# MATHEMATICAL MODELS FOR THE BOUNDARY LAYER FLOW DUE TO A MOVING FLAT PLATE

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**To**

My parents

My wife, Noriah

My children, Hakim, Hafiz, Hanif and Liyana

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# ABSTRACT

The boundary-layer flow over a moving continuous solid surface is important in several engineering processes. For example, materials manufactured by extrusion processes and heat-treated materials travelling between a feed roll and a wind-up roll or on conveyor belt possess the characteristics of a moving continuous surface. In this study, the mathematical model for a boundary layer flow due to a moving flat plate in micropolar fluid is discussed. The plate is moving continuously in the positive $x$-direction with a constant velocity. The governing boundary layer equations are solved numerically using an implicit finite difference scheme. Numerical results presented include the reduced velocity profiles, gyration component profiles and the development of wall shear stress or skin friction for a wide range of material parameter $K$ takes the values, $K = 0$, 0.1, 0.3, 0.5, 1, 3, 5 and 10. The results obtained, when the material parameter $K = 0$ (Newtonian fluid), are in excellence agreement with those obtained for viscous fluids. Further, the wall shear stress increases with increasing $K$. For fixed $K$, the wall shear stress decreases and the gyration component increases with increasing values of $n$, in the range $0 \leq n \leq 1$ where $n$ is a ratio of the gyration vector component and the fluid shear stress at the wall.

# ABSTRAK

Aliran lapisan sempadan terhadap permukaan pejal bergerak sangat penting dalam proses-proses kejuruteraan. Sebagai contoh, ciri-ciri bagi pergerakan suatu permukaan secara berterusan dapat dilihat dalam proses penyemperitan dalam pembuatan bahan-bahan, aliran bahan yang dipanaskan melalui dua pengelek yang diapit atau pergerakan diatas penyampai/penghantar tali sawat. Dalam kajian ini, model matematik bagi aliran lapisan sempadan terhadap plat rata bergerak telah dibincangkan. Satu plat bergerak berterusan dalam arah positif paksi-$x$ dengan halaju tetap. Persamaan lapisan sempadan yang dihasilkan telah diselesaikan secara berangka dengan menggunakan skema beza terhingga tersirat. Keputusan berangka telah diberikan, ini termasuk profil halaju, profile komponen legaran dan perubahan tegasan ricih permukaan dengan mengambil nilai-nilai parameter bahan $K = 0$, 0.1, 0.3, 0.5, 1, 3, 5 and 10. Kajian ini menunjukkan bahawa keputusan bagi masalah dalam bendalir micropolar, keputusan perbandingan dengan bendalir likat adalah sangat memuaskan apabila parameter bahan $K = 0$ (bendalir Newtonan). Seterusnya tegasan ricih permukaan meningkat dengan peningkatan nilai $K$. Untuk nilai $K$ yang ditetapkan, didapati tegasan ricih permukaan menyusut dan kompenan legaran meningkat dengan peningkatan nilai $n$, dalam selang $0 \le n \le 1$, di mana $n$ adalah nisbah kompenan vektor legaran dengan tegasan ricih bendalir pada permukaan.

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# LIST OF ABBREVIATIONS AND SYMBOLS

## Roman Letters

| | | |
|---|---|---|
| $f$ | - | Reduced stream function |
| $f_o$ | - | Reduced stream function for $n = \frac{1}{2}$ |
| $\hat{f}$ | - | Reduced stream function for the classical Blasius boundary-layer flow |
| $g$ | - | Reduced gyration component |
| $g_o$ | - | Reduced gyration component for $n = \frac{1}{2}$ |
| $h_j$ | - | Step size in $\eta$-direction |
| $\zeta$ | - | Microinertia density |
| $K$ | - | Ratio of the gyroviscosity and the fluid viscosity |
| $k_i$ | - | Step size in $\xi$-direction |
| $l$ | - | Length scale |
| $n$ | - | Ratio of the gyration vector component and the fluid shear at a solid boundary |
| $N$ | - | The gyration vector component perpendicular to the x-y plane |
| $p$ | - | Pressure |
| Re | - | Reynolds number, $\frac{\rho U_o l}{\mu}$ |
| $\bar{u}$ | - | Dimensionless velocity $u$ |
| $u$ | - | Fluid velocity component in $x$-direction |
| $U$ | - | Free stream velocity |
| $\bar{v}$ | - | Dimensionless velocity $v$ |

| | | |
|---|---|---|
| $v$ | - | Fluid velocity component in $y$-direction |
| $x$ | - | Coordinate along the plate |
| $\bar{x}$ | - | Dimensionless coordinate $x$ |
| $y$ | - | Coordinate normal to the plate |
| $\bar{y}$ | - | Dimensionless coordinate $y$ |
| $X$, $Y$ | - | Nondimensional streamwise and cross-stream Cartesian coordinates |

**Greek Letters**

| | | |
|---|---|---|
| $\delta$ | - | Boundary layer thickness |
| $\eta$ | - | Pseudo-similarity variable |
| $\hat{\eta}$ | - | Scaled pseudo-similarity variable |
| $\gamma$ | - | Spin-gradient viscosity |
| $\mu$ | - | Dynamic viscosity |
| $\kappa$ | - | Coefficient of gyroviscosity |
| $\upsilon$ | - | Kinematic viscosity |
| $\xi$ | - | Transformed streamwise coordinate |
| $\rho$ | - | Density of the fluid |
| $\psi$ | - | Stream function |
| $\theta$ | - | Momentum thickness |
| $\delta$ | - | Displacement thickness |

**Superscripts**

| | | |
|---|---|---|
| $'$ | - | Differentiation with respect to $\eta$ |
| $\bar{\ }$ | - | Dimensional variables |

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# CHAPTER I

# INTRODUCTION

## 1.1 Introduction

The boundary-layer flow over a moving continuous solid surface is important in many engineering processes. For example, materials manufactured by extrusion processes and heat-treated materials traveling between a feed roll and a wind-up roll or on conveyor belt possess the characteristics of a moving continuous surface. Drag, heat and mass transfer are governed by the structure of the layer, so a detailed knowledge of this structure is necessary to deal with its engineering applications (Fan, 1999). Other examples may be found in continuous casting, glass fiber production, metal extrusion, hot rolling, the cooling and/or drying of paper and textiles, and wire drawing (Altan et al, 1979). The study of heat transfer and the flow field is necessary for determining the quality of the final product of these processes as explained by Karwe and Jaluria (1988). They carried out a numerical simulation of thermal transport associated with a moving flat sheet in material processes.

Our present study will investigate the boundary layer flow due to a moving flat plate in both viscous and micropolar fluids. A micropolar fluid is one which contains suspensions of rigid particles such as blood, liquid crystals, dirty oil and certain colloidal fluids, which exhibits microstructure. The theory of such fluids was first formulated by Eringen (1966). The equations governing the flow of a micropolar fluid involve a microrotation vector and a gyration parameter in addition to the classical velocity vector field. This theory includes the effects of local rotary inertia and couple stresses and is expected to provide a mathematical model for the non-Newtonian behavior observed in certain man-made liquids such as polymeric fluids and in naturally occurring liquids such as animal blood. The theory of thermomicropolar fluids was also developed by Eringen (1972) by extending the theory of micropolar fluids. A comprehensive review of micropolar fluid mechanics was given by Ariman et al (1973), they studied the inadequacy of the classical Navier-Stokes theory to describe rheologically complex fluids such as liquid crystals, animal blood, etc., has led to the development of microcontinuum fluid mechanics as an extension of the classical theory. Many models have been proposed to take into account the mechanically significant microstructure of such fluids.

In our present study, it will also consider the problems of the boundary-layer flow. We derive and solve the full boundary layer equations and the analysis include the pseudo-similarity transformation of the governing equations and the resulting nonlinear equations are then solved using an implicit finite difference scheme, the Keller-box method. The reduced velocity, reduced gyration component and development of wall shear stress or skin friction are shown on graph.

In the next section, we present the research background for the project followed by the objectives and scope and an introduction to the content of this dissertation.

## 1.2 Research Background

Sakiadis (1961) first investigated the boundary-layer flow on a continuous solid surface moving at constant speed. Due to the entrainment of the ambient fluid, this boundary-layer flow is quite different from the Blasius flow past a flat plate. Sakiadis' theoretical predictions for Newtonian fluids were later corroborated experimentally by Tsou et al. (1967). Lee and Davis (1972) investigated the laminar boundary layers on moving continuous surfaces while the turbulent boundary layer on a moving continuous plate was studied by Noor Afzal (1996). Revankar (1989) discussed the problem of heat transfer due to a continuous moving flat surface with variable wall temperature.

Chen and Char (1988) investigated the heat transfer over a continuous stretching surface with suction and blowing. Chiam (1993) studied magnetohydrodynamic boundary layer flow due to a continuous moving flat plate. Gupta et al. (1997) analyzed the heat and mass transfer corresponding to the similarity solution for the boundary layer over a stretching sheet subject to suction or blowing. The problem of viscous variation for a moving flat plate in an incompressible fluid have been investigated by Pop et al. (1992).

Later, Allan (1997) presented the similarity solution of a boundary layer problem over moving surfaces and discussed the idea of nonclassical similarity transformation, which takes into account the effect of the ratio of wall velocity to the free stream velocity. They applied the transformation to the fluid flow over a moving flat plate due to Blasius profile. Recently, Fang (2003) studied the similarity solution for a moving-flat plate thermal boundary layer and Kayvan and Mehdi (2004) considered the local similarity solution for the flow of a "second-grade" viscoelastic fluid above a moving plate.

Peddiesen and McNitt (1970) applied the micropolar boundary layer theory to the problems of steady stagnation point flow, steady flow over a semi-infinite flat plate, and impulsively flow past an infinite flat plate. Numerical results through a finite difference scheme were obtained by them for the first two problems. A similarity solution for boundary layer flow near stagnation point was presented by Ebert (1973). A study of the boundary layer flow of micropolar fluids past a semi-infinite plate was studied by Ahmadi (1976). Gorla (1983) investigated the steady boundary layer flow of a micropolar fluid at a two-dimensional, stagnation point on a moving wall and claimed that the micropolar fluid model is capable of predicting results which exhibit turbulent flow characteristics.

A similarity analysis of the flow and heat transfer past a continuously moving semi-infinite plane in micropolar fluid has been presented by Soundalgekar and Takhar (1983). They consider a steady, two-dimensional flow of a micropolar fluid past a continuously moving flat plate, with a constant velocity in a micropolar fluid medium at rest. Rees and Bassom (1996) have considered the Blasius boundary layer flow of a micropolar fluid over a flat plate. They used the Keller-box method to solve the resulting nonsimilar equations and presented solution for a range of parameters. Perdikis and Raptis (1996) studied the heat transfer of a micropolar fluid in the presence of radiation. Na and Pop (1997) studied the laminar boundary-layer flow of a micropolar fluid over a continuously moving surface through an otherwise quiescent micropolar. The transformed boundary-layer equations are solved numerically for a power-law surface velocity using the Keller-box method.

Raptis (1998) studied numerically the case of a steady two dimensional flow of a micropolar fluid past a continuously moving plate with a constant velocity in the presence of thermal radiation. Kim (2000) studied the unsteady two-dimensional laminar flow of a viscous incompressible electrically conducting fluid in the vicinity of a semi-infinite vertical porous moving plate in the presence of a transverse magnetic field. On the other hand, Chakraborty and Borkakati (2002) studied of the flow of a viscous incompressible electrically conducting fluid on a continuous moving flat plate in the presence of uniform transverse magnetic field. In this study, numerical solutions were obtained using the Runge-Kutta and Shooting Method. Recently, Hassan (2003) analyzed the problem of the effect of suction/injection on the flow of a micropolar fluid past a continuously moving plate in the presence of radiation.

## 1.3 Objectives and Scope

The objectives of this project are:

1. to solve the boundary-layer flow due to a moving flat plate in viscous fluid.
2. to solve the boundary-layer flow due to a moving flat plate in micropolar fluid.

The methodology involved will be:

- carry out the mathematical formulation of the governing equations,
- carry out the finite difference formula,
- develop numerical algorithm and solved numerically using the Keller-box method.

## 1.4 Outline of Dissertation

This thesis is divided into five chapters including this introductory chapter. Section 1.2 presents the research background on the development of research in this area and we present the objectives and scope of this project in Section 1.3.

In Chapter II, we discuss the first problem in viscous fluid. The problem is boundary layer flow due to a moving flat plate in viscous fluid. This chapter will be divided into eight main sections where the first section is the introduction of the problem. A next section contains a discussion on the derivation of the governing equations of the boundary layer flow and details of the numerical method, known as the Keller-box method. One of the basic ideas of the box method, proposed by Keller, is to write the governing system of equations in the form of a first order system. Then Newton's method is used to linearize the resulting nonlinear equations and lastly, the solutions are obtained using the block-elimination method. The Keller-box method used in this study is programmed in Matlab. The complete program is given in Appendix A and the list of the notation of the symbols or variables used in the Matlab is given in Appendix B. The comparison of the present results that obtained by Keller-box method with other results that obtained by other methods are given in Section 2.8. Generally we compare the methodology, once this confirm, then we used the Keller-box method to solve problem in micropolar fluid. The next following chapters of this thesis are Chapter III and IV, which discuss the second problem in micropolar fluid.

Chapter III will be divided into three main sections where the first section is the introduction of the problem. In Section 3.2 contains a discussion on the derivation of the mathematical formulation of the boundary layer flow due to a moving flat plate in micropolar fluid. In Section 3.3 we discuss the transformation of the boundary layer equations.

In Chapter IV, we discuss the details of the numerical solution of the boundary layer flow due to a moving flat plate in micropolar fluid. This chapter will be divided into six main sections where the first section is the introduction of the problem. There is a similarity in Section 2 to 5 in both Chapters II and IV where it describes the sequence of numerical method. The final section contains the results and discussion. Our results for material parameter $K = 0$ (Newtonian fluid) are compared with existing results from the literature for similar problem in viscous fluid and the agreement are found to be very good, and thus, we proceed to get the new results for other values of $K(K \neq 0)$ which are the results in micropolar fluid. The Keller-box method used in this study is programmed in FORTRAN f77. The complete program is given in Appendices C.

The final chapter, namely Chapter VI, which is the concluding chapter, contains the findings of this study and recommendations for future research.