# CONSTRUCTION OF SEASONAL BASED DEPTH DURATION FREQUENCY CURVES

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Dedicated to:

My beloved parents, Nojumuddin Abdul Samad, Hasnah Buang

> My inspired husband, Mohd Zulariffin Md Maarof

My supportive siblings, Nahdzah, Hishamuddin, Izzudin, Ridhauddin

My dedicated lecturers,

My endless spirits

and all my friends.

This is for you.

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#### ABSTRACT

Rainfall characterization is vital for effective management of water resources especially for designing and operating hydraulic structures. In Malaysia, the frequent occurrence of flood especially urban areas has caused tremendous losses and damages to properties and environment. Rainfall generation model is crucial since the adequate long historical data sets are frequently not available. Hence, the derivation of depth-duration frequency (DDF) with valid statistical results is very important in hydrological studies. This study uses four approaches to achieve this. Firstly, based on 40 years hourly rainfall data from the years 1972-2011 for six states in Peninsular Malaysia, appropriate values of Minimum Inter-Event Time (MIT) to separate rainfall data into individual events using auto-correlation and examination analysis approaches are used. Secondly, based on appropriate MIT values, cluster analysis combined with Huff method are used to identify independent rainfall patterns. Thirdly, selected stations taken from each state are used to generate statistical moments of storm characteristic using Monte Carlo-based (MC) approach. Finally, DDF is derived using MC-based and Copula-based approaches. The copulabased DDF is constructed using the conditional Frank Archimedean. The results show that appropriate MIT values for Johor, Melaka, Perak, Kedah, Kelantan and Pahang are 8 hours, 6 hours, 5 hours, 8 hours, 9 hours and 8 hours, respectively. The rainfall patterns constructed show that relatively high rainfall intensities during early and centre parts of the rainfall events are found in all states except Perak, Kelantan and Pahang during the northeast monsoon; relatively high rainfall intensities are detected in the early parts of the rainfall events for Melaka, Perak and Kedah during the southwest monsoon and Johor for inter-monsoons. The results also show that the MC-based method can preserve well the true moments of the storm characteristics. In addition, the MC-based DDF performs better than the Copula-based DDF as the MCbased method shows smaller errors due to the empirical DDF procedure. However, the Copula-based method is more independent and more flexible in constructing the DDF.

#### ABSTRAK

Perincian hujan adalah penting untuk pengurusan sumber air secara efektif terutamanya untuk mereka dan mengendalikan struktur hidraulik. Di Malaysia, kekerapan banjir terutamanya di kawasan bandar menyebabkan kerugian dan kerosakan yang amat besar terhadap harta benda dan alam sekitar. Model penjanaan hujan sangat penting memandangkan set data sejarah selalunya tidak mencukupi. Oleh itu, pemerolehan frekuensi tempoh masa (DDF) dengan keputusan statistik yang sahih sangat penting dalam kajian hidrologi. Oleh hal yang demikian, kajian ini menggunakan empat pendekatan untuk mencapai objektif kajian. Pertama, 40 tahun data curahan hujan daripada tahun 1972 hingga 2011 dari enam negeri di Semenanjung Malaysia digunakan untuk menentukan nilai masa minimum antara peristiwa (MIT) yang sesuai untuk mengasingkan curahan hujan dan peristiwa tersebut dengan menggunakan analisa autokorelasi dan analisa ujian. Kedua, dengan menggunakan nilai MIT yang sesuai, pengenalpastian pola curahan hujan dilakukan dengan menggunakan analisis gugusan statistik yang digabungkan dengan kaedah Huff. Ketiga, penghasilkan ciri-ciri ribut momen statistik berdasarkan penilaian Monte Carlo (MC) dilakukan di stesen yang terpilih. Akhir sekali, penjanaan DDF menggunakan model MC atau Copula dilaksanakan. DDF berdasarkan kebarangkalian Copula dibina menggunakan model Frank Archimedean bersyarat. Keputusan mendapati nilai MIT yang berpatutan bagi Johor, Melaka, Perak, Kedah, Kelantan dan Pahang masing-masing ialah 8 jam, 6 jam, 5 jam, 8 jam, 9 jam dan 8 jam. Pola curahan hujan yang dibina mengesahkan bahawa intensiti curahan hujan yang agak tinggi pada awal dan pertengahan ketika hujan berlaku di semua negeri kecuali Perak, Kelantan dan Pahang ketika monsun timur laut; intensiti curahan hujan yang agak tinggi pada peringkat awal di Melaka, Perak dan Kedah ketika monsun barat daya dan Johor ketika di antara musim. Keputusan juga menunjukkan kaedah MC boleh memelihara momen sebenar ciri-ciri ribut dengan baik. Disamping itu, DDF daripada kaedah MC memberikan prestasi yang lebih baik berbanding DDF daripada kaedah Copula kerana MC mempunyai ralat yang lebih kecil disebabkan oleh kaedah MC dipengaruhi oleh prosedur empirik DDF. Walau bagaimanapun, kaedah Copula lebih bebas dan fleksibel untuk menjana data DDF.

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### LIST OF SYMBOLS

$r_k$	-	Autocorrelation of coefficient of a sample observations
k	-	Lag time
<i>Y</i> <sub>t</sub>	-	The observation at time, t
$\overline{y}$	-	The mean of sample
$F_{ au}$	-	The dimensionless cumulative rainfall
$D_{ au}$	-	The cumulative rainfall depth at time, $t$
$D_t$	-	The total rainfall depth
С	-	Centroid coordinates of the points
D	-	Distance between points and in a Euclidean space
$\chi^{2}$	-	The chi-square test
df	-	The degree of freedom
n <sub>c</sub>	-	Number of column
n <sub>r</sub>	-	Number of row
t	-	Storm duration
b	-	Storm separation time
V	-	Storm depth
$A^2$	-	Anderson Darling
$ ho_{ij}$	-	correlation coefficient in the original space
$ ho_{ij}^{}^{*}$	-	correlation coefficient in the normal space
$\mu_{i},\mu_{j}$	-	Mean of correlated variables
$\sigma_{i},\sigma_{j}$	-	Standard deviation of correlated variables
$x_i, x_j$	-	Correlated variables in original space

$\varphi_{ij}(ullet)$	-	The joint standard normal density function
U	-	The mutually independent standard normal random
L	-	Lower triangular matrix
$F_i^{-1}(ullet)$	-	Marginal cumulative distribution function (CDF) of
		random variable
$\varphi(ullet)$	-	The standard normal CDF
$z_i, z_j$	-	The bivariate standard normal variables
$L^{-1}$	-	The inverse Cholesky transformation
$T_{ij}$	-	Transformation factor
e <sub>i</sub>	-	The percentage error
$e_{mabs}$	-	Mean-absolute percentage error
e <sub>rms</sub>	-	Root-mean-squared percentage error
$\theta_n$	-	Mean and standard deviation of observed data
$ heta_{\!\scriptscriptstyle m,n-m}$	-	Mean and standard deviation of simulated data
$K_T$	-	Frequency factor
V	-	Depth of Empirical-based
$V_{S}$	-	Depth of MC-based
$V_{C}$	-	Depth Of Copula-based
τ	-	Coefficient correlation
$\theta$	-	Parameter of Frank Copula distribution
$\Delta_s$	-	Percentage error between MC-based and Empirical-based
		DDF
$\Delta_c$	-	Percentage error between Copula-based and Empirical-

based DDF

# LIST OF ABBREVIATIONS

MIT	-	Minimum inter-event Time
GM	-	Gamma
GMax	-	Gumbel Maximum
Gmin	-	Gumbel Minimum
LG	-	Lognormal
WB	-	Weibull
A-D	-	Anderson Darling
MC	-	Monte Carlo simulation
DDF	-	Depth-duration-frequency

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### **CHAPTER 1**

### **INTRODUCTION**

### 1.1 Introduction

This chapter discussed the background, problem statement, objectives, significance and scope of the study.

### **1.2** Background of the study

The climate of Malaysia is driven by its equatorial position, extensive coastlines on tropical seas and monsoonal winds with high temperatures, high humidity, relatively light winds, and abundant rainfall throughout the year. There are two seasons of monsoon which are the southwest and northeast monsoons (longest in duration) and two inter-monsoon seasons (shortest duration). Seasonal wind flow patterns and local topographic features determine the precipitation pattern. The northeast monsoon blows from November to March which is responsible for the heavy rains while the period occurs between May and September is a drier period of southwest monsoon. The period between these two monsoons is inter-monsoon seasons which occurs and are marked by heavy rainfall.

The natural results of cyclical monsoons during the local tropical wet season that are characterised by heavy and regular rainfall from roughly October to March are the main cause of floods occurrence. Floods frequently occurred in the Malaysia during the northeast monsoons. For example, northeast monsoon was claimed for heavy rains of up to 350 mm within 24 hours in southern Peninsular Malaysia, specifically Johor, Negeri Sembilan, Melaka and Pahang, causing massive floods within the southern region on December 2006, which were considered as the worst in the history of the southern region of Malaysia. A series of flash floods also hit different areas of Malaysia in November 2010. Recently, in December 2014, the northern and eastern states of Kelantan, Terengganu, Pahang, Perak and Perlis in Peninsular Malaysia were hit by flash floods including some areas in Sabah.

The high atmospheric water vapor can lead to more intense precipitation events. Furthermore, rising temperatures and subsequent increases in atmospheric moisture content may increase the probable maximum precipitation (PMP) or the expected extreme precipitation. Consequently, global warming increases the risk of climatic extremes including floods and damage to infrastructure such as dams, roads and sewer and storm water drainage systems.

Flood has far-reaching impact on the well-being of the population. The flood occurrence in Malaysia will caused the property and structural damage. In addition, the flood occurrence can endanger the lives of humans as the flood victims had to live without clean water and electricity. Some methods of flood control have been practiced since ancient times. These methods include planting vegetation to retain extra water, terracing hillsides to slow flow downhill, and the construction of floodways. Other techniques include the construction of levees, lakes, dams, reservoirs and retention ponds to hold extra water during times of flooding.

The other methods which can be used in controlling floods are managing flood risk in a changing climate. The increase of the intensity and frequency of extreme rainfall in the climate change will increase the region's flood catastrophes, human casualties and economic loss. Extreme climatic events are growing more severe and frequent, calling into question on how to prepare our infrastructure deal with these changes. Therefore, the Intensity-Duration-Frequency (IDF) or Depth-Duration-Frequency (DDF) curves are use in designing the storm water system to deal with these changes. Thus, this study aims to propose an updated model to construct rainfall DDF as it is one of the most commonly used tools in water resources engineering in water management planning. Since the rainfall in Malaysia is governed by the monsoon season, therefore, the study on monsoonal-based should be focused in Malaysia. Hence, this study will also identify the rainfall pattern and generate the rainstorm characteristics. As the study is conducted by event-based, therefore, the suitable method to separate the rainfall into the event is investigated first by using suitable methods.

### **1.3** Statement of the Problem

The impact of a changing climate is already being felt on several hydrological systems both on a regional and sub-regional. All regions affected by climate change, includes Malaysia. The impacts of climate change are an increase in the intensity and frequency of extreme rainfall which can increase the flood risk. To overcome the extreme climatic change is to deal with these changes. Therefore, the storm water systems are designed using rainfall IDF or DDF curves derived from a long and good quality rainfall data.

Deriving the DDF curves is challenging especially for many countries which are having insufficient quantity of rain gauges that provide necessary rainfall information. There are also problems with the readily available rainfall records such as missing data, insufficient length and non-existence of data at locations of interest. Therefore, accurate and reliable prediction of the rainfall process is needed to compensate for the incomplete data. The situation becomes even worst when the rainfall DDF relationships are established on the basis of annual maximum data. Therefore, useful and practical methods would be established to generate the unavailable data by utilizing as much as possible from the limited available data. Therefore, the study on the method of the generation of rainfall characteristics is becoming very important to help the researcher to overcome the existing problem.

The development of DDF curves of an event-based is important as the rainfall events are important in the study of hydraulic structures. In each rainstorm events occurrence, the storm duration, storm depth, inter-event time and rainstorm pattern is available. To obtain valid statistical results from an appropriate identification of the events, choosing a correct method to separate the rainfall into independent event is very important. However, an independent event is hard to recognize from the rainfall records, which is made up of sequential pulses. Minimum period without rainfall or MIT is a typical criterion used to isolate an individual storm event from a long-term rainfall record.

The rainfall event characteristics such as depth and intensity are affected by the choice of MIT value. An erroneous identification of the event may cause invalid statistical results, leading to flawed design and analysis. James (1992 and 1994) stated that the inter-event period strongly affects the amount of runoff generated by a rainstorm such that to estimate flood frequency properly, the storm event must be considered if the event hydrology is to be reliable. Therefore, the suitable method to make a choice of the value of MIT must be further investigated.

#### **1.4** Objectives of the study

The objectives of the study are:

- 1. To determine an appropriate value of Minimum inter-event Time (MIT) to separate the rainfall data into rainfall event.
- 2. To identify the rainfall patterns in Peninsular Malaysia.
- 3. To propose Monte-Carlo simulation(MC-based) with nataf bivariate in generating mean and standard deviation of storm duration, depth and interevent time in deriving the DDF
- 4. To propose Copula approach (Copula-based) in deriving the DDF.
- 5. To compare MC-based DDF and Copula-based DDF with the Empirical-based DDF.

## **1.5** Significance of the Study.

In Malaysia, the frequent occurrence of flood especially within urban areas has caused tremendous losses and damages to properties and the environment. Therefore, to manage the flood risk, the DDF or IDF curves are used in designing the storm water management. The DDF curves are based on historical rainfall time series data and are designed to capture the depth and frequency of precipitation for different durations.

The results of the study undoubtedly have significance contribution in terms of local and regional hydrology. The anticipated impacts of climate change especially increase in rainfall intensity and its frequency appreciates the derivation of DDF curves in this study. It also provides policy makers better information on the adequacy of storm drainage design, for the current climate at the ungauged sites, and the adequacy of the existing storm drainage to cope with the impacts of climate change.

An accurate DDF estimation by using the proposed generation model is very important especially in developing the reliable flood prediction models in the water study. The proposed method of developing the DDF can be an option in deriving the DDF rather than using the empirical method as the traditional method is very time-consuming. (Ariff, N.M et al., 2012). Then, the simulated rainfall data which is developed by the generation model is very important to overcome the unavailable rainfall data problem. The simulated rainfall data is also used as the input in hydrological, agricultural and ecological models.

The event-based analysis is used in the study as the hourly rainfall data consists a very long rainfall data which is not easy to handle in the analysis part. However, the long rainfall data can be divided into the individual rainfall event, which can simplified the calculation of analysis part. Moreover, the rainfall event is regarded to be independent and become very important in hydrology analysis. As the study is an event-based analysis, therefore, the correctness of identification of storm events by using an appropriate MIT value may lead to the significant statistical result. Hence, an accurate and standardized description of storm event properties can be obtained to further understand the rainfall characteristics.

## **1.6** Scope of the Study

As the study focus on the event-based analysis, therefore, this study begins with the determination of the appropriate value of MIT to separate the rainfall into the event by using the two methods which are proposed by Adams and Papa (2000) and Nix (1994). Hence, the study proceeds in identifying the rainfall pattern according to the monsoon seasons in Malaysia by using the statistical cluster analysis combining with the Huff method. The hourly rainfall data from stations in the state of Johor, Melaka, Perak, Kedah, Kelantan and Pahang from years 2007 to 2011 is used in the study.

The study then concentrated on the generation of the statistical moments of storm characteristics by using MC-based with nataf bivariate based on the 30 years rainfall data from years 1972-2001. The best fit of storm characteristics is used as a marginal distribution to fit the rainfall depth and duration. The five distributions used are Gamma (GM), Gumbel Maximum (GMax), Gumbel Minimum (GMin), Lognormal (LG) and Weibull (WB) distributions.

The 40 years rainfall data (1972-2011) were used to derive the DDF by using the MC-based and Copula-based methods. The copula-based DDF is constructed by using the conditional Frank Archimedean. Later, the comparison between the Copula-based DDF and Empirical-based DDF will be examined.

## **1.7** Organization of thesis

This thesis consists of eight chapters. Chapter 2 presents a literature review of the stochastic rainfall models. Chapter 3 presents a detailed description of the selected models, in particular the method of determining an appropriate MIT, identification of rainfall pattern, generation of rainfall characteristics and deriving the DDF. Chapter 4 delivers the result and discussion of the best MIT while Chapter 5 delivers the rainfall pattern of the study area. Chapter 6 and 7 provides the results and discussion on the model's performance in generating the storm characteristics and deriving the DDF. Chapter 8 presents the summary and recommendations for continuing research.

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