NUMERICAL MODELLING AND SIMULATION FOR ONE-DIMENSIONAL FLUID STRUCTURE INTERACTION IN BLOOD FLOW

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requirements for the award of the degree of
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To my beloved father, mother and family

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ABSTRACT

Fluid structure interaction (FSI) needs to be considered in modeling biofluids because the interaction between blood flow and vessel wall is of great clinical interest. However, the interaction between blood flow and vessel wall make FSI problems complex and challenging. Spurious oscillations were observed from numerical solutions and in the case of Bubnov-Galerkin finite element method, the oscillations occurred at relatively high pressure differences. In this thesis, Streamline-Upwind Petrov Galerkin (SUPG) stabilization scheme was formulated to solve one-dimensional FSI problems in blood flow to eliminate the spurious oscillations and to obtain stable numerical solutions for stenotic vessel. A pressure-area constitutive relation to complement the continuity equation and momentum equation was formulated by adopting the collapsible model. The geometry of stenotic vessel consists of single smooth and single irregular stenosis, multi-smooth and multi-irregular stenosis in this thesis. Numerical results show that there are no vessel collapse phenomena in single smooth stenosis and multi-smooth stenosis cases. Vessel collapse phenomena are observed for single-irregular stenosis with 85% cross sectional area amplitude at distal pressure of 47 mmHg while for multi-irregular stenosis with 60% and 85% cross sectional amplitudes at proximal stenosis and distal stenosis respectively, at distal pressure of 36 mmHg. In addition, paradoxical collapse motion along the time phase cycle is obtained in unsteady cases for single irregular stenosis and multi-irregular stenosis with the distal resistance of 2.73 mmHg/(ml/s) and 2.44 mmHg/(ml/s) respectively when sinusoidal pressure variation is applied at the inlet boundary. In conclusion, numerical results show that vessel collapse phenomena occurs when there is supercritical flow at the minimum cross sectional area of the stenotic vessel which is lower than the minimum cross sectional area at static condition and hence lead to the negative transmural pressure at that position.

ABSTRAK

Interaksi struktur bendalir (FSI) perlu diambil kira dalam pemodelan *biofluids* kerana interaksi antara aliran darah dengan dinding salur pembuluh mempunyai kepentingan klinikal yang besar. Walau bagaimanapun, interaksi antara aliran darah dengan dinding salur pembuluh menjadikan masalah FSI rumit dan mencabar. Ayunan palsu telah dapat diperhatikan dari penyelesaian berangka dan dalam kes kaedah unsur terhingga *Bubnov-Galerkin*, ayunan berlaku ketika terdapat perbezaan tekanan yang agak tinggi. Dalam tesis ini, skim penstabilan *Streamline-Upwind Petrov Galerkin* (SUPG) digubal untuk menyelesaikan masalah FSI satu dimensi dalam aliran darah untuk menghapuskan ayunan palsu dan mendapatkan penyelesaian berangka yang stabil bagi salur pembuluh stenosis. Hubungan konstitutif tekanan-kawasan untuk melengkapi persamaan keselanjaran dan persamaan momentum telah diformulasi dengan menggabungkan model boleh runtuh. Geometri salur pembuluh stenosis yang terdiri daripada stenosis tunggal yang seragam dan stenosis tunggal yang tidak seragam, stenosis pelbagai yang seragam dan stenosis pelbagai yang tidak seragam digunakan dalam tesis ini. Keputusan berangka menunjukkan bahawa tiada fenomena keruntuhan salur pembuluh dalam kes stenosis tunggal yang seragam dan stenosis pelbagai yang seragam. Fenomena keruntuhan salur pembuluh telah dikesan dalam kes stenosis tunggal yang tidak seragam dengan 85% amplitud kawasan keratan rentas stenosis pada tekanan distal 47 mmHg sementara dalam kes stenosis pelbagai yang tidak seragam masing-masing dengan 60% dan 85% amplitud keratan rentas pada stenosis proksimal dan stenosis distal, pada tekanan distal di 36 mmHg. Tambahan pula, keruntuhan salur pembuluh paradox di sepanjang masa kitaran fasa telah didapati untuk kes stenosis tunggal yang tidak seragam dan stenosis pelbagai yang tidak seragam dengan rintangan distal masing-masing 2.73 mmHg/(ml/s) dan 2.44 mmHg/(ml/s) apabila variasi tekanan sinusoidal digunakan pada sempadan masuk. Kesimpulannya, keputusan berangka telah membuktikan fenomena keruntuhan salur pembuluh terjadi apabila terdapat aliran superkritikal di kawasan keratan rentas minimum salur pembuluh stenosis adalah kurang daripada kawasan keratan rentas minimum pada keadaan statik dan seterusnya membawa kepada tekanan transmural negatif pada kedudukan tersebut.

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LIST OF ABBREVIATIONS

CFD	-	Computational Fluid Dynamics
FSI	-	Fluid Structure Interaction
SUPG	-	Streamline-Upwind Petrov-Galerkin
DG	-	Discontinuous Galerkin
MUSCL	-	Monotonic Upwind Scheme for Conservation Law
p-*A*	-	Pressure-area

LIST OF SYMBOLS

t	-	Time
x	-	Axial coordinate
A	-	Cross sectional area
u	-	Velocity
c	-	Local wave speed
Q	-	Volumetric flow rate
P	-	Pressure
P_e	-	External Pressure
$P - P_e$	-	Transmural pressure
α	-	Momentum correction factor
ρ	-	Fluid density
κ	-	Viscosity friction coefficient
μ	-	Fluid dynamic viscosity
β	-	Vessel stiffness for *p*-*A* Model 1
A_o	-	Cross sectional area of the flow at reference pressure P_o
h_o	-	Vessel's wall thickness at reference pressure P_o
v	-	Poisson ratio
E	-	Young's elastic modulus
L	-	Vessel length
A_{in}, A_{ou}	-	Flow cross sectional area at the inlet and outlet
K_p	-	Vessel stiffness for *p*-*A* Model 2
R	-	Mean flow radius
n_1 , n_2	-	Tube law exponents
F_{frict} , F_f	-	Lumped loss in the flow

f_L	-	Major laminar loss
D_e	-	Hydraulic diameter
K_{sep}	-	Separation loss coefficient
A_{th}	-	Throat area of stenosis
L_s	-	Length of separation region
x_{sep}	-	Separation point
D_o	-	Nominal vessel diameter
$A_o(x)$	-	Vessel area variation
A_{oo}	-	Nominal vessel area
$\lambda_A(x)$	-	Stenosis shape function
λ_{Ao}	-	Cross sectional area reduction amplitude
x_s	-	Starting point of stenosis
x_E	-	Stopping point of the stenosis
$K_p(x)$	-	Vessel stiffness variation
K_{po}	-	Nominal vessel stiffness
$\lambda_K(x)$	-	Stiffness variation amplitude
f	-	Frequency
P_1	-	Perfusion / inlet pressure
P_2	-	Distal / outlet pressure
$\lambda_{Ao1}, \lambda_{Ao2}$	-	Cross sectional area reduction amplitude for multi-stenosis
$\lambda_{Ko1}, \lambda_{Ko2}$	-	Stiffness reduction amplitude for multi-stenosis
x_{sep1}, x_{sep2}	-	Separation point for multi-stenosis
A_{th1}, A_{th2}	-	Throat area for multi-stenosis
x_{s1}, x_{s2}	-	Starting point for multi-stenosis
x_{E1}, x_{E2}	-	Stopping point for multi-stenosis
L_{BS}	-	Length between stenosis
U	-	Matrix of dependent variables
F	-	Flux matrix
B	-	Force matrix (conservation form)
H	-	Jacobian of flux vectors
w	-	Weighting function
N_i	-	Linear shape function

$\mathbf{P}(\mathbf{w})$	-	Operator applied to the test function
$\mathbf{R}(\mathbf{U})$	-	Residual of one-dimensional FSI equations
$\boldsymbol{\tau}$	-	Stabilization parameters
$\mathbf{S}(\mathbf{U})$	-	Force matrix (quasi-linear form)
$\boldsymbol{\lambda}$, $\boldsymbol{\Lambda}$	-	Eigenvalues of $\mathbf{H}$
$\mathbf{L}$, $\mathbf{L}_{\mathbf{1},\mathbf{2}}$	-	Left-eigenvectors of $\mathbf{H}$
C_{TI}	-	Time-independent compatibility conditions
C_{TD}	-	Time-dependent compatibility conditions
S	-	Speed index
R_{dis}	-	Distal resistance
A_{min}	-	Minimum cross sectional area
P_{min}	-	Minimum pressure
S_{max}	-	Maximum speed index
$\bar{Q}$	-	Average Flow Rate
$S1, S2$	-	Vessel position for multi-stenosis
A_{S1}, A_{S2}	-	Cross sectional area at vessel position $S1, S2$
P_{S1}, P_{S2}	-	Pressure at vessel position $S1, S2$
S_{S1}, S_{S2}	-	Speed index at vessel position $S1, S2$

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CHAPTER 1

INTRODUCTION

1.1 Research Background

Fluid structure interaction (FSI) is encountered and applicable in many different branches of engineering and science. For example, FSI is a crucial consideration in the design of many engineering systems such as aircraft and bridges. In general, FSI is defined as the interaction between the deformable structures with an internal or surrounding fluid flow. Such deformation can be either stable or oscillatory. The deformation of the structure contributes to the changes in boundary conditions of the fluid flow. Fluid flows encountered in our daily life include amongst others meteorological phenomena, environmental hazards, processes in human body such as blood flow and breathing.

FSI is more often considered in modelling biofluids because the interaction between the blood flow and vessel wall is of great clinical interest, for example, in studying cardiovascular diseases which are a major cause of death in developed countries (Mortazavinia *et al.*, 2012). The consideration of the interaction between blood flow and vessel wall had seldom being consideration in the previous studies due to the difficulty in solving the coupled fluid and solid equations (Zhang *et al.*, 2003). Although the assumptions of the rigid wall surfaces yield results that are reasonable accurate, there are still have some considerations to be taken into such that the elastic nature of the arterial wall, stresses on the arterial wall that play crucial role in arterial disease as well as the material property alterations with the development of the atherosclerotic lesion. (Kanyanta *et al.*, 2009; Friedman *et al.*, 2010; Siogkas *et al.*, 2011).

Recent studies about the effect of rigid wall and FSI on flow distributions in arterial modelling had been carried out. The axial velocities of rigid wall are higher compared to the ones in compliant model. Such situation is explained through mass conservation theory where the internal fluid pressure exerted on the vessel wall pushes the vessel wall outward consistently and slows fluid flow due to the flow area expansion. These findings showed that incorporating FSI has significant effects on blood flow characteristics, yet FSI models are computationally expensive when the arterial geometry is highly complicated. (Lee and Xu, 2002; Siogkas *et al.*, 2011; Mortazavinia *et al.*, 2012; He *et al.*, 2016).

1.2 Problem Statement

FSI describes the wave propagation in arteries driven by the pulsatile blood flow. From theoretical point of view, such problems are complex and challenging due to high nonlinearity of the problem. Two-dimensional and three-dimensional mathematical models are solved with the aid of the commercial software or black-box solvers, yet there are some considerations such as added mass effect, coupling conditions between fluid and structure and suitable boundary conditions to avoid the wave reflection influence the numerical stability.

Despite of the numerical stability obtained from the physical problems, there is wiggling phenomena observed in computational fluid dynamics (CFD) problems especially flow with high Peclet number or high Reynolds number. Same phenomenon is expected for one-dimensional FSI blood flow problems for relatively high pressure differences. Besides, numerical formulation and simulation become complicated to include the geometrical variation of the vessel such as the spatial variation of area and corresponding stiffness resulting from the attempt to model the stenosis. Moreover, flow in stenotic vessel is further complicated when there is choked flow or flow transition where vessel collapse is observed.

Thus, sets of governing equations together with suitable boundary conditions are important to study the flow behavior in straight and stenotic vessel. Numerical technique and formulation with oscillations free is significant in ensuring the attainment of reliable information and numerical results.

1.3 Objectives of Research

The objectives of this research are specified as follows:

1. To develop numerical method based on finite element method with Streamline-Upwind Petrov-Galerkin (SUPG) stabilization scheme to solve one-dimensional blood flow in a stenosed artery.
2. To determine the effect of geometry to the flow behavior by including the irregular shape and multi-stenosis geometry.
3. To determine the effect of area reduction amplitude to the flow behavior in smooth and irregular stenosis.
4. To determine the effect of distal pressure to the flow behavior in smooth and irregular stenosis.
5. To identify the physiological conditions for vessel collapse phenomena in stenotic vessel.

1.4 Scope of Research

The scope of this study is on the numerical modelling and simulation in one-dimensional FSI blood flow cases. One-dimensional, incompressible, Newtonian flow is considered in this study. Continuity equation, momentum equation and pressure-area constitutive relation are coupled and solved numerically with the employment of compatibility conditions at the boundary nodes. Besides, finite element method with SUPG stabilization formulation is employed as space discretization and first-order forward difference is employed as time discretization. For straight vessel, two types of pressure-area constitutive relations are coupled together with continuity equation and momentum equation, that are, nonlinear elastic model and collapsible model, which are termed as $p - A$ Model 1 and $p - A$ Model 2 respectively. Pressure differences for p-A Model 1 range from 400 Pa to 2500 Pa while pressure differences for p-A Model 2 range from 10 mmHg to 45 mmHg.

For stenotic vessel, one-dimensional, incompressible Newtonian flow with frictional losses is considered. Collapsible model is applied as pressure-area constitutive relation to describe the flow in stenotic vessel and capture the vessel collapse phenomena. Four different geometry of stenosis are discussed, which are single smooth stenosis, single irregular stenosis, multi-smooth stenosis and multi-irregular stenosis. For single stenosis and multi-stenosis cases, cross sectional area reduction amplitude vary from 60% to 85% and stiffness reduction amplitude is set 10. Perfusion pressure is set at 100 mmHg. Distal pressure is varying from 47 mmHg to 70 mmHg for single stenosis cases and 36 mmHg to 60 mmHg for multi-stenosis cases.

1.5 Significance of Research

First of all, SUPG stabilization scheme is formulated to solve one-dimensional FSI in straight and stenotic vessel. This study is significant as this would be the first application of SUPG in the study of one-dimensional FSI blood flow problems. Besides, one-dimensional FSI governing equations are solved numerically with the employment of compatibility conditions to minimize the wave reflection at the boundary. Compatibility conditions and SUPG stabilization term are derived from the characteristic system, which emphasizing the physical nature of the problem. With the approach that proposed in this study, the understanding on the characteristic flow behavior, physiological conditions to induce vessel collapse, relationship between cross sectional area, volumetric flow rate and pressure of the flow are observed.

1.6 Outline of Thesis

This thesis consists of seven chapters, including this introduction chapter. Chapter 1 introduces the general information about the thesis, including the research background, problem statement, objectives, scopes and significance of this study. Chapter 2 presents the literature review about FSI in blood flow. The chapter begins with the difficulty and challenges of considering FSI in two-dimensional and three-dimensional mathematical models which then contributes to the research on one-dimensional FSI model. Then, the governing equations and numerical works on one-dimensional FSI in blood flow are detailed. Chapter 3 discusses about the mathematical models which are used throughout the study, including the governing equations, initial conditions and boundary conditions.

Chapter 4 deliberates about the numerical technique and formulation in the study. The chapter begins with finite element method, followed by the SUPG formulation. The formulation of SUPG involves the stabilization term, which is correspond in adding the diffusion along the characteristic direction with the appropriate stabilization parameter. Nonreflecting boundary conditions which known as compatibility conditions are derived from the characteristic system of governing equations to minimize the outgoing characteristic waves at the boundary. Eigenvalues and left-eigenvectors of the Jacobian flux vectors are solved from the method of characteristics system in order to get the time-independent and time-dependent compatibility conditions. The coupled governing equations are discretized into matrix form and solved with Newton-Raphson nonlinear iterative solver. The stability criterion is discussed and an algorithm code is presented at the end of the chapter.

Chapter 5 discusses about the steady flow in the straight vessel, followed by stenotic vessel. Bubnov-Galerkin finite element is formulated for *p-A* Model 1, as in the work reported in Sochi (2015). However, when Bubnov-Galerkin finite element is applied to the higher pressure difference, that is, in the range higher than reported in Sochi (2015), spurious oscillations occur. In confirming the occurrence of the oscillations, *p-A* Model 2 is studied. Similar phenomenon is observed for both *p-A* models. Hence, SUPG stabilization scheme is formulated and the numerical results are validated with the analytical solutions for both *p-A* models. The analytical solution for *p-A* Model 1 is taken from Sochi (2015) while the analytical solution for *p-A* Model 2 is derived. Then SUPG formulation is extended to stenotic vessel and the numerical results are compared with the numerical works in Downing and Ku (1997). SUPG numerical results are shown to eliminate spurious oscillations obtained from Bubnov-Galerkin finite element formulation and provide reliable information of the flow. Afterward, SUPG formulation is extended to parametric variations study with the irregular geometry of stenosis. Four cases are studied and vessel collapse conditions are identified for single irregular stenosis and multi-irregular stenosis. The relationship between vessel collapse, pressure and speed index is discussed.

Chapter 6 discusses about the unsteady flow in stenotic vessel. Sinusoidal time variations are applied at the inlet boundary to mimic the pressure variation of 120/80 mmHg. Initially, SUPG unsteady numerical results are compared with Downing and Ku (1997). Parametric variations study is concerned as in previous chapter. Steady flow numerical results in previous chapter are applied as the initial values of unsteady flow cases in this chapter. Effect of distal resistance for all the cases are plotted and discussed. Furthermore, vessel conditions for each phase cycle are demonstrated by the plotting of the pressure distributions along the stenotic vessel throughout the phase cycle. Finally, the thesis ends with Chapter 7. Summary of research and some suggestions for future works are stated.

REFERENCES

Acosta, S., Puelz, C., Rivière, B., Penny, D. J. and Rusin, C. G. (2015). Numerical method of characteristics for one-dimensional blood flow. *Journal of Computational Physics,* 294**,** 96-109.

Anderson, J. (1995). *Computational Fluid Dynamics,* McGraw-Hill Education.

Andersson, C. and Ahl, D. (2011). *Fluid Structure Interaction - Evaluation of two coupling techniques.* Master Thesis, School of Information Science, Computer and Electrical Engineering, Halmstad University, Sweden.

Astorino, M., Chouly, F. and Fernández, M. A. (2009). An added-mass free semi-implicit coupling scheme for fluid-structure interaction. *Comptes Rendus Mathematique,* 347 (1-2)**,** 99-104.

Astorino, M., Chouly, F. and Fernández, M. A. (2010). Robin Based Semi-Implicit Coupling in Fluid-Structure Interaction: Stability Analysis and Numerics. *SIAM Journal on Scientific Computing,* 31 (6)**,** 4041-4065.

Badia, S., Nobile, F. and Vergara, C. (2008). Fluid-structure partitioned procedures based on Robin transmission conditions. *Journal of Computational Physics,* 227 (14)**,** 7027-7051.

Barnard, A. C. L., Hunt, W. A., Timlake, W. P. and Varley, E. (1966). A Theory of Fluid Flow in Compliant Tubes. *Biophysical Journal,* 6 (6)**,** 717-724.

Bathe, K. J. and Ledezma, G. A. (2007). Benchmark problems for incompressible fluid flows with structural interactions. *Computers & Structures,* 85 (11-14)**,** 628-644.

Bathe, K. J. and Zhang, H. (2004). Finite element developments for general fluid flows with structural interactions. *International Journal for Numerical Methods in Engineering,* 60 (1)**,** 213-232.

Benra, F.-K., Dohmen, H. J., Pei, J., Schuster, S. and Wan, B. (2011). A Comparison of One-Way and Two-Way Coupling Methods for Numerical Analysis of Fluid-Structure Interactions. *Journal of Applied Mathematics,* 2011**,** 1-16.

Bochev, P. B., Gunzburger, M. D. and Shadid, J. N. (2004). Stability of the SUPG finite element method for transient advection–diffusion problems. *Computer Methods in Applied Mechanics and Engineering,* 193 (23–26)**,** 2301-2323.

Boileau, E., Nithiarasu, P., Blanco, P. J., Müller, L. O., Fossan, F. E., Hellevik, L. R., Donders, W. P., Huberts, W., Willemet, M. and Alastruey, J. (2015). A benchmark study of numerical schemes for one‐dimensional arterial blood flow modelling. *International Journal for Numerical Methods in Biomedical Engineering,* 31 (10)**,** 1-33.

Brooks, A. N. and Hughes, T. J. R. (1982). Streamline upwind/Petrov-Galerkin formulations for convection dominated flows with particular emphasis on the incompressible Navier-Stokes equations. *Computer Methods in Applied Mechanics and Engineering,* 32 (1–3)**,** 199-259.

Bukac, M., Canic, S., Glowinski, R., Tambaca, J. and Quaini, A. (2013). Fluid-structure interaction in blood flow capturing non-zero longitudinal structure displacement. *Journal of Computational Physics,* 235**,** 515-541.

Burman, E. (2010). Consistent SUPG-method for transient transport problems: Stability and convergence. *Computer Methods in Applied Mechanics and Engineering,* 199 (17–20)**,** 1114-1123.

Canic, S. and Kim, E. H. (2003). Mathematical analysis of the quasilinear effects in a hyperbolic model blood flow through compliant axi-symmetric vessels. *Mathematical Methods in the Applied Sciences,* 26 (14)**,** 1161-1186.

Canic, S., Mikelic, A. and Tambaca, J. (2005). A two-dimensional effective model describing fluid-structure interaction in blood flow: analysis, simulation and experimental validation. *Comptes Rendus Mecanique,* 333 (12)**,** 867-883.

Canic, S. (2006). Fluid Structure Interaction in Blood flow. *Proceedings of the MSRI/AWM Workshop ``The Legacy of Ladizhenskaya and Oleinik."*, Berkeley, CA, 11-15.

Causin, P., Gerbeau, J. F. and Nobile, F. (2005). Added-mass effect in the design of partitioned algorithms for fluid-structure problems. *Computer Methods in Applied Mechanics and Engineering,* 194 (42-44)**,** 4506-4527.

Christie, I., Griffiths, D. F., Mitchell, A. R. and Zienkiewicz, O. C. (1976). Finite element methods for second order differential equations with significant first derivatives. *International Journal for Numerical Methods in Engineering,* 10 (6)**,** 1389-1396.

Codina, R. (1993). A discontinuity-capturing crosswind-dissipation for the finite element solution of the convection-diffusion equation. *Computer Methods in Applied Mechanics and Engineering,* 110 (3–4)**,** 325-342.

Codina, R. (2000). On stabilized finite element methods for linear systems of convection–diffusion-reaction equations. *Computer Methods in Applied Mechanics and Engineering,* 188 (1–3)**,** 61-82.

De Frutos, J., García-Archilla, B., John, V. and Novo, J. (2014). An adaptive SUPG method for evolutionary convection–diffusion equations. *Computer Methods in Applied Mechanics and Engineering,* 273**,** 219-237.

De Sampaio, P. a. B. and Coutinho, A. L. G. A. (2001). A natural derivation of discontinuity capturing operator for convection–diffusion problems. *Computer Methods in Applied Mechanics and Engineering,* 190 (46–47)**,** 6291-6308.

Degroote, J. (2013). Partitioned Simulation of Fluid-Structure Interaction. *Archives of Computational Methods in Engineering,* 20 (3)**,** 185-238.

Degroote, J., Bruggeman, P., Haelterman, R. and Vierendeels, J. (2008). Stability of a coupling technique for partitioned solvers in FSI applications. *Computers & Structures,* 86 (23-24)**,** 2224-2234.

Degroote, J., Haelterman, R., Annerel, S., Bruggeman, P. and Vierendeels, J. (2010). Performance of partitioned procedures in fluid–structure interaction. *Computers & Structures,* 88 (7–8)**,** 446-457.

Donea, J. and Huerta, A. (2003). *Finite Element Methods for Flow Problems,* England, John Wiley & Sons Ltd.

Downing, J. M. (1995). Flow through a compliant stenotic artery: a parametric evaluation. Armstrong Laboratory.

Downing, J. M. and Ku, D. N. (1997). Effects of Frictional Losses and Pulsatile Flow on the Collapse of Stenotic Arteries. *Journal of Biomechanical Engineering,* 119 (3)**,** 317-324.

Elad, D. and Kamm, R. D. (1989). Parametric Evaluation of Forced Expiration Using a Numerical Model. *Journal of Biomechanical Engineering,* 111 (3)**,** 192-199.

Elad, D., Kamm, R. D. and Shapiro, A. H. (1989). Steady compressible flow in collapsible tubes: application to forced expiration. *Journal of Fluid Mechanics,* 203**,** 401-418.

Elad, D., Katz, D., Kimmel, E. and Einav, S. (1991). Numerical schemes for unsteady fluid flow through collapsible tubes. *Journal of Biomedical Engineering,* 13 (1)**,** 10-18.

Fernández, M. A., Gerbeau, J. F. and Grandmont, C. (2006). A projection algorithm for fluid–structure interaction problems with strong added-mass effect. *Comptes Rendus Mathematique,* 342 (4)**,** 279-284.

Ferrari, F. B., Wolosker, N., Rosoky, R. A., D'ippolito, G., Wolosker, A. M. B. and Puech-Leão, P. (2004). Natural history of stenosis in the iliac arteries in patients with intermittent claudication undergoing clinical treatment. *Revista do Hospital das Clínicas,* 59**,** 341-348.

Formaggia, L., Nobile, F., Quarteroni, A. and Veneziani, A. (1999). Multiscale modelling of the circulatory system: a preliminary analysis. *Computing and Visualization in Science,* 2 (2-3)**,** 75-83.

Formaggia, L., Gerbeau, J. F., Nobile, F. and Quarteroni, A. (2001). On the coupling of 3D and 1D Navier-Stokes equations for flow problems in compliant vessels. *Computer Methods in Applied Mechanics and Engineering,* 191 (6-7)**,** 561-582.

Formaggia, L., Lamponi, D. and Quarteroni, A. (2003). One-dimensional models for blood flow in arteries. *Journal of Engineering Mathematics,* 47 (3-4)**,** 251-276.

Formaggia, L., Moura, A. and Nobile, F. (2007). On the stability of the coupling of 3D and 1D fluid-structure interaction models for blood flow simulations. *Esaim-Mathematical Modelling and Numerical Analysis-Modelisation Mathematique Et Analyse Numerique,* 41 (4)**,** 743-769.

Friedman, M. H., Krams, R. and Chandran, K. B. (2010). Flow Interactions with Cells and Tissues: Cardiovascular Flows and Fluid-Structure Interactions. *Annals of Biomedical Engineering,* 38 (3)**,** 1178-1187.

Giordano, M. and Magi, V. (2006). Petrov–Galerkin finite element stabilization for two-phase flows. *International Journal for Numerical Methods in Fluids,* 51 (9-10)**,** 1117-1129.

Gresho, P. M. and Lee, R. L. (1981). Don't suppress the wiggles—They're telling you something! *Computers & Fluids,* 9 (2)**,** 223-253.

Guidoboni, G., Glowinski, R., Cavallini, N. and Canic, S. (2009a). Stable loosely-coupled-type algorithm for fluid-structure interaction in blood flow. *Journal of Computational Physics,* 228 (18)**,** 6916-6937.

Guidoboni, G., Glowinski, R., Cavallini, N., Canic, S. and Lapin, S. (2009b). A kinematically coupled time-splitting scheme for fluid-structure interaction in blood flow. *Applied Mathematics Letters,* 22 (5)**,** 684-688.

He, F., Hua, L. and Gao, L. (2016). Effects of wall condition on flow distributions in arterial modeling: comparison of rigid, dynamic, and compliant walls. *Journal of Mechanical Science and Technology,* 30 (3)**,** 1471-1475.

Heil, M., Hazel, A. L. and Boyle, J. (2008). Solvers for large-displacement fluid–structure interaction problems: segregated versus monolithic approaches. *Computational Mechanics,* 43 (1)**,** 91-101.

Heinrich, J. C., Huyakorn, P. S., Zienkiewicz, O. C. and Mitchell, A. R. (1977). An 'upwind' finite element scheme for two-dimensional convective transport equation. *International Journal for Numerical Methods in Engineering,* 11 (1)**,** 131-143.

Hou, G., Wang, J. and Layton, A. (2012). Numerical Methods for Fluid-Structure Interaction - A Review. *Communications in Computational Physics*, 12 (2)**,** 337-377.

Hughes, T. J. R. and Mallet, M. (1986). A new finite element formulation for computational fluid dynamics: IV. A discontinuity-capturing operator for multidimensional advective-diffusive systems. *Computer Methods in Applied Mechanics and Engineering,* 58 (3)**,** 329-336.

Hughes, T. J. R., Mallet, M. and Akira, M. (1986). A new finite element formulation for computational fluid dynamics: II. Beyond SUPG. *Computer Methods in Applied Mechanics and Engineering,* 54 (3)**,** 341-355.

Hughes, T. J. R., Scovazzi, G. and Tezduyar, T. E. (2008). Stabilized Methods for Compressible Flows. *Journal of Scientific Computing,* 43 (3)**,** 343-368.

Idelsohn, S. R., Del Pin, F., Rossi, R. and Onate, E. (2009). Fluid-structure interaction problems with strong added-mass effect. *International Journal for Numerical Methods in Engineering,* 80 (10)**,** 1261-1294.

Janela, J., Moura, A. and Sequeira, A. (2010a). A 3D non-Newtonian fluid-structure interaction model for blood flow in arteries. *Journal of Computational and Applied Mathematics,* 234 (9)**,** 2783-2791.

Janela, J., Moura, A. and Sequeira, A. (2010b). Absorbing boundary conditions for a 3D non-Newtonian fluid-structure interaction model for blood flow in arteries. *International Journal of Engineering Science,* 48 (11)**,** 1332-1349.

Ji, J., Kobayashi, S., Morikawa, H., Tang, D. and Ku, D. N. (2008). Influences of External Pressure on Flow and Deformation in Arterial Stenosis Model. *Journal of Biomechanical Science and Engineering,* 3 (2)**,** 75-84.

Ji, J., Kobayashi, S., Morikawa, H., Tang, D. and Ku, D. N. (2009). Influence of Cyclic Change of Distal Resistance on Flow and Deformation in Arterial Stenosis Model. *In:* Lim, C. T. and Goh, J. C. H. (eds.) *13th International Conference on Biomedical Engineering: ICBME 2008 3–6 December 2008 Singapore*. Berlin, Heidelberg: Springer Berlin Heidelberg.

Ji, J., Kobayashi, S., Morikawa, H., Tang, D. and Ku, D. N. (2010). Diastolic Predominant Flow in Compliant Coronary Stenosis Model. *Journal of Biomechanical Science and Engineering,* 5 (4)**,** 303-313.

Ji, J., Toubaru, S., Kobayashi, S., Morikawa, H., Tang, D. and Ku, D. N. (2011). Flow and Deformation in a Multi-Component Arterial Stenosis Model. *Journal of Biomechanical Science and Engineering,* 6 (2)**,** 79-88.

Kamm, R. D. and Shapiro, A. H. (1979). Unsteady Flow in a Collapsible Tube Subjected to External Pressure or Body Forces. *Journal of Fluid Mechanics,* 95**,** 1-78.

Kanyanta, V., Ivankovic, A. and Karac, A. (2009). Validation of a fluid-structure interaction numerical model for predicting flow transients in arteries. *Journal of Biomechanics,* 42 (11)**,** 1705-1712.

Kececioglu, I., Mcclurken, M. E., Kamm, R. D. and Shapiro, A. H. (1981). Steady, supercritical flow in collapsible tubes. Part 1. Experimental observations. *Journal of Fluid Mechanics,* 109**,** 367-389.

Kelly, D. W., Nakazawa, S., Zienkiewicz, O. C. and Heinrich, J. C. (1980). A note on upwinding and anisotropic balancing dissipation in finite element approximations to convective diffusion problems. *International Journal for Numerical Methods in Engineering,* 15 (11)**,** 1705-1711.

Kimmel, E., Kamm, R. D. and Shapiro, A. H. (1988). Numerical Solutions for Steady and Unsteady Flow in a Model of the Pulmonary Airways. *Journal of Biomechanical Engineering,* 110 (4)**,** 292-299.

Kobayashi, S., Tang, D. and Ku, D. N. (2004). Collapse in High-Grade Stenosis during Pulsatile Flow Experiments. *JSME International Journal Series C Mechanical Systems, Machine Elements and Manufacturing,* 47 (4)**,** 1010-1018.

Kolachalama, V. B., Bressloff, N. W., Nair, P. B. and Shearman, C. P. (2007). Predictive haemodynamics in a one-dimensional human carotid artery bifurcation. Part I: Application to stent design. *IEEE Trans Biomed Eng,* 54 (5)**,** 802-812.

Ku, D. N., Zeigler, M. N. and Downing, J. M. (1990). One-Dimensional Steady Inviscid Flow Through a Stenotic Collapsible Tube. *Journal of Biomechanical Engineering,* 112 (4)**,** 444-450.

Le Beau, G. J., Ray, S. E., Aliabadi, S. K. and Tezduyar, T. E. (1993). SUPG finite element computation of compressible flows with the entropy and conservation variables formulations. *Computer Methods in Applied Mechanics and Engineering,* 104 (3)**,** 397-422.

Lee, C. J., Zhang, Y., Takao, H., Murayama, Y. and Qian, Y. (2013). A fluid–structure interaction study using patient-specific ruptured and unruptured aneurysm: The effect of aneurysm morphology, hypertension and elasticity. *Journal of Biomechanics,* 46 (14)**,** 2402-2410.

Lee, J. and Smith, N. (2008). Development and application of a one-dimensional blood flow model for microvascular networks. *Proceedings of the Institution of Mechanical Engineers Part H-Journal of Engineering in Medicine,* 222(4)**,** 487-511.

Lee, K. W. and Xu, X. Y. (2002). Modelling of flow and wall behaviour in a mildly stenosed tube. *Medical Engineering & Physics,* 24 (9)**,** 575-586.

Mates, R. E., Gupta, R. L., Bell, A. C. and Klocke, F. J. (1978). Fluid dynamics of coronary artery stenosis. *Circulation Research,* 42 (1)**,** 152-162.

Mcclurken, M. E., Kececioglu, I., Kamm, R. D. and Shapiro, A. H. (1981). Steady, supercritical flow in collapsible tubes. Part 2. Theoretical studies. *Journal of Fluid Mechanics,* 109**,** 391-415.

Melicher, V. and Gajdosik, V. (2008). A numerical solution of a one-dimensional blood flow model—moving grid approach. *Journal of Computational and Applied Mathematics,* 215 (2)**,** 512-520.

Michler, C., Hulshoff, S. J., Van Brummelen, E. H. and De Borst, R. (2004). A monolithic approach to fluid-structure interaction. *Computers & Fluids,* 33 (5-6)**,** 839-848.

Mortazavinia, Z., Goshtasbi, R. E., Emdad, H., Sharifkazemi, M. B., Zare, A. and Mehdizadeh, A. R. (2012). Study of pulsatile non-Newtonian blood flow through abdominal aorta and renal arteries incorporating fluid structure interaction. *J Biomed Phys Eng,* 2(3)**,** 93-102.

Nisam, P. R., Binu, L. S. and Sukesh, A. K. (2009). One Dimensional Modeling and Computation of Blood Flow and Pressure of a Stented Artery. *2009 3rd International Conference on Bioinformatics and Biomedical Engineering.* 11-13 June 2009. Beijing, China, 1-5.

Nobile, F. (2001). *Numerical Approximation of Fluid Structure Interaction with Application to Hemodynamics.* PhD Thesis, EPFL, Switzerland.

Nobile, F. and Vergara, C. (2008). An effective fluid-structure interaction formulation for vascular dynamics by generalized Robin conditions. *Siam Journal on Scientific Computing,* 30 (2)**,** 731-763.

Olufsen, M. S., Peskin, C. S., Kim, W. Y., Pedersen, E. M., Nadim, A. and Larsen, J. (2000). Numerical Simulation and Experimental Validation of Blood Flow in Arteries with Structured-Tree Outflow Conditions. *Annals of Biomedical Engineering,* 28 (11)**,** 1281-1299.

Pain, C. C., Eaton, M. D., Smedley-Stevenson, R. P., Goddard, A. J. H., Piggott, M. D. and De Oliveira, C. R. E. (2006). Streamline upwind Petrov–Galerkin methods for the steady-state Boltzmann transport equation. *Computer Methods in Applied Mechanics and Engineering,* 195 (33–36)**,** 4448-4472.

Quarteroni, A. and Formaggia, L. (2003). Mathematical Modelling and Numerical Simulation of the Cardiovascular System. *Modelling of Living Systems, Handbook of Numerical Analysis Series.* Amsterdam: Elsevier.

Raghu, R. and Taylor, C. A. (2011). Verification of a one-dimensional finite element method for modeling blood flow in the cardiovascular system incorporating a viscoelastic wall model. *Finite Elements in Analysis and Design,* 47 (6)**,** 586-592.

Raghu, R., Vignon-Clementel, I. E., Figueroa, C. A. and Taylor, C. A. (2011). Comparative Study of Viscoelastic Arterial Wall Models in Nonlinear One-Dimensional Finite Element Simulations of Blood Flow. *Journal of Biomechanical Engineering,* 133 (8)**,** 1-11.

Saito, D., Kusachi, S., Nishiyama, O., Yasuhara, K., Yoshida, H. and Haraoka, S. (1980). Increase in the calculated resistance of anatomically fixed stenosis in vitro in association with decrease in distal resistance. *Acta medica Okayama,* 34 (2)**,** 109-115.

Saito, M., Ikenaga, Y., Matsukawa, M., Watanabe, Y., Asada, T. and Lagrée, P.-Y. (2011). One-Dimensional Model for Propagation of a Pressure Wave in a Model of the Human Arterial Network: Comparison of Theoretical and Experimental Results. *Journal of Biomechanical Engineering,* 133 (12)**,** 1-9.

Sevilla, R., Hassan, O. and Morgan, K. (2013). An analysis of the performance of a high-order stabilised finite element method for simulating compressible flows. *Computer Methods in Applied Mechanics and Engineering,* 253**,** 15-27.

Shapiro, A. H. (1977). Steady Flow in Collapsible Tubes. *Journal of Biomechanical Engineering,* 99 (3)**,** 126-147.

Sherwin, S. J., Formaggia, L., Peiró, J. and Franke, V. (2003a). Computational modelling of 1D blood flow with variable mechanical properties and its application to the simulation of wave propagation in the human arterial system. *International Journal for Numerical Methods in Fluids,* 43 (6-7)**,** 673-700.

Sherwin, S. J., Franke, V., Peiró, J. and Parker, K. (2003b). One-dimensional modelling of a vascular network in space-time variables. *Journal of Engineering Mathematics,* 47 (3-4)**,** 217-250.

Sieber, M. S. G. (2001). *Numerical Simulation of Fluid-Structure Interaction Using Loose Coupling Methods.* Doctoral Thesis, Technical University Darmstadt,Germany.

Siogkas, P. K., Sakellarios , A. I., Exarchos, T. P., Stefanou, K., Fotiadis, D. I., Naka, K., Michalis, L., Filipovicn. and Parodi, O. (2011). Blood flow in arterial segments: rigid vs. deformable walls simulations. *Journal of the Serbian Society for Computational Mechanics,* 5 **,** 69-77.

Sochi, T. (2013). *One-Dimensional Navier-Stokes Finite Element Flow Model*. Imaging Sciences and Biomedical Engineering, King's College London, St Thomas' Hospital, London, England.

Sochi, T. (2014). The flow of Newtonian and power law fluids in elastic tubes. *International Journal of Non-Linear Mechanics,* 67**,** 245-250.

Sochi, T. (2015). Navier-Stokes Flow in Cylindrical Elastic Tubes. *Journal of Applied Fluid Mechanics ,* 8 (2)**,** 181-188.

Soulaimani, A. and Fortin, M. (1994). Finite element solution of compressible viscous flows using conservative variables. *Computer Methods in Applied Mechanics and Engineering,* 118 (3–4)**,** 319-350.

Steele, B. N. (2007). Using one-dimensional finite element analysis to estimate differential pressure of renal artery stenoses. *2007 Computers in Cardiology,* 30 Sept - 3 Oct 2007. 391-394.

Tang, D., Yang, C., Kobayashi, S. and Ku, D. N. (2001). Steady Flow and Wall Compression in Stenotic Arteries: A Three-Dimensional Thick-Wall Model With Fluid–Wall Interactions. *Journal of Biomechanical Engineering,* 123 (6)**,** 548-557.

Tang, D., Yang, C., Kobayashi, S., Zheng, J. and Vito, R. (2003). Effect of Stenosis Asymmetry on Blood Flow and Artery Compression: A Three-Dimensional Fluid-Structure Interaction Model. *Annals of Biomedical Engineering,* 31 (10)**,** 1182-1193.

Tang, D., Yang, C., Kobayashi, S., Zhang, J., Woodard, P. K., Teng, Z., Billar, K., Bach, R. and Ku, D. K. (2009). 3D MRI-Based Anisotropic FSI Models With Cyclic Bending for Human Coronary Atherosclerotic Plaque Mechanical Analysis. *Journal of Biomechanical Engineering,* 131**,** 1-11.

Tang, D. L., Yang, C., Kobayashi, S. and Ku, D. N. (2004). Effect of a lipid pool on stress/strain distributions in stenotic arteries: 3-D fluid-structure interactions (FSI) models. *Journal of Biomechanical Engineering-Transactions of the Asme,* 126 (3)**,** 363-370.

Tezduyar, T. E. and Osawa, Y. (2000). Finite element stabilization parameters computed from element matrices and vectors. *Computer Methods in Applied Mechanics and Engineering,* 190 (3–4)**,** 411-430.

Tezduyar, T. E. and Park, Y. J. (1986). Discontinuity-capturing finite element formulations for nonlinear convection-diffusion-reaction equations. *Computer Methods in Applied Mechanics and Engineering,* 59 (3)**,** 307-325.

Tezduyar, T. E. and Senga, M. (2006). Stabilization and shock-capturing parameters in SUPG formulation of compressible flows. *Computer Methods in Applied Mechanics and Engineering,* 195 (13–16)**,** 1621-1632.

Thompson, K. W. (1987). Time dependent boundary conditions for hyperbolic systems. *Journal of Computational Physics,* 68 (1)**,** 1-24.

Van Brummelen, E. H. (2009). Added Mass Effects of Compressible and Incompressible Flows in Fluid-Structure Interaction. *Journal of Applied Mechanics-Transactions of the Asme,* 76 (2)**,** 1-11.

Van De Vosse, F. N. and Stergiopulos, N. (2011). Pulse wave propagation in the arterial tree. *Annual Review of Fluid Mechanics,* 43**,** 467-499.

Vesely, O., Novakova, L. and Adamec, J. (2015). Experimental study of effect of stenosis geometry on flow parameters. *EPJ Web of Conferences,* 92**,** 1-5.

Vesely, O., Novakova, L. and Adamec, J. (2016). Experimental study of effect of stenosis geometry on pressure loss for periodic flow. *EPJ Web of Conferences,* 114**,** 1-4.

Vierendeels, J., Lanoye, L., Degroote, J. and Verdonck, P. (2007). Implicit coupling of partitioned fluid-structure interaction problems with reduced order models. *Computers & Structures,* 85 (11-14)**,** 970-976.

Vignon, I. E. and Taylor, C. A. (2004). Outflow boundary conditions for one-dimensional finite element modeling of blood flow and pressure waves in arteries. *Wave Motion,* 39 (4)**,** 361-374.

Waite, L. and Fine, J. M. (2007). *Applied Biofluid Mechanics*, McGraw-Hill Education.

Wang, X., Fullana, J.-M. and Lagrée, P.Y. (2014). Verification and comparison of four numerical schemes for a 1D viscoelastic blood flow model. *Computer Methods in Biomechanics and Biomedical Engineering,* 18 (15)**,** 1704-1725.

Westerhof, N., Stergiopulos, N. and Noble, M. I. M. (2004). *Snapshots of Hemodynamics: An aid for clinical research and graduate education*, Springer US.

Whittaker, R. J., Heil, M., Jensen, O. E. and Waters, S. L. (2010a). Predicting the onset of high-frequency self-excited oscillations in elastic-walled tubes.

Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences, 466 **,** 3635-3657.

Whittaker, R. J., Heil, M., Jensen, O. E. and Waters, S. L. (2010b). A Rational Derivation of a Tube Law from Shell Theory. *The Quarterly Journal of Mechanics and Applied Mathematics,* 63 (4)**,** 465-496.

Yilmaz, F. and Gundogdu, M. Y. (2008). A critical review on blood flow in large arteries; relevance to blood rheology, viscosity models, and physiologic conditions. *Korea-Australia Rheology Journal,* 20 (4)**,** 197-211.

Zhang, H., Zhang, X., Ji, S., Guo, Y., Ledezma, G., Elabbasi, N. and Decougny, H. (2003). Recent development of fluid–structure interaction capabilities in the ADINA system. *Computers & Structures,* 81 (8–11)**,** 1071-1085.