AN EXPLORATORY STUDY ON GOAL PROGRAMMING AS AN ALTERNATIVE METHOD TO DEVELOP PREDICTION EQUATIONS

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Especially for my loving parents, dad Lau Heng Tiong and mum Loi Kiik Hee and my young brother, Lau Chik Muan.

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ABSTRACT

One of the most promising techniques for multiple objective decision analysis is goal programming. Goal programming is a powerful tool which draws upon the highly developed and tested technique of linear programming, but provides a simultaneous solution to a complex system of competing objectives. Least squares method in regression analysis is also a popular technique used in decision making. It is an approach used in the study of relations between variables, particularly for the purpose of understanding how one variable depends on one or more other variables. However, one of the main problems is that the method of least squares is biased by extreme cases. This study proposes goal programming as an alternative to analyze such problems. The analysis were done by using QM for Windows and MINITAB software package.

ABSTRAK

Pengaturcaraan gol adalah satu kaedah yang paling berkesan dalam penganalisisan keputusan objektif berganda. Ia juga merupakan suatu teknik yang lebih baik berbanding pengaturcaraan linear dalam penyelesaian serentak untuk sistem kompleks. Kaedah kuasa dua terkecil dalam analisis regresi juga adalah satu teknik yang terkenal dalam membuat keputusan. Kaedah ini mengkaji hubungan antara pemboleubah terutama dalam memahami bagaimana satu pemboleubah bersandar kepada satu atau lebih pembolehubah yang lain. Bagaimanapun, masalah utama bagi kaedah kuasa dua terkecil ialah pengaruh kes ekstrim. Kajian ini mencadangkan pengaturcaraan gol sebagai kaedah alternatif untuk mengatasi masalah tersebut. Kajian ini menggunakan program QM for Window dan MINITAB dalam analisis.

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LIST OF SYMBOLS

a	-	Intercept of response variable, y
b	-	Estimate regression coefficient
$\boldsymbol{b}$	-	Vector of b
e	-	Error / residual term
GP	-	Goal programming
i	-	$1, 2, \ldots, m$
iqr	-	Interquartile range
j	-	$1, 2, \ldots, n$
k	-	$1, 2, \ldots, K$
K	-	The total number of preemptive priority factors
L	-	Minimize function
LS	-	Least squares
MAD	-	Mean absolute deviation
MAPE	-	Mean absolute percentage error
MPE	-	Mean percentage error
MSE	-	Mean square error
n	-	Number of prediction / forecasts
p	-	The total number of system constraints
$p(x)$	-	Polynomial function / fitting curve
QM	-	Quantitative method
RMSE	-	Root mean square error
s	-	$1, 2, \ldots, S$
S	-	The total number of the decision and deviational variables where $S = n + 2m$
X	-	Independent (predictor) variable
Y	-	Dependent (response) variable

$\sum_{i=1}^{n}(Y_i - \hat{Y}_i)$ - Sum of errors

Z - Objective function

β_x - Parameter of regression equation

i' - The pivot row

s' - The pivot column

x^k - Independent variable of k

$\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$ - Sum of the squares of the error / least squares / ordinary least squares

a_{ij} - The coefficient associated with variable j in the ith goal

A_i - Artificial variable

b_i - The associated right hand side value

C_b - The column for assigning the preemptive priority factors and weights to the basic variables

C_j - The row for assigning the preemptive priority factors and weights to the basic and nonbasic variables

E_i - Excess or surplus variable

g_k - level of achievement of the goal in priority k, where $\boldsymbol{g} = (g_1, g_2, \ldots, g_k)$

P_0 - Super priority factor/artificial objective function

P_k - The priority factor of the kth goal

$r_{k,s}$ - The index number for priority k under sth basic or nonbasic variable

S_i - Slack variable

u_i - The function of preemptive factors and weights associated with the ith basic variable

v_s - The function of preemptive priority factors and weights associated with the sth basic or nonbasic variable

x_b - Basic variable

x_j - The jth decision variable

$y_{i,s}$ - Element in the ith row under the sth basic or nonbasic variable. That is, the coefficient of the sth basic or nonbasic variable in goal i

$Z_j - C_j$ - The index row

d_i^- - Negative deviational variable from ith goal (underachievement)

d_i^+ - Positive deviational variable from *i*th goal (overachievement)

W_i^- - Positive numerical weight assigned to the negative deviational variable, d_i^- of the *i*th constant

W_i^+ - Positive numerical weight assigned to the positive deviational variable, d_i^+ of the *i*th constant

x_i^* - The optimal value of decision variables

$\hat{b}_i$ - New value of b_i

$\hat{g}_k$ - New value of g_k

$\hat{r}_{k,s}$ - New value of $r_{k,s}$

$\hat{u}_i$ - New value of u_i

$\hat{v}_s$ - New value of v_s

$\hat{y}_i$ - Predicted / estimated value

$\hat{y}_{i,s}$ - New value of $y_{i,s}$

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CHAPTER 1

RESEARCH FRAMEWORK

1.0 Introduction

Many decision problems involve multiple objectives. Often, these objectives are conflicting with each other. A number of techniques have been proposed for multiple-objective decision making. One of the most promising techniques for multiple objective decision analysis is goal programming. Goal programming is a powerful tool which draws upon the highly developed and tested technique of linear programming, but provides a simultaneous solution to a complex system of competing objectives (Lee,1981). Goal programming can handle decision problems having a single goal with multiple subgoals.

Generally, many decision problems in organizations which involve multiple objectives are not easy to analyze by optimization techniques such as linear programming. Multiple-criteria decision making (MCDM) or multiple-objective decision making (MODM) has been a popular topic of management science during the past decade. A number of different approaches of MCDM or MODM have been proposed, such as multiattribute utility theory, multiple-objective linear programming, goal programming, compromise programming and various heuristics which are methods based on rules that are developed through experience. Goal programming is among the best widely accepted and applied technique. The primary reason for the wide popularity of goal programming appears to be its underlying philosophy of "satisficing" (Lee and Shim, 1986).

Nobel laureate Herbert A. Simon (1981) suggested that the satisficing approach, rather than optimizing is based on the concept of bounded rationality. This approach has emerged as a pragmatic methodology of decision making.

1.1 Research Background

A regression model is a mathematical equation that describes the relationship between two or more variables. The dependent variable is the one being explained, and the independent variables are the ones used to explain the variation in the dependent variable.

Regression techniques are associated with the fitting of straight lines, curves, or surfaces, to set of observations, where the fit is for one reason or another imperfect. The straight line is the simplest curve that can be fitted to a set of n paired observations $(x_1, y_1), (x_2, y_2) \ldots (x_n, y_n)$. The least squares method is the most frequently used procedure for obtaining a linear function. A problem of fitting occurs only if the fit is for some reason imperfect. To be a statistical problem there must be some random element present in the data which leads to this inexactitude of fit. It is the nature of this random element that determines the appropriate method of fitting, that is of estimating the constants or parameters in the equation.

In simple linear regression analysis, the estimated regression model is $\hat{Y} = a + bX$ (Y denotes the predicted dependent variable and X denotes the independent variable). In multiple regressions, the estimated regression model is $\hat{Y} = a + \sum_{i=1}^{n} b_i X_i$ (Y denotes the predicted dependent variable and X_i denotes the independent variables). Although the method of least squares is one of the best known and probably widely utilized methods employed in the analyses of making predictions or forecasts of the future, most previous efforts in this area however, suffer from several disadvantages. According to Campbell (1972), one of the main

problems is that the method of least squares is biased by extreme cases. The current study proposes goal programming as an alternative to analyze such problems.

1.2 Objectives of the Study

The following are objectives of this study:

i. To develop prediction equations using least squares method.
ii. To develop prediction equations using goal programming.
iii. To compare the accuracy of goal programming and least squares method.

1.3 Importance of the Study

Many prediction equations have been obtained using the least squares method. These equations have been used in various areas such as educational system planning, financial planning and economic policy analysis. This study explores goal programming as an alternative method to produce prediction equations. This is because goal programming is a widely accepted and applied technique in multiple objective decision analysis.

1.4 Scopes of the Study

This study focuses on the use of the linear goal programming method to produce prediction equations in regression analysis problems. Only three data sets are considered. The first set consists of only one independent variable, the second set has two independent variables while the third set has three independent variables.

1.5 Thesis Organization

This dissertation is organized into six chapters. Chapter 1 discusses the research framework. It begins with the introduction to goal programming and the least squares method. The objectives, importance and scope of this study are also presented.

Chapter 2 reviews the least squares method and goal programming. First, the least squares line and multiple regression least squares will be reviewed. Then, the modeling of the goal programming will be discussed. The discussion starts with the background of goal programming. Formulation and methodology of the goal programming model are also presented. Finally, some complications and post optimality analysis in goal programming are explained.

Chapter 3 begins with a discussion on outliers in a data set. In this chapter analysis of data sets using the least squares methods are carried out. Chapter 4 presents on the analysis of the same data sets using the goal programming model.

In chapter 5, comparison between the least squares and goal programming are made.

Chapter 6 summarizes and concludes the whole study and makes some suggestions for future investigation.