (1987) highlighted that the ability to problem pose is complimentary to the child's problem solving skills. It is evident that we need to broaden the types of problems experiences presented to the children so that they could assign multiple meanings of diverse structural and operational complexities associated with the formal mathematical computations. Children too should be encouraged to develop flexibility in their use of language to describe quantitative comparison to help them create a broader range of problems (English, 1998).

REFERENCES

- Blum, W., & Niss, M. (1991). Applied Mathematical Problem Solving, Modeling, Applications and Links to Other Subjects: State Trends and Issues in Mathematics Instruction. Educational Studies in Mathematics, 22, 37-68.
- Brown, S.A. & Walter, M.I. (1993). *Problem Posing: Reflections and Application* Hillsdale, NJ: Erlbaum.
- Carpenter, T. P., Moser, J.M., & Romberg, T. (1982). *Addition and Subtraction: Developmental Perspectives*. Hillsdale, NJ: Lawrence Erlbaum Associates.
- De Corte, E. Verschaffel, L.and DeWin, L. (1985). Influence of Rewording Verbal Problems on Children's Problem Representations and Solutions. *Journal of Educational Psychology*, 77. 460-67
- Dickerson, V. M. (1999). The Impact of Problem-posing Instruction on the Mathematical Problem-solving Achievement of Seventh Graders. Unpublished Ph. D. Dissertation. Emory University.
- English, L.D. (1998). Children's Problem Posing Within Formal and Informal Contexts. *Journal for Research in Mathematics Education*, 29(1), 83-106.
- English, L.D. (1997). Promoting A Problem-Posing Classroom. *Teaching Children Mathematics*, 4(3), 172-180.

- English, L. D. and Halford, G.S. (1995). *Mathematics Education: Models and Processes*. New Jersey: Lawrence Erlbaum Associates.
- Fuson, K.C. (1992). Research on Whole Number Additions and Subtraction. In D.A. Grows (Ed). Handbook of Research on Mathematics Teaching and Learning: A Project of NCTM. 243-275. New York: Macmillan.
- Hashimoto, Y. (1997). The Methods of Fostering Creativity Through Mathematical Problem Solving. *International Reviews on Mathematical Education*, ZDM, 3. 86-87.
- Hashimoto, Y. (1987). Classroom Practice of Problem Solving in Japanese Elementary Schools. In J.P. Becker & T. Miwa (eds.). *Proceedings of the U.S. Japan Seminar on Mathematical Problem Solving* (pp. 94-119). Carbondale, IL: Southern Illinois University.
- Kementerian Pendidikan Malaysia(1998). *Huraian Sukatan Pelajaran Matematik Tahun 5 KBSR*. Pusat Perkembangan Kurikulum.
- Kementerian Pendidikan Malaysia (2001). *Huraian Sukatan Pelajaran Matematik Tahun 4.* Kurikulum Bersepadu Sekolah Rendah. Kuala Lumpur: PPK.
- Kilpatrick, J. (1987). Problem Formulating: Where Do Good Problems Come From? . In A. H. Schoenfeld (Ed.), *Cognitive Science and Mathematics Education*. Hillsdale, NJ: Erlbaum.
- Leung, S.S. (1993). The Relation of Mathematical Knowledge And Creative Thinking To The Mathematical Problem Posing of Prospective Elementary School Teachers On Tasks Differing In Numerical Information Content. Unpublished Ph.D Dissertation, University of Pittsburg, USA
- Leung, S. S. (1997). On the Role of Creative Thinking in Problem Posing. . *International Reviews on Mathematical Education*, ZDM, 3. 81-85.
- Lowrie, T. & Whitland, J. (2000). Problem Posing as a Tool for Learning and Assessment in the Primary School. *Proceedings of the 24th. Conference of the International Group for the Psychology of Mathematics Education.* Hiroshima University. July 23-27 2000. 3. 3-247..
- National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, Virginia.
- National Council of Teachers of Mathematics. (1991). *Professional Standards For Teaching Mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (1989). Curriculum and Evaluation Standards for School Mathematics. Reston, VA: Author.

- Resnick, L. B. (1983). A Developmental Theory of Number Understanding. In Ginsburg, H. P. (Ed.). *The Development of Mathematical Thinking*. N. Y. Academic Press. 109-155.
- Silver, E.A. (1994). On Mathematical Problem Posing. For the Learning of Mathematics. 14(1). 19-28.
- Silver, E.A. (1994). The Posing of Division Problems by Preservice Elementary School Teachers: Conceptual Knowledge and Contextual Connections. APaper presented at The Annual Meeting of the American Educational Research Association (New Orleans, LA, April 1994).
- Verschaffel, L. & De Corte, E. (1996). Number and Arithmetic. In Bishop, A. J. (Ed.) International *Handbook of Mathematics Education*. Nederlands, Kluwer Academic Publishers. 99-137.
- Wirtz, R. & Kahn, E. (1982). Another Look At Application in Elementary School Mathematics. *Arithmetic Teache*, 30(1), 21-25.
- Yeap, B. & Berindeer Kaur (2000). Analyzing Children's Problem-posing Responses. Proceedings of the 24th. Conference of the International Group for the Psychology of Mathematics Education. Hiroshima University. July 23-27 2000. 1. 1-194.