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An application of algorithms of Adams and Gear methods on boundary layer convective heat transfer with pressure gradient using Homotopy Perturbation Method (HPM) over a flat plate

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Graphical abstract
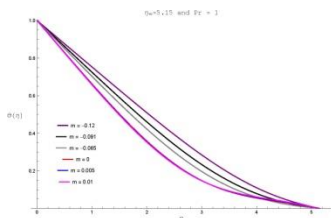
Abstract

Boundary layer flow of convective heat transfer with pressure gradient over a flat plate is solved with an application of algorithms of Adams Method (AM) and Gear Method (GM) using Homotopy Perturbation Method (HPM). The distributions of temperature and velocity in the boundary layer are examined, particularly on the influences due to Prandtl number (Pr) and pressure gradient (m). Consequently, the equations of momentum and energy are resolved concurrently. These HPM outcomes have been compared with the previous published work in the literature; and these are found to be in good agreement with the results obtained from numerical methods.

Keywords: Adams Method (AM), Gear Method (GM), Homotopy Perturbation Method (HPM), pressure gradient parameter, convective heat transfer

Abstrak

Aliran lapisan sempadan bagi pemindahan haba perolakan dengan kecerunan tekanan di atas plat rata diselesaikan dengan penggunaan Kaedah Adam dan Kaedah Gear melalui Kaedah Usikan Homotopi.Taburan suhu dan halaju dalam lapisan sempadan dikaji, terutamanya terhadap pengaruh bilangan Prandtl (Pr) dan kecerunan tekanan (m). Untuk memenuhi matlamat ini, persamaan tidak linear momentum dan tenaga diselesaikan secara serentak. Keputusan-keputusan HPM telah dibandingkan dengan kerja-kerja penerbitan terdahulu dalam kesusasteraan; dan keputusan ini didapati selari dengan keputusan-keputusan yang diperoleh melalui kaedah berangka.

Kata kunci: Kaedah Adams, Kaedah Gear,Kaedah Usikan Homotopi, parameter kecerunan tekanan, pemindahan haba perolakan

1.0 INTRODUCTION

Nothing is perfect in this world. So, there is always a room for improvement. This is a case of reinvestigation of the problem posed by Fathizadeh and Rashidi in[1]. For this purpose the two main algorithms, the Adams and Gear methods have been used in [2]. Recently an attempt have been made to solve the same problem with laminar boundary layer over a permeable surface with convective boundary condition using HAM by Shagaiya and Daniel in [3] and their reported results were not similar as Fathizadeh and Rashidi in [1]. Importance of a boundary layer flow cannot be avoided in various areas of fluid mechanics since it reveals the motion of a viscous fluid closed to a body.

In recent past, researchers have discussed the boundary layer flow convection heat and mass transfer over a flat plate in[4]–[10], boundary layer flow and mass transfer with a stretching or shrinking sheet in [11]–[14], as similarity solutions for flow and heat transfer over a permeable surface with convective boundary condition in [15]. The Homotopy Perturbation Method (HPM) is a novel and effective method, and has been successfully applied to solve various nonlinear complicated engineering problems that cannot be solved by analytical method used by Ji-Huan [16]–[18], Cai *et al.* [19], Cveticanin [20], El-Shahed [21], Abbasbandy [22] and Belendez *et al.* [23]. Ji-Huan and others have built up further this technique for diverse non linear problems [24]–[27].

Researchers have implemented some other approximation techniques like Variational Iteration Method (VIM), Adomian Decomposition Method (ADM) and Homotopy Analysis Method (HAM) effectively. Yulita Molliq *et al.* [28] have obtained the analytical solutions to fractional heat and wave like equations with variable coefficients with the help of VIM successfully. We are considering HPM in our study.

The present work deals with an application of HPM using the algorithms of Adams and Gear methods on boundary layer convective heat transfer with pressure gradient over a flat plate. This study is motivated by the different results for pressure gradient (m) reported in Cebeci and Bradshaw [29], Shagaiya and Daniel in [3] and Fathizadeh and Rashidi in [1].

2.0 METHODOLOGY

It is a composition of three steps, first one is basics of HPM, second one is mathematical formulation and third one is boundary layer flow over a flat plate.

2.1 Basics of HPM

The fundamental concepts of this technique are given as follows:
Consider the nonlinear differential equation

$$A(u) - f(r) = 0, \quad r \in \Omega \tag{1}$$

with boundary conditions

$$B(u, \partial u/\partial n) = 0, \quad r \in \Gamma \tag{2}$$

where A is a differential operator, B is an operator, $f(r)$ is an analytic function, Γ is the domain Ω boundary. A can be divided into L linear and N non linear, therefore, Eq.(1) is of the form:

$$L(u) + N(u) - f(r) = 0 \tag{3}$$

By the homotopy method[29], a homotopy $v(r, P): \Omega \times [0,1] \to R$ is constructed, which satisfies

$$H(v, p) = (1 - p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0,$$
$$p \in [0,1], \quad r \in \Omega \tag{4}$$

or

$$H(v, p) = L(v) - L(u_0) + pL(u_0) + p[N(v) - f(r)] = 0, \tag{5}$$

where $p \in [0,1]$ is a parameter which is embedded, u_0 is the initial approximated solution of Eq.(1), where the boundary conditions are fulfilled. Clearly, from Eq. (4 or 5), H takes the forms

$$H(v, 0) = L(v) - L(u_0) = 0 \tag{6}$$

$$H(v, 1) = A(v) - f(r) = 0 \tag{7}$$

the transformation of p from 0 to 1 is referred to $v(r, p)$, from $u_0(r)$ to $u(r)$. Topologically, this is known as deformation, besides $L(v) - L(u_0)$, $A(v) - f(r)$ are termed homotopic. In this study, the embedding parameter p as a small parameter and assumed that the solution of Eq. (4) or Eq. (5) can be written as a power series in p:

$$v = v_0 + pv_1 + p^2 v_2 + \cdots \tag{8}$$

Setting $p = 1$ results in the approximate solution of Eq.(1):

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \cdots \tag{9}$$

The coupling of the perturbation method and the homotopy method is called the homotopy perturbation method, which has eliminated limitations of the traditional perturbation methods. On the other hand, the proposed technique can take full advantage of the traditional perturbation techniques.

2.2 Mathematical Formulation

The Navier-Stokes equation is considered for the boundary layer flow over a flat plate with a pressure gradient term. Mathematical formulation for the Navier-Stokes equations become under the suppositions [30]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{10}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + v\frac{\partial^2 u}{\partial y^2} \tag{11}$$

and

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = -\frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{12}$$

where u and v are the velocity components in $x-$ and $y-$directions respectively, v is the kinematic fluid viscosity, ρ is the fluid density, μ is the coefficient of fluid viscosity, λ is the relaxation time, T is the temperature, κ is the fluid thermal conductivity and c_p is the specific heat. Now, the stream function $\psi(x,y)$ is introduced as:

$$u = \frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x} \tag{13}$$

For an external flow $-\frac{1}{\rho}\frac{dp}{dx}$ can be replaced by $U_\infty\frac{dU_\infty}{dx}$

where as in relations with Eq. (13), the Eq. (10) is identically satisfied and the Eqs. (11) and (12) are reduced to the following forms:

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = U_\infty\frac{dU_\infty}{dx} + v\frac{\partial^3 \psi}{\partial y^3} \tag{14}$$

and

$$\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{15}$$

Here, we have introduced the dimensionless variables η and ψ as:

$$\eta = y\sqrt{\frac{U_\infty}{vx}}, \psi = f(\eta)\sqrt{vxU_\infty}, \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty} \text{ and}$$
$$\left\{U_\infty = Cx^m, \quad m = \frac{x}{U_\infty}\frac{dU_\infty}{dx}\right\} \tag{16}$$

Based on Eq. (16), we have used similarity transformation to reduce the governing differential equations Eq. (14) and Eq. (15) to an ordinary non-linear differential equations Eq. (17) and Eq. (18) respectively.

$$f''' + \left(\frac{m+1}{2}\right)ff'' + m(1-(f')^2) = 0 \tag{17}$$

$$\theta'' = \frac{Pr(m+1)}{2}f\theta' = 0 \tag{18}$$

where f is related to the velocity (u) by $f' = \frac{u}{U_\infty}$. The reference velocity is the free stream velocity of forced convection [1] and $Pr = \frac{\mu c_p}{\kappa}$ is the Prandtl number [31]. The boundary conditions are obtained from the similarity variables.

$$f(0) = 0, f'(0) = 0, f'(\eta_\infty) = 1, \theta(0) = 1, \theta(\eta_\infty) = 0. \tag{19}$$

2.3 Boundary Layer Flow Over a Flat Plate

In accordance to HPM technique, then Eq.(17) and Eq. (18)become:

$$(1-p)(f'''-f_0''') + p\left(f''' + \left(\frac{m+1}{2}\right)ff'' + m(1-f'^2)\right) = 0 \tag{20}$$

$$(1-p)(\theta''-\theta_0'') + p\left(\theta'' + \left(\frac{Pr(m+1)}{2}\right)f\theta'\right) = 0 \tag{21}$$

$$f = f_0 + pf_1 + p^2f_2 + \cdots \tag{22}$$

$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 + \cdots \tag{23}$$

Assuming $f''' = 0, \theta'' = 0$, and substituting f from Eq. (22) into Eq. (20) and θ from Eq. (23) into Eq. (21) after some simplification, rearrangement and equating the similar terms based on powers of $p-$terms, since $p \in [0,1]$ is an embedded parameter for approximation solution and assumed that the solution can be written as a power series in p, we have:

$$p^0: f_0''' = 0,$$
$$f_0(0) = 0, \ f_0'(0) = 0, \ f_0'(\eta_\infty) = 1,$$
$$\theta_0'' = 0,$$
$$\theta_0(0) = 1, \ \theta_0(\eta_\infty) = 0. \tag{24}$$

$$p^1: f_1''' = -\left(\frac{m+1}{2}\right)f_0f_0'' - m(1-(f_0')^2),$$
$$f_1(0) = 0, \ f_1'(0) = 0, \ f_1'(\eta_\infty) = 0,$$
$$\theta_1'' = -\left(\frac{Pr(m+1)}{2}\right)f_0\theta_0',$$
$$\theta_1(0) = 0, \ \theta_1(\eta_\infty) = 0. \tag{25}$$

$$p^2: f_2''' = -\left(\frac{m+1}{2}\right)(f_0f_1'' + f_1f_0'') + 2mf_0'f_1',$$
$$f_2(0) = 0, \ f_2'(0) = 0, \ f_2'(\eta_\infty) = 0,$$
$$\theta_2'' = -\left(\frac{Pr(m+1)}{2}\right)(f_0\theta_1' + f_1\theta_0'),$$
$$\theta_2(0) = 0, \ \theta_2(\eta_\infty) = 0. \tag{26}$$

$$p^3: f_3''' = -\left(\frac{m+1}{2}\right)(f_0f_2'' + f_1f_1'' + f_2f_0'')$$
$$+ m(2f_0'f_2' + (f_1')^2)$$
$$f_3(0) = 0, \ f_3'(0) = 0, \ f_3'(\eta_\infty) = 0,$$
$$\theta_3'' = -\left(\frac{Pr(m+1)}{2}\right)(f_0\theta_2' + f_1\theta_1' + f_2\theta_0'),$$
$$\theta_3(0) = 0, \ \theta_3(\eta_\infty) = 0. \tag{27}$$

$$p^4: f_4''' = -\left(\frac{m+1}{2}\right)(f_3 f_0'' + f_2 f_1'' + f_1 f_2'' + f_0 f_3'')$$
$$+2m(f_1' f_2' + f_0' f_3')$$
$$f_4(0) = 0, \quad f_4'(0) = 0, \quad f_4'(\eta_\infty) = 0, \qquad (28)$$
$$\theta_4'' = -\left(\frac{\Pr(m+1)}{2}\right)(f_3 \theta_0' + f_2 \theta_1' + f_1 \theta_2' + f_0 \theta_3'),$$
$$\theta_4(0) = 0, \quad \theta_4(\eta_\infty) = 0.$$

Solving Eqs. (24)-(28):

$$f_0 = \frac{1}{2\eta_\infty}(\eta^2) \qquad (29)$$

$$f_1 = \frac{1}{480\eta_\infty^2}(-2\eta^5 + 5\eta^2\eta_\infty^3 + 6\eta^5 m - 80\eta^3 m\eta_\infty^2 \qquad (30)$$
$$+105\eta^2 m\eta_\infty^3)$$

$$f_2 = \frac{1}{161280\eta_\infty^3}(11\eta^8 - 28\eta^5\eta_\infty^3 + 26\eta^2\eta_\infty^6 \qquad (31)$$
$$+27\eta^8 m^2 - 896\eta^6 m^2\eta_\infty^2 + 1764\eta^5 m^2\eta_\infty^3 + \cdots)$$

$$f_3 = \frac{1}{1277337600\eta_\infty^4}(-1500\eta^{11} + 5445\eta^8\eta_\infty^3 \qquad (32)$$
$$-5742\eta^5\eta_\infty^6 + 825\eta^2\eta_\infty^9 + 3348\eta^{11} m^3 + \cdots)$$

$$f_4 = \frac{1}{27897053184000\eta_\infty^5}(557940\eta^{14} - 2730000\eta^{11}\eta_\infty^3 \qquad (33)$$
$$+4317885\eta^8\eta_\infty^6 - 1861860\eta^5\eta_\infty^9 + \cdots)$$

$$\theta_0 = \frac{1}{\eta_\infty}(-\eta + \eta_\infty) \qquad (34)$$

$$\theta_1 = \frac{1}{48\eta_\infty^2}(\eta^4 m\Pr - \eta\eta_\infty^3 m\Pr + \eta^4\Pr - \eta\eta_\infty^3\Pr) \qquad (35)$$

$$\theta_2 = \frac{1}{80640\eta_\infty^3}(-40\eta^7 m^2\Pr^2 + 35\eta^4 m^2\Pr^2\eta_\infty^3 \qquad (36)$$
$$+5\eta m^2\Pr^2\eta_\infty^6 + 12\eta^7 m^2\Pr + \cdots)$$

$$\theta_3 = \frac{1}{58060800\eta_\infty^4}(560\eta^{10} m^3\Pr^3 - 600\eta^7 m^3\Pr^3\eta_\infty^3 \qquad (37)$$
$$-75\eta^4 m^3\Pr^3\eta_\infty^6 + 115\eta m^3\Pr^3\eta_\infty^9 + \cdots)$$

$$\theta_4 = \frac{1}{2789705318400\eta_\infty^5}(-431200\eta^{13} m^4\Pr^4 \qquad (38)$$
$$-1724800\eta^{13} m^3\Pr^4 - 2587200\eta^{13} m^2\Pr^4 + \cdots)$$

3.0 RESULTS AND DISCUSSIONS

The value of η_∞ has its impact on the boundary layer thickness. The work of Cebeci [28] and Bird [32] reported the values of η_∞ as 8 and 5.64 for both situations when pressure gradient $m = 0$ for velocity profile and energy profile as Prandtl number $Pr = 1$. In Esmaeilpour and Ganji [9] the solution for the boundary layer flow with no pressure gradient, the η_∞ is chosen as 5 in generating the velocity and temperature. In our case, η_∞ has been taken 5.25 and 5.15 for the velocity and temperature profiles respectively.

Table 1 Different values for m when $\eta_\infty = 5.25$

η	$f(\eta)$ NM	Fathizadeh and Rashidi [1]	Amber et al. $\eta_\infty = 5.25$ HPM		
				m	
		0	0	−0.065	−0.11
0	0	0	0	0	0
0.2	0.006641	0.007793	0.00679	0.00377	0.00068
0.4	0.026676	0.029386	0.02717	0.01545	0.00333
0.6	0.059722	0.064757	0.06112	0.03553	0.00882
0.8	0.106108	0.113849	0.10856	0.06452	0.01804
1	0.165572	0.176556	0.16939	0.10288	0.03184
1.2	0.237949	0.252703	0.24341	0.15104	0.05111
1.4	0.329815	0.342031	0.33032	0.20938	0.07667
1.6	0.420321	0.444188	0.42976	0.27820	0.10938
1.8	0.529518	0.558712	0.54123	0.35773	0.15001
2	0.650024	0.685028	0.66410	0.44811	0.19929
2.2	0.781193	0.822444	0.79764	0.54934	0.25792
2.4	0.922290	0.970148	0.94102	0.66133	0.32648
2.6	1.072506	1.127221	1.09331	0.78384	0.40547
2.8	1.230977	1.292647	1.25349	0.91651	0.49525
3	1.396808	1.465334	1.42052	1.05885	0.59606
3.2	1.569095	1.644142	1.59336	1.21023	0.70799
3.4	1.746950	1.827919	1.77101	1.36994	0.83097
3.6	1.929525	2.015541	1.95253	1.53717	0.96476
3.8	2.116030	2.205961	2.13713	1.71108	1.10899
4	2.305746	2.398258	2.32418	1.89078	1.26311
4.2	2.498040	2.591683	2.51323	2.07542	1.42645
4.4	2.692361	2.785712	2.70404	2.26419	1.59826
4.6	2.882480	2.980074	2.89655	2.45635	1.77768
4.8	3.085321	3.174772	3.09086	2.65127	1.96382
5	3.283274	3.370069	3.28711	2.84841	2.15577

The aim of this section is to analyze the effects of various physical parameters on the function of non–Newtonian (Navier-Stokes equations) fluid, velocity and temperature distributions. The validation of the present method using homotopy perturbation method is checked with the results of the function of non–Newtonian fluid obtained by Fathizadeh and Rashidi [1] and the numerical results reported in it, in Table 1, when pressure gradient parameter $m = 0$ have been taken. Thus it can be observed in fourth column of the Table 1, are the results obtained in this paper, these seemed to be better than the results reported in Fathizadeh and Rashidi work [1] shown in the third column of Table 1, these results are more closed to the numerical (NM) results in the second column. Note that the values in second and third columns have been taken from [1]. For the better representation of the function of the non–Newtonian fluid, two other columns for different values of m have been given.

Table 2 For the different values of m when $\eta_\infty = 5.25$

η	$f'(\eta)$ NM	Fathizadeh and Rashidi [1]	Amber et al. $\eta_\infty = 5.25$ HPM m		
		0	0	−0.065	−0.11
0	0	0	0	0	0
0.2	0.066408	0.070328	0.067951	0.038199	0.00761
0.4	0.132764	0.140606	0.135847	0.078978	0.01962
0.6	0.198937	0.210705	0.203544	0.122279	0.036026
0.8	0.264709	0.28041	0.270803	0.167995	0.056813
1	0.32978	0.349425	0.337298	0.215963	0.081951
1.2	0.393776	0.417375	0.402616	0.265954	0.111382
1.4	0.456262	0.483811	0.466272	0.31767	0.145008
1.6	0.516757	0.548225	0.527725	0.370736	0.182683
1.8	0.574758	0.610057	0.586395	0.424703	0.224197
2	0.629766	0.668719	0.641694	0.479051	0.269266
2.2	0.68131	0.723611	0.693055	0.533196	0.317521
2.4	0.728982	0.77415	0.739966	0.586506	0.368501
2.6	0.772455	0.819799	0.782012	0.638321	0.421655
2.8	0.81151	0.860099	0.818912	0.68798	0.476342
3	0.846044	0.894707	0.850554	0.73485	0.531848
3.2	0.876081	0.923428	0.877026	0.778362	0.5874
3.4	0.901761	0.946255	0.898635	0.818047	0.642195
3.6	0.92333	0.963397	0.915908	0.853565	0.695433
3.8	0.941118	0.975307	0.929578	0.884736	0.746349
4	0.955518	0.982693	0.940539	0.911549	0.794251
4.2	0.966957	0.986519	0.94978	0.934168	0.838553
4.4	0.975871	0.987979	0.958285	0.952908	0.878796
4.6	0.982684	0.988443	0.966911	0.968204	0.914666
4.8	0.98779	0.98937	0.976249	0.98055	0.945998
5	0.991542	0.992164	0.986487	0.990439	0.972766

Table 2, are the results for velocity profile $f'(\eta)$ for the different values of pressure gradient parameter m at $\eta_\infty = 5.25$. Thus it is seen in the second and fourth columns of Table 2, are in close agreement with those published previously in Fathizadeh and Rashidi [1] in the third column. Note that the values in second and third columns have been taken from [1]. Rest of the columns have been given for the better representation of velocity profile $f'(\eta)$.
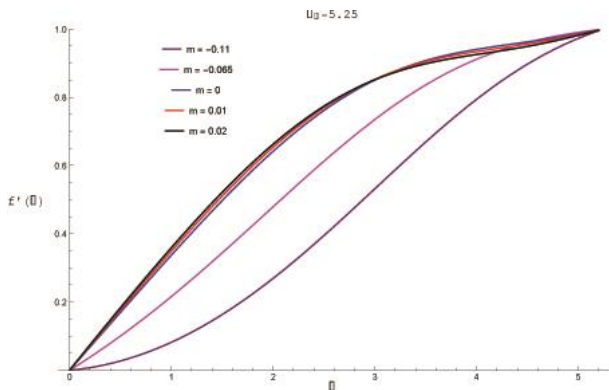

Figure 1 Velocity profile for $f'(\eta)$ for the different values of $-0.11 \leq m \leq 0.02$ when $\eta_\infty = 5.25$
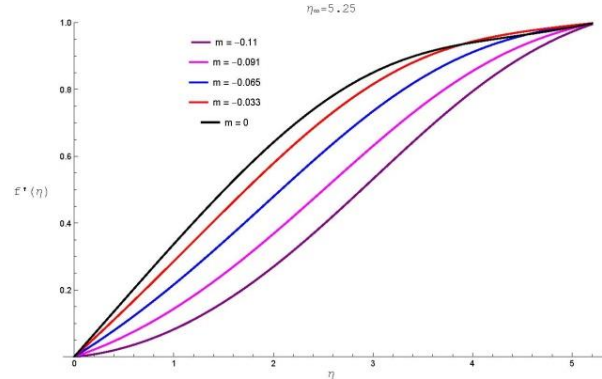

Figure 2 Velocity profile for $f'(\eta)$ for the different values of $-0.11 \leq m \leq 0$ when $\eta_\infty = 5.25$

Figures 1 and 2 show that the velocity profiles increase with increasing of m and consequently, the momentum boundary layer thickness becomes thinner and thinner.

Table 3 For the different values of m when $\eta_\infty = 5.15$

η	$\theta(\eta)$ NM	Fathizadeh and Rashidi [1]	Amber et al. $\eta_\infty = 5.15$ Pr HPM for m	
			1	0.5
		0	0	0
0	1	1	1	1
0.2	0.933592	0.929826	0.932044	0.944699
0.4	0.867236	0.859703	0.864143	0.88942
0.6	0.801063	0.789759	0.796441	0.83422
0.8	0.735291	0.720208	0.729173	0.779194
1	0.67022	0.651349	0.662666	0.724473
1.2	0.606224	0.583556	0.597329	0.670223
1.4	0.543738	0.517278	0.533644	0.616645
1.6	0.483243	0.453023	0.47215	0.563966
1.8	0.425242	0.391351	0.413418	0.512438
2	0.370234	0.332848	0.358032	0.462332
2.2	0.31869	0.278112	0.306551	0.413927
2.4	0.271018	0.227721	0.259476	0.367502
2.6	0.227545	0.182208	0.217212	0.323329
2.8	0.18849	0.142023	0.180029	0.281655
3	0.143955	0.107501	0.148027	0.242696
3.2	0.123918	0.078823	0.121109	0.206624
3.4	0.088239	0.055982	0.098964	0.173554
3.6	0.06667	0.038753	0.081069	0.143536
3.8	0.058882	0.026666	0.066707	0.116553
4	0.033043	0.018998	0.055013	0.09251
4.2	0.031482	0.014772	0.045054	0.071243
4.4	0.024129	0.012791	0.03592	0.052526
4.6	0.017317	0.011692	0.026858	0.036077
4.8	0.012211	0.010045	0.017398	0.021588
5	0.008458	0.006501	0.007485	0.00874

Table 3, are the results for energy profile $\theta(\eta)$ for the pressure gradient parameter $m = 0$ when Prandtl number $Pr = 1$ at $\eta_\infty = 5.15$. Thus it is seen in the second and fourth columns of Table 3, are in close agreement with those published previously in Fathizadeh and Rashidi [1] in the third column. The fifth column are the results of $\theta(\eta)$ taking $m = 0$ When $Pr = 0.5$ at $\eta_\infty = 5.15$. Note that the second and third columns have been taken same as in [1].
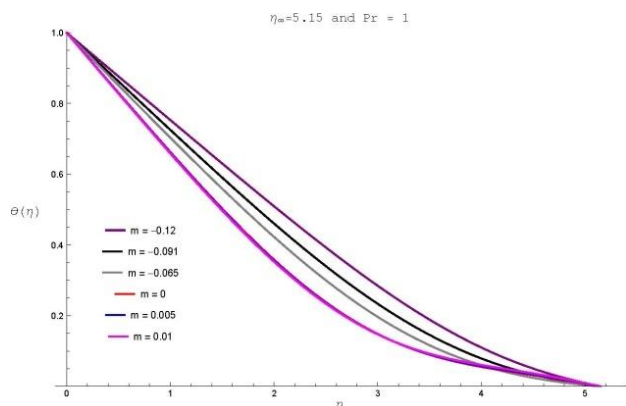

Figure 3 Energy profile for $\theta(\eta)$ for $-0.12 \leq m \leq 0.01$ when $\eta_\infty = 5.15$ and $Pr = 1$
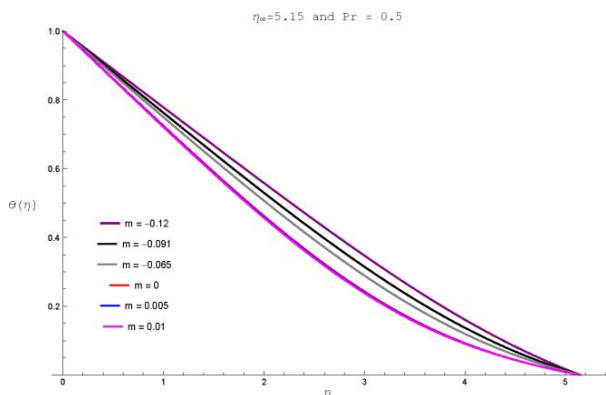

Figure 4 Energy profile for $\theta(\eta)$ for $-0.12 \leq m \leq 0.01$ when $\eta_\infty = 5.15$ and $Pr = 0.5$

Figures 3 and 4 show that the temperature profiles decrease with increasing m and hence the thermal boundary layer thickness becomes thinner and thinner. Where as in comparison of above figures, 3 have taken with $Pr = 1$ and 4 have taken with $Pr = 0.5$ which shows the thermal boundary layer thickness is more thinner with the higher value of Pr.

4.0 CONCLUSION

In this work we have calculated more values of velocity (i.e f_0, f_1, f_2, f_3 and f_4) for the better approximation of the system. The energy and momentum equations with pressure gradient are solved with an application of HPM using algorithms of Adams and Gear methods. The results generated are found to be in good agreement with those results

which are numerically acquired. Using HPM technique, for velocity profile the range of admissible pressure gradient (m) was -0.11 to 0.02 (i.e. $-0.11 \leq m \leq 0.02$). For velocity and energy profiles of the value η_∞ have been taken to be 5.25 and 5.15 when the Prandtl numbers (Pr) are 1 and 0.5, for energy profile the range of pressure gradient (m) has obtained as -0.12 to 0.01 (i.e $-0.12 \leq m \leq 0.01$). The momentum and thermal boundary layer thicknesses decrease with an increase in the value of pressure gradient. It could be interesting in future work to have a comparison and validation of this work with another approximation method known as Variational Iteration Method (VIM).

Acknowledgement

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