INTEGRATING ALGEBRAIC THINKING IN PROBLEM-BASED LEARNING AMONG SECONDARY SCHOOL STUDENTS

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My beloved Papaku Mustaffa bin Long, Mamaku Hasna @ Suriya binti Hj Hasan, my siblings Norhuda binti Mustaffa, Amiri bin Mustaffa and Nadirah binti Mustaffa and my family. My beloved best friend Nadira binti Ismail and Makcik Hairunnahar binti Ayob. Millions of thanks for all the love, guidance, sacrifice and du'a.

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ABSTRACT

Algebraic thinking is a very important skill that should be mastered by students at an early stage before learning algebra. However, algebraic thinking is not emphasized in learning algebra. Therefore, this study aims to develop a framework that integrates students' algebraic thinking into the problem-based learning (PBL) process. Three different learning approaches; the conventional approach (CA), the integration of algebraic thinking (AT) and the PBL approach with the integration of algebraic thinking (ATPBL); were implemented in three different boarding schools (*SBP*) involving 85 participants in a quasi-experimental study. Results showed that the algebraic thinking process of students in ATPBL sessions was enhanced. A parametric test using MANCOVA revealed that the students from the ATPBL group performed better in exploring relationships, generalizing and formalizing, reasoning about and with representations, and using algebra as a tool compared to the students from the AT group. The ATPBL group performed significantly better in the manipulation of symbols and procedures, exploring relationships, generalizing and formalizing, reasoning about and with representations, and using algebra as a tool compared to the students from the CA group. Next, there was significant difference in students' algebraic thinking in the AT group compared to the CA group. Qualitative data from a learning task, namely, *PBMAThinking* (which consisted of teaching notes, self-readings, reflections, self-evaluations, evaluation of scenario problems, task-based interview transcripts, and task-based interview notes) were used to explore the acquisition of algebraic thinking into the ATPBL group. In summary, this study suggests that the PBL approach with the integration of algebraic thinking is able to enhance algebraic thinking among *SBP* students at lower secondary level. Accordingly, the framework that integrates students' algebraic thinking with the PBL process is expected to assist teachers in enhancing the effectiveness of teaching and learning of algebra and can potentially serve as a basis for developing algebraic thinking among *SBP* students, particularly at lower secondary level. Therefore, a conclusion was reached that algebraic thinking should be emphasized in the teaching process when learning algebra.

ABSTRAK

Pemikiran algebra adalah kemahiran yang sangat penting yang harus dikuasai oleh pelajar pada tahap awal sebelum mempelajari algebra. Walau bagaimanapun, pemikiran algebra tidak ditekankan dalam pembelajaran algebra. Oleh itu, kajian ini bertujuan untuk membangunkan satu rangka kerja yang mengintegrasikan pemikiran algebra pelajar dalam proses pembelajaran berasaskan masalah (PBL). Tiga pendekatan pembelajaran yang berbeza; pendekatan konvensional (CA), integrasi pemikiran algebra (AT) dan pendekatan PBL dengan integrasi pemikiran algebra (ATPBL) telah dilaksanakan di tiga buah sekolah berasrama penuh (SBP) yang berbeza yang melibatkan 85 orang pelajar dalam kajian kuasi-eksperimen. Dapatan menunjukkan bahawa proses pemikiran algebra pelajar dalam sesi ATPBL dipertingkatkan. Ujian parametrik menggunakan MANCOVA menunjukkan bahawa pencapaian pelajar dari kumpulan ATPBL meningkat dalam meneroka hubungan, generalisasi dan formalisasi, pemikiran tentang perwakilan dan menggunakan algebra sebagai alat berbanding dengan pelajar dari kumpulan AT. Kumpulan ATPBL menunjukkan prestasi yang lebih baik dalam manipulasi simbol dan prosedur, meneroka hubungan, generalisasi dan formalisasi, pemikiran tentang perwakilan dan menggunakan algebra sebagai alat berbanding dengan pelajar dari kumpulan CA. Seterusnya, terdapat perbezaan yang signifikan dalam pemikiran algebra pelajar dalam kumpulan AT berbanding dengan kumpulan CA. Data kualitatif dari tugasan pembelajaran iaitu *PBMAThinking* (yang terdiri daripada nota pengajaran, pembacaan kendiri, refleksi, penilaian kendiri, penilaian masalah senario, transkrip temu bual berasaskan tugasan, dan nota temu bual berasaskan tugasan) telah digunakan untuk meneroka pemerolehan pemikiran algebra ke dalam kumpulan ATPBL. Secara ringkasnya, kajian ini menunjukkan pendekatan PBL dengan integrasi pemikiran algebra dapat meningkatkan pemikiran algebra dalam kalangan pelajar SBP di peringkat menengah rendah. Sehubungan itu, rangka kerja yang mengintergrasi pemikiran algebra dalam kalangan pelajar melalui PBL dijangka dapat membantu guru dalam meningkatkan keberkesanan pengajaran dan pembelajaran algebra serta berfungsi untuk membangunkan pemikiran algebra dalam kalangan pelajar SBP khususnya di peringkat menengah rendah. Oleh yang demikian, pemikiran algebra haruslah diberi penekanan dalam proses pembelajaran dan pengajaran algebra.

TABLE OF CONTENTS

CHAPTER	TITLE	PAGE
	DECLARATION	ii
	DEDICATION	iii
	ACKNOWLEDGEMENT	iv
	ABSTRACT	v
	ABSTRAK	vi
	TABLE OF CONTENTS	vii
	LIST OF TABLES	xiv
	LIST OF FIGURES	xvii
	LIST OF ABBREVIATIONS	xviii
	LIST OF SYMBOLS	xix
	LIST OF APPENDICES	xx
1	**INTRODUCTION**	**1**
	1.1 Introduction	1
	1.2 Research Background	3
	1.2.1 Algebraic Thinking	7
	1.2.2 Problem-based Learning (PBL)	9
	1.2.3 Boarding schools *(SBP)*	12
	1.3 Problem Statement	13
	1.4 Research Objectives	16
	1.5 Research Questions	17
	1.6 Hypotheses	18
	1.7 Theoretical Framework	19
	1.8 Conceptual Framework	21
	1.9 Importance of Study	23

	1.10	Scope of the Study	25
	1.11	Limitations of the Study	26
	1.12	Operational Definitions	27
		1.12.1 Algebra	27
		1.12.2 Algebraic thinking	28
		1.12.3 Conventional Approach (CA)	28
		1.12.4 Integration of Algebraic Thinking (AT)	29
		1.12.5 Learning Task Integration of Algebraic Thinking (*AThinking*)	29
		1.12.6 Problem-Based Learning (PBL)	29
		1.12.7 Problem-Based Learning (PBL) Approach with the Integration of Algebraic Thinking (ATPBL)	30
		1.12.8 Scenario Problem	30
		1.12.9 Learning Task through Problem-Based Learning (PBL) Approach with the Integration of Algebraic Thinking (ATPBL) (*PBMAThinking*)	31
		1.12.10 Boarding School (*Sekolah Berasrama Penuh*) (*SBP*)	31
		1.12.11 Lower Secondary School Students	32
		1.12.12 Characteristics of Algebraic Thinking	32
	1.13	Summary	33
2		**LITERATURE REVIEW**	**35**
	2.1	Introduction	35
	2.2	Learning Theories	35
		2.2.1 Cognitive Development Theory	36
		2.2.2 Social Constructivism Theory	38
	2.3	Algebra	40
	2.4	Learning Algebra	42
	2.5	Algebraic Thinking	48
		2.5.1 Frameworks of Algebraic Thinking	52
		2.5.2 Algebraic Thinking in Secondary Schools	60

	2.6	Problem-Based Learning (PBL)	64
		2.6.1 Application of Problem-Based Learning (PBL) across fields	65
		2.6.2 Characteristics of Problem-Based Learning (PBL)	67
		2.6.3 Models of Problem-Based Learning (PBL)	73
		2.6.4 Problem-Based Learning (PBL) in Mathematics	74
		2.6.5 Implementation of Problem-Based Learning (PBL) in Teaching and Learning Mathematics in Malaysia	77
		2.6.6 The Impact of Problem-Based Learning (PBL) on the Cognitive Domain	78
	2.7	Study on Boarding School (*SBP*) Students	80
	2.8	Summary	82
3	**METHODOLOGY**		**84**
	3.1	Introduction	84
	3.2	Research Design	84
		3.2.1 Handling the Threats of Validity	88
	3.3	Population and Samples	93
		3.3.1 Sampling	93
	3.4	Research Procedure for the Study	94
		3.4.1 Research Procedures for Conventional Approach (CA) Group	94
		3.4.2 Experimental Procedures for Integration of Algebraic Thinking (AT) Group	97
		3.4.3 Experimental Procedures for Problem-Based Learning (PBL) Approach with the Integration of Algebraic Thinking (ATPBL) Group	101
		3.4.4 Summary of the Research Procedures of Three Groups	110
	3.5	Research Instruments	112

	3.5.1	Algebraic Thinking Test	113
	3.5.2	Scenario Problems of Problem-Based Learning (PBL)	119
	3.5.3	Task-Based Interview Protocol	120
	3.5.4	Learning Task Integration of Algebraic Thinking (AT) (*AThinking*)	121
	3.5.5	Learning Task Problem-Based Learning (PBL) Approach with the Integration of Algebraic Thinking (ATPBL) (*PBMAThinking*)	123
3.6	The Pilot Study		126
	3.6.1	Reliability of Instrument	127
		3.6.1.1 Algebraic Thinking Test	127
		3.6.1.2 Learning Task of Integration of Algebraic Thinking (AT) (*AThinking*)	128
		3.6.1.3 Learning Task of Problem-Based Learning (PBL) Approach with the Integration of Algebraic Thinking (ATPBL) (*PBMAThinking*)	129
	3.6.2	Inter-rater Reliability	130
	3.6.3	Validity of Instruments	132
		3.6.3.1 Content Validity Ratio	139
3.7	Data Analysis		140
	3.7.1	Analysis of Characteristics of Algebraic Thinking	140
	3.7.2	Analysis of Characteristics of Algebraic Thinking Before and After Taught with Different Approach	141
	3.7.3	Analysis of Characteristics of Algebraic Thinking Among Groups	141
	3.7.4	Analysis to Determine Students' Way of Thinking Algebraically while Learning Algebra through Problem-Based Learning	

(PBL) Approach and Formulating the Framework 143
3.7.4.1 Process of Analyzing Data 150
3.7.5 Triangulation 154
3.8 Summary 154

4 DATA ANALYSIS AND RESULTS 155
4.1 Introduction 155
4.2 Participants' Demographic Details 155
4.3 Characteristics of Algebraic Thinking of Students in Learning Algebra 156
4.4 Assumption for Running Inferential Statistics 160
4.4.1 Assumption for Running T-Test 160
4.4.2 Assumptions for Running MANCOVA 161
4.5 Differences in Algebraic Thinking 165
4.5.1 Difference in Algebraic Thinking in Students before and After Being Taught Using the Conventional Approach (CA) 166
4.5.2 Difference in Algebraic Thinking in Students before and After Being Taught Using the Integration of Algebraic Thinking (AT) 167
4.5.3 Difference in Algebraic Thinking in Students Before and After Being Taught Using the Problem-Based Learning (PBL) Approach with the Integration of Algebraic Thinking (ATPBL) 168
4.6 Students' Algebraic Thinking in Learning Algebra Using the Conventional Approach (CA) compared to the Integration of Algebraic Thinking (AT) 169
4.7 Difference in Algebraic Thinking for Students Taught Using the Conventional Approach (CA) Compared to the Problem-Based Learning (PBL) Approach with Integration of Algebraic Thinking (ATPBL) 169

	4.8	Difference in Algebraic Thinking in Students Taught Using the Integration of Algebraic Thinking (AT) Compared to the Problem-Based Learning (PBL) Approach with the Integration of Algebraic Thinking (ATPBL)	170
	4.9	Students' Algebraic Thinking among Three Groups; the Conventional Approach (CA), the Integration of Algebraic Thinking (AT) and the Problem-Based Learning (PBL) Approach with the Integration of Algebraic Thinking (ATPBL) Groups	171
		4.9.1 Overall Result on Students' Algebraic Thinking among Groups	174
	4.10	Students Think Algebraically while Learning Algebra through Problem-Based Learning (PBL)	176
		4.10.1 Analysis Based on Participants	177
		4.10.2 Summary of Analysis Based on Participants	203
		4.10.3 Analysis Based on the Phase in Problem-Based Learning (PBL) Process	205
		4.10.4 Summary of Analysis Based on the Phase in Problem-Based Learning (PBL) Process	219
		4.10.5 Conclusion to the Integrating Algebraic Thinking through Problem-Based Learning (PBL) Process	220
		4.10.6 Framework for Integrating Students' Algebraic Thinking Through the Problem-Based Learning (PBL) Approach	221
	4.11	Summary	228
5	**DISCUSSIONS, CONCLUSIONS, AND RECOMMENDATIONS**		**229**
	5.1	Introduction	229
	5.2	Overview of the Study	229
	5.3	Characteristics of Algebraic Thinking	231

5.4	The Differences in Students' Algebraic Thinking in All Groups	234
5.5	Difference in Students' Algebraic Thinking among Approaches	237
5.6	Students' way of thinking while learning Algebra through the Problem-Based Learning (PBL) Approach with the Integration of Algebraic Thinking (ATPBL)	239
5.7	A Framework that Integrates Students' Algebraic Thinking into the Problem-Based Learning (PBL) Process	242
5.8	Conclusion	246
5.9	Contributions to the Body of Knowledge and Closing the Research Gap	248
5.10	Implications of the Study	249
	5.10.1 Students	250
	5.10.2 Teachers	251
	5.10.3 Ministry of Education	252
	5.10.4 Other educators	252
5.11	Limitations of the Study	252
5.12	Recommendations for Future Study	253
5.13	Summary	255
REFERENCES		**256**
Appendices A- R		291-349

LIST OF TABLES

TABLE NO.	TITLE	PAGE
2.1	Framework of algebraic thinking Walkoe (2013)	53
2.2	Framework of algebraic thinking Driscoll *et al*. (2003).	53
2.3	Framework of algebraic thinking Lins (1992).	54
2.4	Framework of algebraic thinking Kieran (1996).	55
2.5	Comparison of the frameworks of algebraic thinking	56
2.6	Various approaches to develop algebraic thinking	61
2.7	Topics in mathematics implemented through PBL	75
2.8	Study on PBL in Teaching and Learning in mathematics in Malaysia	77
2.9	The impact to cognitive domain through PBL	79
2.10	Chronology of the establishment boarding schools (*SBP)*	81
3.1	Research design	86
3.2	Suggestions for controlling extraneous variables	92
3.3	A KNL template	109
3.4	Summary of the research procedures of three groups	111
3.5	The research objectives and instruments	112
3.6a	Validation of questions for algebraic thinking test before being finalized (Manipulation of symbols and procedures)	114
3.6b	Validation of questions for algebraic thinking test before being finalized (Exploring relationships)	115
3.6c	Validation of questions for algebraic thinking test before being finalized (Generalizing and formalizing)	116
3.6d	Validation of questions for algebraic thinking test before being finalized (Using algebra as a tool)	116

3.6e	Validation of questions for algebraic thinking test before being finalized (Reasoning about and with representations)	117
3.6f	Validation of questions for algebraic thinking test before being finalized (Connecting representations)	118
3.7	Construction of learning task *AThinking*	122
3.8	Construction of learning task ATPBL *(PBMAThinking)*	123
3.9	Learning task AT *(AThinking)*	128
3.10	The observed agreement with two experts in mathematics	131
3.11	Interpretation of Kappa	132
3.12	Scenario Problem 1 Mapped With Algebraic Thinking	136
3.13	Scenario Problem 2 Mapped With Algebraic Thinking	137
3.14	CVR Value for scenario problems, *AThinking* learning task and *PBMAThinking* learning task.	139
3.15	Techniques to enhance trustworthiness	149
4.1	Table of distribution of sample in different groups.	156
4.2	Comparison of the characteristics of algebraic thinking between the three groups	157
4.3	Comparison of the characteristics of algebraic thinking among the three groups for the post-test	159
4.4	Paired samples t-test for before and after CA	166
4.5	The effect size value for dependent means	166
4.6	Paired sample t-test for before and after AT	167
4.7	Effect size value for dependent means	167
4.8	Paired sample t-test for before and after in ATPBL	168
4.9	The effect size value for dependent means	168
4.10	Comparison between CA and AT	169
4.11	Comparison between CA with ATPBL	170
4.12	Comparison between AT with ATPBL	171
4.13	Comparison among CA, AT and ATPBL groups	172
4.14	Percentage changes in characteristics of algebraic thinking	174
4.15	Summary of the quantitative results and findings	175

4.16	Algebraic thinking of four participants in *PBMAThinking*, task based interview transcript and task based interview notes	205
4.17	Characteristics of algebraic thinking in Scenario Problem 1	222
4.18	Characteristics of algebraic thinking in Scenario Problem 2	223

LIST OF FIGURES

FIGURE NO.	TITLE	PAGE
1.1	Theoretical Framework	21
1.2	Conceptual Framework	23
2.1	Example of a question from the Form Two textbook in Malaysia	44
2.2	Example of a word problem from the Malaysian Form Two textbook.	45
3.1	Summary of research procedures in CA group	97
3.2	Summary of procedures for experiment in AT group	100
3.3	The learning process of PBL based on the model developed by Tan (2003)	102
3.4	Summary of procedures for experiment in ATPBL	107
3.5	First Draft of Scenario Problem 1	134
3.6	The second draft of Scenario Problem 1	135
3.7	Excerpt from Scenario Problem 2	137
3.8	Process of analyzing the data that led to the formulation of a framework to integrate students' algebraic thinking into the PBL learning process.	153
4.1	The learning task ATPBL according to phase and learning process	206
4.2	Learning process of PBL based on two scenario problems	225
4.3	Framework for integrating students' algebraic thinking through the PBL process	227
5.1	Framework for implementing ATPBL among different groups	245

LIST OF ABBREVIATIONS

CA	-	Conventional Approach
AT	-	Integration of algebraic thinking
ATPBL	-	Problem-based learning approach with the integration of algebraic thinking
SBP	-	*Sekolah Berasrama Penuh*/ Boarding School
RPH	-	*Rancangan Pengajaran Harian*
MSP	-	Manipulation of symbols and procedures
ER	-	Exploring relationships
GF	-	Generalizing and formalizing
UA	-	Using Algebra as a tool
RA	-	Reasoning about and with representations
CR	-	Connecting representations
JU	-	*Jurulatih Utama*
BMI	-	Body Mass Index
PBL	-	Problem-based learning
TIMSS	-	Trends in International Mathematics and Science Study
UPSR	-	*Ujian Penilaian Sekolah Rendah*
PMR	-	*Penilaian Menengah Rendah*
PT3	-	*Pentaksiran Tingkatan 3*
ml	-	Mililiter

LIST OF SYMBOLS

P_o	-	Observed Agreement
P_e	-	Expected Agreement
M	-	Mass
σ	-	Variance
d	-	Sample effect size (Cohen)

LIST OF APPENDICES

APPENDIX	TITLE	PAGE
A1	Letter of permission from the EPRD	291
A2	Letter of permission from Perak State Education Department	292
A3	Letter of permission from management department of boarding schools	293
B	Algebraic Thinking Test	294
C	Statutory Review of Task based Interview Transcript; Task based Interview Notes and Students work in Learning Task ATPBL for Reliability Indices Cohen Kappa	301
D1	Validation of Algebraic Thinking Test	302
D2	Validation of Learning Task AT (*AThinking*)	305
D3	Validation of Learning Task ATPBL (*PBMAThinking*)	308
D4	Validation of Scenario Problems	310
E	Statutory Review of Translation Bahasa Melayu to English; Task based Interview Transcript, Task based Interview Notes and Students work in Learning Task ATPBL	315
F1	Results Pre-test and Post-test for CA Group	316
F2	Results Pre-test and Post-test for AT Group	317
F3	Results Pre-test and Post-test for ATPBL Group	318
G1	Results for MANCOVA Multivariate Tests	319
G2	Results for MANCOVA Test of Between-Subjects	

	Effects	320
G3	Assumptions of MANCOVA analysis	322
G4	Tests of normality	324
H1	Scenario Problem 1	325
H2	Scenario Problem 2	328
I1	Patterns of algebraic thinking Sina	329
I2	Patterns of algebraic thinking Fira	330
I3	Patterns of algebraic thinking Finah	331
I4	Patterns of algebraic thinking Ariamuh	332
J	Task-based Interview Protocol *Senario Masalah* 1 dan 2	333
K	Participants checking the validity	335
L	Sample of Analysis of Interview Data	336
M1	Sample of Analysis of of Students' work Learning Task ATPBL (*PBMAThinking*)	338
M2	The codebook used for coding data	339
N	Models of PBL	343
O1	The final version of the algebraic thinking test	345
O2	Comparison of specifications for PBL problems	346
P	Overview of research questions, instruments, design and data analysis	347
Q	Algebraic thinking in every phase for both scenario problems	348
R	List of Publications, Competition and Participation	349

CHAPTER 1

INTRODUCTION

1.1 Introduction

Algebra is an important topic in mathematics at all levels, from elementary school to university (Booker and Windsor, 2010; Cai, Lew, Morris, Moyer, Ng, and Schmittau, 2005; Eccius-Wellmann, 2012; Napaphun, 2012; Walkoe, 2013). Algebra is widely applied in various fields, such as economics, food and beverages, banking, business, chemistry and many other areas (Eccius-Wellmann, 2012; Grandau, 2013). It is interrelated with other mathematical topics such as statistics and geometry. It is also the gateway for entering university and is the foundation of advanced mathematics (Barbosa and Vale, 2015; Cai and Knuth, 2011; Drijvers, Doorman, Kirschner, Hoogveld and Boon, 2014; Walkoe, 2013). Algebra is required to develop science, technology, art and humanity as well as to solve everyday problems (Abonyi and Nweke, 2014). Reforming the teaching and learning of algebra, which started at least 4,000 years ago, is a major challenge that involves all levels of education worldwide (Katz, 1997). Therefore, it is important for students to understand the concept and application of algebra as well as its current development.

Learning algebra is pivotal in schools. It is a developed skill and knowledge, which is often used to solve daily problems. Companies apply algebra to work out their yearly expenditure for their annual budgets. Algebra is also applied in various stores to predict the demand for a particular product and subsequently to place orders. It is also applied in banking transactions, such as interest and instalment loans, as well as in the calculation of annual taxable income (Egodawatte, 2011). However, for decades, algebra has been presented as a set of procedures and principles that are isolated from the rest of mathematics. This has led to misconceptions among students that algebra is not related to the solving of daily problems (Fendel, Resek, Fraser and Alper, 1997).

In algebra, variables can be applied to represent, analyze, and perform calculation problems (Abonyi and Nweke, 2014). Algebra has been used as a tool to generalize patterns, as well as to justify and manipulate symbols (Banerjee, 2011). However, the reasons for difficulties in learning algebra are concentrated in the manipulation of symbols and procedures as well as understanding conceptual of content of algebra (Chazan, 1996). Furthermore, students' achievement in algebra is still low and poor knowledge has been demonstrated in simplifying algebraic expressions (Abonyi and Nweke, 2014). In addition, most students fail to connect arithmetic with algebra (Alagic and Emery, 2003; Alghtani and Abdulhamied, 2010; Banerjee, 2011; Cai and Moyer, 2008; Napaphun, 2012; Noyce Foundation, 2009; Witzel, 2005; Wang, 2015).

Algebra is used to determine, analyze and solve equations involving expressions and relations (Lew, 2004). However, it is more than solving equations or simplifying expressions (Zeller and Barzel, 2010). It is a way of thinking (Lew, 2004). Developing algebraic thinking in earlier grades enables a deeper understanding of the concept of mathematics, including algebra (Booker and Windsor, 2010; Cai and Knuth, 2011; Cai and Moyer, 2008; Lew, 2004). Early algebraization requires a way of thinking that involves justifying, predicting, proving, solving problems, generalizing, noticing the structure, modelling, studying changes, and analysing the relationships between quantities (Cai and Knuth, 2011). By incorporating algebraic thinking skills earlier in the curriculum, students' success rate in algebra will be increased (Lew, 2004; Ralston, 2013). These skills are required in order for the students to be able to think algebraically. However, algebraic thinking is not promoted (Siew, Geofrey and Lee, 2016) and emphasized in the Malaysian mathematics curriculum.

The mathematics curriculum plays a significant role in the formation of basic algebraic concepts and the development of algebraic thinking (Dikkartin and Uyangor, 2012). Students can understand algebraic symbolism in the early development of algebraic thinking (Cai and Knuth, 2011). Algebraic thinking is a process by which the students express and build mathematical relationships practically (Soares, Blanton and Kaput, 2006). According to Kieran (2004), algebraic thinking is a way for students to focus on relations, operations, alphanumeric characters, representing and solving problems as well as refocusing the meaning of the equals sign. The foundation of algebraic thinking is developed as the student becomes able to make connections of patterns with the real world (McGarvey, 2012). This is aligned with the principles of problem-based learning (PBL). According to Mason, Graham and Johnston-Wilder (2005), algebraic thinking develops through the combination of four strands of algebra with mathematical themes and mathematical powers within conjecture conditions. Korea, China, the USA, and Canada are among the countries that have their own curriculum for algebraic thinking skills, which leads to better performance in algebra compared to Malaysia. The mathematics curriculum in Malaysia is only focused on thinking and reasoning in general. However, some elements of algebraic thinking do exist. Therefore, study is required to identify the appropriate strategy to make the learning of algebra more comprehensive and effective (Cai, Lew, Morris, Moyer, Ng and Schmittau, 2005). To encourage students to practice such thinking, proper teaching and learning activities should be designed, and one potential strategy is PBL. PBL is the one of the best approaches that emphasizes problem as a starting point, followed by student-centered and teacher as a facilitator in the learning process. It is also proven that PBL provide positive impact in student's achievement in mathematics. However, there are scarce of studies pertaining algebraic thinking in PBL, whereby algebraic thinking is important as a foundation of success in learning algebra.

1.2 Research Background

Wang (2015) conducted a review that addressed the factors that contribute to difficulties in learning algebra. These difficulties consist of algebra content, cognitive gap, teaching issues, learning matters and transition knowledge. Students have difficulties in representing an unknown quantity in an equation using a letter. Furthermore, students are unable to generalize in algebra based on given arithmetical rules and operations. They often misinterpret the equals sign as a formal symbol for equivalence (e.g. 3 + 4 = 2 + 5). The students thought that the answer is in the right side, which is 2 + 5. Moreover, students are unable to solve word problems. They find it difficult to formulate an equation from a word problem. This shows that the difficulties of learning algebra are still being studied.

Study by Alibali, Stephens, Brown, Kao and Nathan (2014) reported that their participants were fairly successful in solving algebraic equations. They found that the students found it difficult to integrate the mathematical operations and were weak in multiplication of equations. Their findings also reported that the students had difficulties in solving and symbolizing multiple operations in problem solving. These Walkoes (2013) emphasized these characteristics in manipulation of symbols and procedures and connecting representations. The most important part of teaching and learning is that the knowledge taught is relevant and is retained by the learner, and this places emphasis on the role of the teacher to accomplish the learning outcomes (Adu and Olaoye, 2015). They simply require correct answers from the students instead of revealing the beauty of mathematics, especially algebra. Furthermore, they do not encourage the students to think algebraically. Hence, the students would not fully understand the transition from arithmetic to algebra.

The traditional approach for teaching algebra, which involved writing and solving equations according to the rules of mathematics, works better for students with higher levels of academic ability compared to those at lower levels (Abramovich, 2005). This approach is taught procedurally and in isolation from other mathematical domains and the real world (Kaput, 2000a). Teachers should improvise the method of teaching rather than reminding the students of step-by-step processes, because students are unable to remember algorithms for long periods (Dougherty, Bryant, Bryant, Darrough and Pfannenstiel, 2014; Xin, Wiles and Lin, 2008).

Based on previous studies, algebraic thinking focuses more on the achievement of algebra itself. This does not represent the true understanding concept of algebra. Furthermore, the finding does not entail all the characteristics of algebraic thinking. Most of the studies promote to enhance the ability of the students to make generalizations. Generalization and justification commonly influences each other (Ellis, 2007) as mentioned in the framework ofWalkoes. However, it is explicitly introduced in the KSSM syllabus for Malaysian students. It involves addition, subtraction, division, and multiplication of integers. Conversely, generalizations in patterns should be emphasized (Callejo and Zapatera, 2017). Patterns play an important role in introducing algebra (Zazkis and Liljedahl, 2002). More significantly, algebraic thinking is about finding and using generalization in patterns that should be introduced in elementary years (Tirosh, Tsamir, Levenson, Barkai, and Tabach, 2017).

In teaching algebra, teachers prefer to follow the curriculum in textbooks (Wang, 2015). Malaysian teachers, in particular, rely on textbooks and practice problem-solving as the learning strategies. However, the exercises and questions in Malaysian algebra textbooks only emphasise procedures (Singh, 2003), and this leads the students to solve the problems without thinking properly (Istikomah and Mohamad, 2013). This is also due to the nature of the problems being too artificial and unrelated to daily life. Furthermore, the questions and exercises in the textbooks do not lead the students to think algebraically. This is a mismatch with the nature of learning algebra, which requires students to emphasize critical and reasoning thinking in solving mathematical problems. Although the mathematics curriculum in Malaysia emphasizes reasoning and thinking for learning mathematics (*Kementerian Pendidikan Malaysia*, 2000), it is more concerned with mathematical and logical thinking. Students are encouraged to estimate, predict, and investigate using concrete materials, calculators, and computers. In logical thinking, students are able to evaluate, predict, and argue. In reality, students need to be encouraged to use algebraic thinking skills in the mathematics curriculum so that they will be able to relate algebra with real-life situations. They will also be able to understand the concepts, find patterns, and generalize. Furthermore, students are able to solve problems, justify, and prove mathematically as emphasized based on the framework of Walkoe.

Examination results provide an indicator of how students learn algebra. The result of the *Sijil Pelajaran Malaysia* (*SPM*) shows that many students fail to perform algebraic manipulations (*Lembaga Peperiksaan Malaysia*, 2003a, 2003b, 2008, 2010a). The ability of performing algebraic manipulation is related to the characteristics of algebraic thinking based on Walkoe's framework, which is the manipulation of symbols and procedures. Yet there is no specific measurement to characterise algebraic thinking. In addition, achievement in algebra has been decreasing, as shown in the results of the Trends in International Mathematics and Science Study (TIMSS) in 2011 (Mullis, Martin, Foy, and Arora, 2011) and 2015 (Mullis, Martin, Foy and Arora, 2015) and the Programme for International Student Assessment (PISA) in 2012. Therefore, this study focuses on algebra as the subject matter. Malaysian teachers apply drill and practice in teaching mathematics (Sam, 2003; Sam and Yong, 2006; Saleh and Hussin, 2011). They believe that this approach is the most efficient due to familiarization with repeated routine problems (Zanzali, 2012). Therefore, students are unable to think analytically to solve problems (Jing, Tarmizi, Bakar and Aralas, 2017). Hence, teachers should work towards adopting various teaching and learning approaches instead of teaching at a fast pace to cover the syllabus. Moreover, teachers also need to encourage the students to use thinking and reasoning in solving problems.

The challenge has always been to find ways of teaching algebra so as to make the students learn with understanding (Kaput, 2000a). Hence, there is a need to explore various teaching strategies in teaching algebra (Wang, 2015). Middle school students are able to understand the connections between algebra and other mathematical concepts based on the constructivist approach, which promotes procedural knowledge, and conceptual knowledge in the algebra strand (Ross and Willson, 2012). Sixth grade students (11-12 years old) learn to make conjoined texts of patterns found in number arrays regarding recursive patterns (Zolkower and Shreyar, 2007).

Nowadays, teachers should choose the appropriate method to enhance multiple competencies, especially algebraic thinking among students. Many studies (Booker and Windsor, 2010; Windsor and Norton, 2011; Windsor, 2008) have been concerned with the development of algebraic thinking through the problem-solving approach. In real settings in schools, algebraic thinking is assessed through tests and examinations. However, thinking skills should be assessed while the students are learning. The best approach should be on in which the teachers are able to fulfil the syllabus requirements and develop students' behaviour (Idris, 2001). Some teachers regard problem-solving as an alternative teaching approach to develop algebraic thinking in the classroom.

1.2.1 Algebraic Thinking

Based on previous study, teachers are only able to identify the students' thinking in general, instead of identifying it in specific mathematical domains. This limits teachers' ability to identify the level of students' algebraic thinking (Walkoe, 2013). Therefore, there is a need to identify the particular mathematical domains, such as characteristics of algebraic thinking.

Algebraic thinking is the thinking that requires a person to make a connection between arithmetic and algebra (Banerjee, 2011). Express generality, recognizing and analyzing patterns or articulating structures is complicated and problematic for students, but these abilities are important in mathematical thinking. Furthermore, the students are unable to identify algebra as a tool for problem-solving due to the fact that generalization is implicit in algebra (Guti´errez, Mavrikis and Pearce, 2008). In teaching and learning mathematics, arithmetic is usually taught earlier than algebra. However, arithmetic should be taught alongside algebra, as this would prepare the students for the more complex algebraic concepts (Radford, 2014). Furthermore, the students understand that the equals sign is a symbol of calculation. They think that an equals sign must always be followed by an answer (Napaphun, 2012), which is a concrete value. However, in finding patterns, it is not necessarily a value. It can be a variable in the same sequence, which is highlighted in exploring the relationships of characteristics as stated in Walkoe's framework.

According to Booker and Windsor (2010), algebraic thinking engages students across all strands of mathematics. Therefore, it should be implemented at middle and upper primary level. This enables students to obtain concrete algebraic thinking in secondary school when they are exposed to practical problems using models and situations at early stages of learning. However, according to Kieran (2004), algebraic thinking should start in earlier grades and should involve analyzing and exploring the relationship, justifying, problem-solving, predicting, and generalizing. Furthermore, algebraic thinking is part of a school subject that can be connected to students' everyday lives and materials with mathematical symbols (Soares, Blanton and Kaput, 2006).

In Malaysia, there are still very few empirical studies of algebraic thinking in middle school. Siew *et al*. (2016) reported that 8^{th} grade students achieved significantly higher mean scores in algebraic thinking compared to a control group using DragonBox 12. Furthermore, the researchers observed pre-algebraic thinking. Findings from a study by Gan (2008) in Malaysia investigated how primary school pupils solve pre-algebraic problems and identified their pre-algebraic thinking based on their solution processes. Moreover, Lian and Yew (2011) proposed a framework for pre-algebraic thinking to enhance generalization. However, study by Lian, Meng and Idris (2009) reported that pre-service teachers performed lower at the relational and multistructural levels in algebraic thinking. Study on algebraic thinking has been implemented at various levels, such as middle school, primary, and pre-service teachers. However, there is a particular need to study algebraic thinking for middle school students. This is based on the theory of Piaget, who emphasized that at the formal operational stage, which relates to students aged eleven years and above, children can learn more abstract concepts. This stage is important, as children struggle to construct more complex concepts and solve concrete and abstract problems. They are able to think deductively or inductively and to prove theories or laws of mathematics. They also are able to make generalizations or conclusions. .

Other than that, using problems in PBL may involve reasoning and making generalizations, which is aligned with Walkoe's point of view that focus should be on algebraic thinking and reasoning (Kalaivani and Tarmizi, 2014).

Algebraic thinking consists of three components; general activities of algebra; transformational ("rule-based") activities; and global, meta-level, mathematical activities (Kieran, 1996). In 2013, Walkoe expanded Kieran's model to encompass manipulating symbols and procedures, exploring relationships, generalizing and formalizing, using algebra as a tool, reasoning about and with representations, and connecting representations. Exploring relationships involves examining how structures or quantities relate to each other as well as to conjunctions.

Generalizing and formalizing are components of the general activities of algebra. However, exploring relationship is separated from generalizing and formalizing. The extensions in Walkoe's framework include functions and their representations. An example of category generalizing and formalizing is that the student is able to identify and find a formula for the linear pattern. Meanwhile, students' ability to think about the relationship of independent and dependent variables of the function falls within the category of exploring relationships. However, there is a need for students to be able to reason across the representation or to make a connection with the representation especially the ability to connect patterns with real world. This is related to one of the characteristics of PBL. Walkoe's extension of Kieran's work demonstrates more specifically the features of algebraic thinking.

1.2.2 Problem-based Learning (PBL)

To enhance thinking skills among students, the learning process should involve feedback on students' own learning, information-seeking behaviour, problem-solving skills, and group processes (Tan, 2009). PBL is practice-based; the students must practice solving problems and not just learn about the problem-solving itself (Jonassen, 2011).

In a PBL process, scaffolding helps to develop cognitive connections such as analytical thinking (Tan, 2003). PBL applies real life problems to improve students' analytical thinking, higher level thinking abilities, and problem solving (Hatısaru and Küçükturan, 2009a). These real life problems have specific answers and PBL is concerned with how the students interpret, plot, and plan to solve the problems in groups. Teachers often discover that students accomplish meaningful and permanent learning when solving real life problems. A study conducted by Kalaivani and Tarmizi (2014) applied the algebra domain and reviewed higher order thinking skills, and the researchers suggested that teachers should focus on algebraic thinking in order to empower the teaching and learning of algebra.

PBL has the ability to construct students' knowledge and understanding of any particular concept. In addition, it can enhance higher order thinking skills. Problem solving and the PBL approach are in parallel with the characteristics of algebraic thinking. However, the components of algebraic thinking embraces algebra as a tool for functions and mathematical modelling; these components seek, express, and generalize patterns and rules in real world contexts, and apply them in the problem-based learning process. Furthermore, PBL provides multidisciplinary learning and life skills in addition to problem-solving skills.

Students are presented with real life problems to encourage their thinking skills, decision-making skills, investigation, and inquiry through PBL (Lang and Evans, 2006). Students are encouraged through real life situations that lead towards self-directed learning that is meaningful and relevant (Nargundkar, Samaddar and Mukhopadhyay, 2014). They can anchor their learning through these tasks and feel in control of the learning process through the constructivist approach (Savery and Duffy, 2001).

Multidisciplinary learning consists of interconnecting disciplines, while life skills consist of self-direct learning, information mining, collaborative learning, reflective thinking, peer teaching, and evaluation. The process of PBL helps the students in enhancing connectivity between prior knowledge or experience and the real-world context, theories, other people's perceptions, as well as new facts and ideas. However, to achieve these skills, facilitating, coaching, and mediating are indispensable (Tan, 2003). Moreover, questions arise as to what the students have learnt in PBL and how they learn during group discussion and self-directed learning (Yew and Schmidt, 2012), specifically in learning algebra. Furthermore, PBL is still not widely practiced in secondary schools in Malaysia (Nasir, 2016). Moreover, Kalaivani and Tarmizi (2014) suggest the need to focus on algebraic thinking using PBL approach.

Therefore, study on developing algebraic thinking through PBL should be conducted to meet the challenges of the twenty-first century. Based on previous studies, students who underwent intervention have improved in acquisition of characteristics of algebraic thinking. However, not all previous studies focused on most of the characteristics of algebraic thinking, and it highlight the process to gain the characteristics of algebraic thinking was not highlighted. Furthermore, algebra was also not applied in the context of real life where PBL approach is employed. PBL is the one of best approaches that emphasize on student-centered, which gives students the chance to discover knowledge in a meaningful and applicable way (Liza, Karomiah and Abdullah, 2011). Based on a preliminary study conducted by the researcher, the study should be implemented with boarding school (*SBP*) students, since they have demonstrated low levels of performance of algebraic thinking. Furthermore, PBL is particularly well suited to higher achievers (Gallagher and Gallagher, 2013; Sak, 2004). Higher achieving students have been shown to perform better than lower achieving students in all scaffolding situations (Samsonov, Pedersen and Hill, 2006). Since these students are selected for their excellent performance in academic tests such as the *Ujian Penilaian Sekolah Rendah (UPSR),* a general examination administered to Malaysian students at the age of 12, and *Penilaian Menengah Rendah (PMR)/ Pentaksiran Tingkatan 3 (PT3),* administered at the age of 15, they have high performance in cognitive ability.

1.2.3 Boarding schools *(SBP)*

Boarding schools (*Sekolah Berasrama Penuh: SBP*) in Malaysia are schools at which the students are fully residential in hostels. They have fixed schedules for students' activities from early morning until bedtime. Boarding school entry is under the management of the Ministry of Education (*Kementerian Pelajaran Malaysia: KPM*). It is based on excellent academic achievement in the *Ujian Penilaian Sekolah Rendah* (*UPSR*) to enter at Form One level and in the *Penilaian Menengah Rendah* (*PMR*) or *Pentaksiran Tingkatan 3* (*PT3*) to enter at Form Four level and is supported by co-academic achievements such as debate or scrabble or related to achievements in academic or co-curriculum activities such as badminton, soccer or other sports (Ilias, 2012; Khalidah, Rohani and Mashitah, 2014; Muhriz, Abidin, Abdullah and Jan, 2011).

Many studies have been done involving boarding school (*SBP*) students, including examining their levels of anxiety, depression (Mahfar, Aslan, Noah, Ahmad and Jaafar, 2014), perceptions towards good teachers, and attitudes towards Physics (Daud, Karim, Hassan and Rahman, 2015), as well as study in the affective domain (Ghani, Yaacob, Ahmad, Aman and Isa, 2010; Mokhtar, Mohd Jailani, Tamuri and Abdul Ghani, 2011). However, there has been a lack of study that explores cognitive ability in *SBP* students, specifically in terms of mathematics and algebraic thinking. *SBP* students are highly able in terms of cognitive skills and excellent academic performance. However, their characteristics of thinking, such as algebraic thinking, have not yet been explored in detail.

Prior to the main study, a preliminary study was carried out to identify the algebraic thinking among 205 Form Two students from seven boarding schools in Perak. Boarding schools in Perak include schools from the top, middle and bottom rank among boarding schools in Malaysia. The data showed that only 0.98% scored grade A for the Algebraic Thinking Test. Conversely, 111 out of the 205 students (54.15%) scored grade F. Moreover, 31 students (15.12%) scored grade D. In the Algebraic Thinking Test, most of the students were unable to answer the questions that required them to use algebra as a tool to explore relationships as stated in Walkoes' framework. More importantly, the students were unable to answer the questions that required them to use reasoning (Mustaffa, Ismail, Tasir and Said, 2016). Therefore, there is a need to implement this study among *SBP* students.

1.3 Problem Statement

Study on mathematics education has reported that many students perceive algebra as difficult (Egodawatte, 2011; Kalaivani and Tarmizi, 2014) and abstract (Booth, Barbieri, Eyer and Paré-Blagoev, 2014; Puteh and Khalin, 2016; Xin, Zhang, Park, Tom, Whipple and Si, 2011). Students have also demonstrated misconceptions in solving problems that involve variables (Booth *et al*., 2014; Egodawatte, 2011; Wasserman, 2014; Yantz, 2013). Algebra is one of the components in mathematics that deal with operations or manipulation of symbols and variables. Algebra requires students to solve equations procedurally instead of emphasizing their thinking process (Wong, 2005). In particular, students learning algebra are unable to connect arithmetic with algebra (Banerjee, 2011; Khalid and Noor, 2012; Zeller and Barzel, 2010). Students are unable to perform successfully in learning algebra through the conventional approach (Abonyi and Nweke, 2014), as this approach prevents them from developing the ability to think towards solving algebraic problems (Wong, 2005).

According to Xin *et al* (2008), students should be introduced to thinking algebraically before learning formal algebra. Algebraic thinking is a combination of understanding patterns and functions using mathematical models; it involves analyzing changes and representing situations with symbols in algebra (Vennebush, Marquez, and Larsen, 2005). To solve algebraic problems, it is important for students to understand and be able to think algebraically, but many students are unable to recognize the characteristics of algebraic thinking in the process of learning, whether directly or indirectly. Some parts of algebraic thinking are inherent in the local curriculum but have not been taught explicitly (Adni, 2012). Hence it is important to make algebra meaningful and applicable to learn.

A number of researchers have investigated the algebraic thinking of middle school students (Adni, 2012; Ayalon and Even, 2013; Booth *et al*., 2014; Hitt, Saboya and Cortés Zavala, 2015; Johanning, 2004; Lee and Freiman, 2006; Lew, 2004; Li, Peng, and Song, 2011; Noss *et al*., 2012; Siew *et al*., 2016; Steele and Johanning, 2004; Trezise and Reeve, 2014; Walkowiak, 2014; Zeller and Barzel, 2010). However, not all the characteristics of algebraic thinking have been demonstrated among students. Early algebra emphasizes algebraic thinking, which involves the understanding of arithmetic relationships, generalizing and recognizing variable structure. Early algebra should be differentiated from typical algebra in terms of contents, subjects, and teaching methods (Lee and Pang, 2012). The development of algebraic thinking requires students to (1) help themselves to make a smooth transition between arithmetic and algebra and (2) appreciate the usefulness of generalized algebraic approach in solving various problems (Cai and Moyer, 2008; Cai *et al*., 2005).

Study on algebraic thinking should be conducted to all students including high achievers specifically boarding school students. There has been a lack of study on the implementation of algebraic thinking among boarding school students. Boarding schools select students who demonstrate excellent academic and co-academic abilities. All boarding school students are required to stay in the hostels provided and their food and drinks are provided for them. These students are assets to the country; they are being sponsored and are closely monitored so that they can help the country in return when they have successfully completed their studies and gained powerful positions.

It is important to study algebraic thinking because it could make the learning of algebra more comprehensive and allow the development of an algebraic perspective of mathematics. Furthermore, algebraic thinking is able to develop a deeper understanding of the underlying structure of mathematics, dealing with generalizations and ways of thinking that allow results to be expressed across a range of problem forms rather than simply finding a particular answer to a series of individual problems. The importance of algebraic thinking in learning algebra will guide teachers in teaching algebra effectively as well as enhance students' algebraic thinking. Mathematics teachers should have alternative ways to teach algebra; for example, by conducting real-life activities to diminish the disconnection of arithmetic with algebra (Alghtani and Abdulhamied, 2010). Another strategy, as indicated by the literature, is to use PBL, an approach that focuses on the development of thinking. This method has been perceived to be able to facilitate students' knowledge construction and reasoning skills because it uses real-world problems as the starting point in the learning process.

This study will compare three different learning approaches; the conventional approach, the integration of algebraic thinking approach and the PBL approach with integration of algebraic thinking. The researcher will identify the characteristics of students' algebraic thinking in three different learning approaches in order to determine the existence of algebraic thinking among the students before and after the intervention. Knowing these characteristics will indicate the need to emphasize algebraic thinking in learning algebra. Therefore, the students will be able to understand and apply the concept of algebra and connect it to real situations. In addition, the foundation of algebraic thinking is developed as the students are able to make connections of patterns with the real world (McGarvey, 2012). This shows that algebraic thinking is aligned with the principles of PBL. In this study, algebraic thinking is integrated with the PBL process, namely, problem-based learning approach with integration of algebraic thinking (ATPBL). Both AT and PBL are not widely used in the local school setting. The focus of ATPBL is to improve algebraic thinking and provide a process of learning in algebra. This demands a high level of cognitive skills and we believe that not all students can handle this kind of learning. This pertains to a reformation process of the teaching and learning of algebra. It is suggested that this innovation should be tested on students in the high-achieving schools, such as boarding school (*SBP*) students, since *SBP* students are among the most excellent students in the country. They would appear to be more ready to cope with AT and ATPBL. Moreover, PBL is required to carry out independent work, having the convenience to access material and apparatus. These students are independent learners and would be exposed to non-routine problems. Many studies have been conducted with the participation of boarding school (*SBP*) students. However, there is a lack of empirical studies in Malaysia on the cognitive domain, specifically in the field of algebraic thinking. Given the above points, the researcher intends to integrate algebraic thinking through PBL. The cohesion between the two strategies is expected to have positive effects in improving students' thoughts about learning algebra.

1.4 Research Objectives

The objectives of the study are as follows;

(i) To analyze the characteristics of algebraic thinking of lower secondary boarding school (*SBP*) students in learning algebra;

  (a) through the conventional approach (CA).

  (b) with the integration of algebraic thinking (AT).

  (c) through the problem-based learning (PBL) approach with the integration of algebraic thinking (ATPBL).

(ii) To compare the characteristics of students' algebraic thinking in learning algebra between;

  (a) CA and AT.

  (b) CA and ATPBL.

  (c) AT and ATPBL.

(iii) To determine the students' way of thinking algebraically while learning algebra through the ATPBL.

(iv) To develop a framework that integrates students' algebraic thinking with the PBL learning process.

1.5 Research Questions

The study is conducted to investigate;

(i) What are the characteristics of algebraic thinking of lower secondary boarding school (*SBP*) students in learning algebra;

(a) through the conventional approach (CA)?

(b) with the integration of algebraic thinking (AT) before and after the intervention?

(c) through the PBL approach with the integration of algebraic thinking (ATPBL)?

(ii) Is there any significant difference in algebraic thinking for *SBP* students before and after being taught;

(a) by the conventional approach (CA)?

(b) with the integration of algebraic thinking (AT)?

(c) through the PBL approach with the integration of algebraic thinking (ATPBL)?

(iii) Is there any significant difference in algebraic thinking for *SBP* students taught;

(a) by the conventional approach (CA) compared to the integration of algebraic thinking (AT)?

(b) by conventional approach (CA) compared to the PBL approach with the integration of algebraic thinking (ATPBL)?

(c) with the integration of algebraic thinking (AT) compared to the PBL approach with the integration algebraic thinking (ATPBL)?

(iv) How do students think algebraically while learning algebra through the PBL approach with the integration of algebraic thinking (ATPBL)?

(v) What is the framework that integrates students' algebraic thinking with the PBL learning process?

1.6 Hypotheses

Based on the research questions, the researcher has put forward several null hypotheses (H_0) built on the significance level $\alpha = 0.05$ as follows;

H_{01} There is no significant difference in students' algebraic thinking before and after being taught using CA.

H_{02} There is no significant difference in students' algebraic thinking before and after being taught using AT.

H_{03} There is no significant difference in students' algebraic thinking before and after being taught using ATPBL.

H_{04} There is no significant difference in students' algebraic thinking after being taught using CA compared to AT.

H_{05} There is no significant difference in students' algebraic thinking after being taught using CA compared to ATPBL.

H_{06} There is no significant difference in students' algebraic thinking after being taught using AT compared to ATPBL.

1.7 Theoretical Framework

Constructivism consists of various theories that assimilate the behaviourist and cognitivist theories (Amineh and Asl, 2015). Constructivism functions as a theory of education in which the teacher encourages students to construct their own knowledge and implement it practically (Mvududu and Thiel-Burgess, 2012). Constructivism supports the cognitive development and social constructivism theory in this study. There are various theories that support the learning of algebra, such as Piaget's cognitive development theory, Bruner's stages of representation, Diene's six–stage theory, Gagne's theory of instruction and Ausubel's meaningful verbal learning. However, this study applies Piaget's (1970) cognitive development theory. Piaget stated that children aged 12 to 18 years old are within the formal operational stage. They are able to think inductively and deductively in proving mathematical theorems and rules and making generalizations or conclusions. In this stage, children also are able to use mathematical concepts to represent abstract concepts, solve and justify problems and relate certain concepts with other concepts.

This study focuses on identifying the characteristics of algebraic thinking in learning algebra. The characteristics of algebraic thinking are based on the framework developed by Walkoe (2013), which is based on Piaget's cognitive development theory. The characteristics of algebraic thinking are the manipulation of symbols and procedures, exploring relationships, generalizing and formalizing, using algebra as a tool, reasoning about and with representations and connecting representations. The characteristics of algebraic thinking relate to the formal operational stage in Piaget's theory, since this study was carried out for Form Two students (aged 14 years).

At this stage, students are not reliant on concrete materials, since they are able to think abstractly and solve complex problems. There are two processes of adaption, which are interrelated; assimilation and accommodation. Assimilation is a process of transferring new experience into an existing schema, while accommodation is a process of restructuring towards a mental structure to fulfil the requirements of the new experience. Both processes are used simultaneously and alternately throughout life (Powell and Kalina, 2009; Huitt and Hummel, 2003; Saad, 2002). In these two processes, students are able to transfer their knowledge of algebra into arithmetic. They are also able to solve problems through procedural thinking and to think algebraically.

Constructivism is the process of learning that explains how knowledge is constructed in the human mind. Teachers are unable to transfer knowledge to students in a perfect situation. Therefore, students need to construct their own knowledge according to their own experiences and abilities. According to Cottrill (2003), a person constructs knowledge himself or herself in the same environment. However, not all students are able to construct knowledge by themselves. They need teachers and peers to help them with some tasks. Scaffolding is a process that occurs when a learner is unable to solve a problem; an adult or an experienced peer helps the learner with some tasks until he or she is able to work independently (Moalosi, 2013). Learning algebra focuses more on mathematical symbols and the rules for manipulating these symbols. It also focuses on solving equations and expressions. Therefore, there is a need for a proper learning approach to assist in the process of learning algebra to enhance students' algebraic thinking.

This study applies PBL as a learning process for learning algebra. The roots of PBL are in Vygotsky's (1978) social constructivism theory. Social constructivism encourages learners to share their ideas and work collaboratively to solve different problems (Moalosi, 2013). The focus of social constructivism is on learning rather than performance which is in line with PBL. PBL is a process of meaningful and experiential learning that helps students to become actively involved in learning by providing them with real world problems to be solved (Hmelo-Silver, 2004). Using the social constructivist approach for teaching and learning, PBL requires students to be responsible for their own learning; to know about problems and to be able to define and solve them using appropriate learning materials (Goltz, Hietapelto, Reinsch, and Tyrell, 2008). Figure 1.1 illustrates that algebraic thinking is integrated with the learning process of PBL. Algebra should be taught in an effective manner so that students are able to apply the abstract concept in a meaningful way. The learning process of PBL is based on the work of Tan (2003). All the characteristics of algebraic thinking are integrated into the PBL learning process.

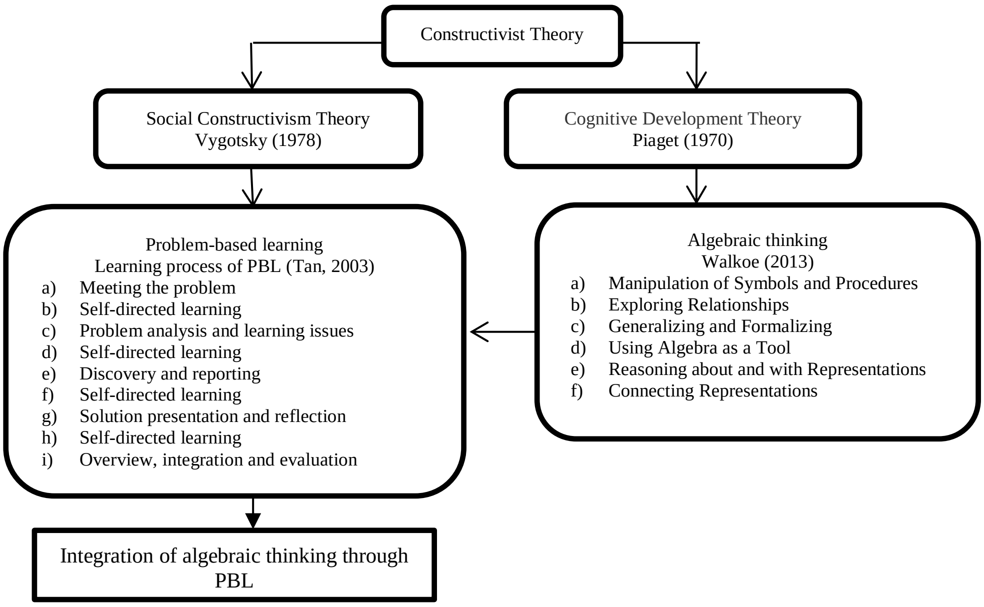

Figure 1.1 Theoretical Framework

1.8 Conceptual Framework

The conceptual framework for this study serves to show how all the theories are organized to achieve the study objectives. This study analyzed students' algebraic thinking using three different approaches, namely the conventional approach (CA), integration of algebraic thinking (AT) and the PBL approach with integration of algebraic thinking (ATPBL). The two study interventions are first the integration of algebraic thinking, and second, the PBL approach with integration of algebraic thinking. The framework of algebraic thinking put forward by Walkoe (2013) was adopted in both interventions because its description of algebraic thinking is sufficiently detailed and comprehensive to be implemented for secondary level students. Furthermore, it is suitable for the syllabus of Form Two students in Malaysia. Most of the characteristics of algebraic thinking in Walkoe's (2013) framework, namely the manipulation of symbols and procedures, exploring relationships, using algebra as a tool, reasoning about and with representations and connecting representations, are explicitly taught to Form Two students, the only exceptions being generalizing and formalizing. However, not all of these construct are addressed explicitly, such as justifying, proving, thinking about or with representations of functions such as graphs, table and situations and using one representation to reason about another. The strength of Walkoe's (2013) framework is it expansion and extension of the framework developed by Kieran (1996).

Walkoe's (2013) framework is applicable for secondary school students' manipulation of symbols and procedures, using algebra as a tool and connecting representations. However, reasoning about and with representations, exploring relationships and generalizing and formalizing can be further enhanced through a suitable approach. It is important to identify the characteristics of algebraic thinking among students as well as guidance provided by teachers and discussion with peers. This can lead the students to make connections between arithmetic and algebra as well as to think algebraically rather than focusing on computational fluency. Walkoe's (2013) framework consists of manipulation of symbols and procedures, exploring relationships, generalizing and formalizing, using algebra as a tool, reasoning about and with representations and connecting representations.

Both interventions were implemented for Form Two students using the *AThinking* learning task for the AT group and the *PBMAThinking* learning task for the ATPBL group. The ATPBL group was given two scenario problems in the learning process based on Tan's model (2003) of PBL. Tan's model is suitable to be applied for any subject matter, as it emphasizes problem-solving skills and new areas of learning. It also consists of multiple perspectives. The problems encourage solutions from various subjects and topics, emphasizing cross-disciplinary knowledge. In this study, Scenario Problem 1 was about designing a t-shirt, and involved the choice of t-shirt, selection of suppliers and profit and loss of selling the t-shirt See Appendix H1). This problem involved discussion and decision-making among the students. This scenario problem required knowledge of business and linear equations. Meanwhile, Scenario Problem 2 had gaps in information and knowledge as the problem trigger. A problem situation can involve insufficient of data and incomplete information (Tan, 2003). In this study, Scenario Problem 2 was about diet for male and female students aged fourteen. It involved the construction of a menu to encourage healthy eating habits to reduce obesity among students, which required the integration of algebraic thinking (See Appendix H2). Solving this scenario problem required the knowledge of dietary needs and linear equations.

The results showed that the biggest impact occurred through the PBL approach with integration of algebraic thinking. Therefore, the integration of algebraic thinking in the learning process of PBL leads to the formulation of a framework for developing algebraic thinking in PBL. The framework integrates students' algebraic thinking with the PBL learning process to improve characteristics of algebraic thinking among lower secondary boarding school (*SBP*) students in the learning process of algebra. In comparison, the other frameworks emphasized on generalization that does not involve most of the characteristics, and does not identify the process to gain the characteristics of algebraic thinking.

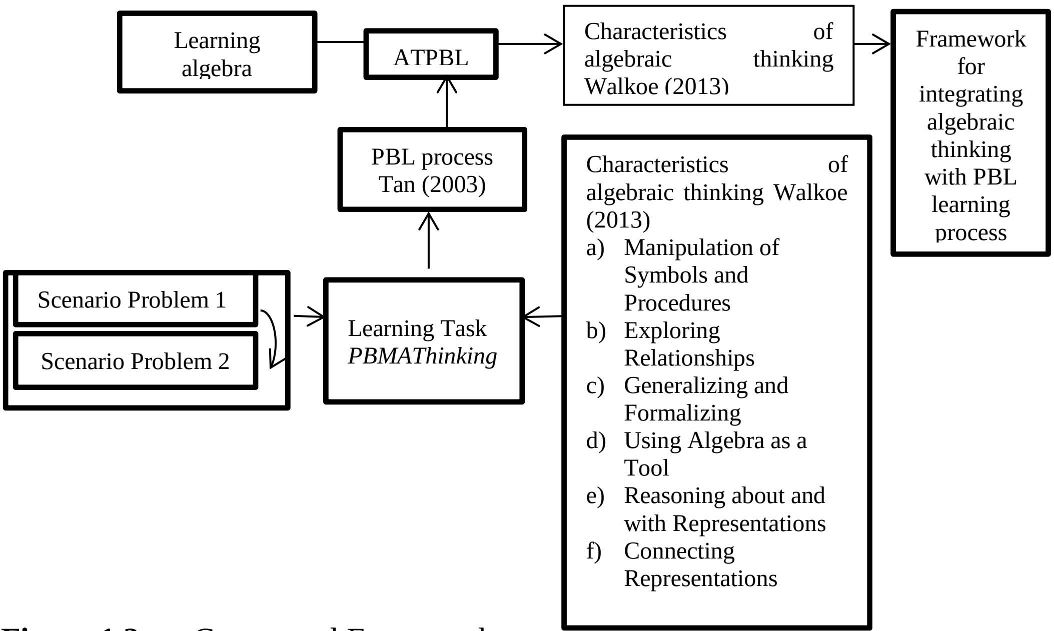

Figure 1.2 Conceptual Framework

1.9 Importance of Study

This study proposes a framework that integrates students' algebraic thinking into the PBL learning process. This proposed framework will help teachers to enhance students' algebraic thinking. To achieve high scores in algebraic thinking tests, algebraic thinking through PBL has to be emphasized from the outset and in every phase in PBL. Students who are new to the PBL process would be unable to succeed in scenario problem one. To enhance algebraic thinking in PBL, there should be training at an early stage. Students with medium or high algebraic thinking are able to manage in PBL, and their algebraic thinking will be improved. If students have low algebraic thinking and are newly exposed to the learning process of PBL, it will take time. In summary, PBL is able to enhance algebraic thinking. However, the students have to work through more than one scenario problem. The proposed framework integrates students' algebraic thinking (manipulation of symbols and procedures, exploring relationships, generalizing and formalizing, using algebra as a tool, reasoning about and with representations and connecting representations) into every phase of the PBL learning process.

Furthermore, teaching and learning algebra using an approach that emphasizes algebraic thinking helps teachers and students to notice the characteristics of algebraic thinking in the algebra domain. Hence, teachers can provide appropriate teaching aids.

Students are able to connect arithmetic to algebra through the PBL approach with the integration of algebraic thinking. This is due to how the students think algebraically. Furthermore, the students are able to connect algebra with real-life situations. Therefore, the students will realize that learning algebra is meaningful and applicable to real life.

The findings of this study will demonstrate the effectiveness of teaching and learning algebra from different approaches. In addition, it will also contribute to the learning process to be implemented in mathematics, especially in algebra, through various teaching and learning approaches.

It is hoped that this study will help educators in secondary schools in Malaysia to better utilize PBL as an instructional strategy to enhance algebraic thinking. However, it is not intended to be generalized to all topics in mathematics. It is only applicable to examine algebra topics and the characteristics of algebraic thinking. This study also contributes to the existing literature on PBL and potentially has an impact on algebraic thinking in mathematics.

The theoretical framework, research methodology, and findings can be used as guidelines and references as well as ideas for other researchers who are interested in algebraic thinking and PBL. It will also be beneficial for higher education institutes, mathematics educations and other relevant parties to implement algebraic thinking through PBL. Furthermore, it offers guidelines for the curriculum in Malaysia to develop its own characteristics of algebraic thinking.

1.10 Scope of the Study

This study aims to derive a framework for integrating algebraic thinking through PBL that incorporates the characteristics of algebraic thinking and the learning process of PBL. It is designed to provide a learning environment of algebraic thinking through PBL.

In this study, the characteristics of algebraic thinking of lower secondary boarding school (*SBP*) students have been identified with respect to;

i. Manipulation of symbols and procedures
ii. Exploring relationships
iii. Generalizing and formalizing
iv. Using algebra as a tool
v. Reasoning about and with representations
vi. Connecting representations

The development of algebraic thinking using problem-solving is well documented in the literature. However, algebraic thinking is not sufficiently taught. In this study, the researcher focused on the characteristics of algebraic thinking within three learning approaches; the conventional approach (CA), the integration of algebraic thinking (AT) and the PBL approach with integration of algebraic thinking (ATPBL). The PBL approach is widely discussed in the literature. It starts the learning process with problems and requires the use of facilitation and scaffolding. In terms of sample and population, this study involved Form Two boarding school (*SBP*) students in Perak. Three different schools were selected to form a control group consisting of twenty-five students and two experimental groups; the AT group and the ATPBL group, each comprizing thirty students. The topic studied was algebra, specifically Linear Equations.

1.11 Limitations of the Study

This study used techniques which involved collecting and analyzing verbal protocols. The data was obtained from the students' work during the *PBMAThinking* learning tasks. One limitation of this method is that the process of collecting, coding, and analyzing the verbal protocol data requires a lot of work (Cai, 1995; Gan and Ghazali, 2007). Therefore, the involvement of a large number of participants is not feasible for this study. Hence, the results of this study are merely indicative and can only be used to describe the patterns of the participants involved in this study.

The second limitation is the limited scope of the algebra topic, since only the Linear Equations topic was used in this study to suit the ability of lower secondary school students, as discussed in the problem statement. Therefore, it is important to recognize that the practical transferability (or generalizability) of the findings is constrained by the nature of the algebra topic chosen to be included in the study instrument.

The third limitation is the small number of scenario problems used in this study. This is because the scenario problems used were based on the syllabus and there was a limited time frame to collect data from the three groups. Furthermore, the task-based interviews were very cognitively demanding for the participants.

Moreover, the time allocated in the curriculum to cover Linear Equations is only five weeks. In this study, only two scenario problems were used in the learning process, because giving too many problems might tire the students and thus affect their thought processes (Payne, 1994).

The fourth limitation is related to the methodology of this study, particularly verbal protocol analysis, which involved the use of participants' own verbal reports as data. The validity of verbal reports as data may be doubted, as they inevitably involve selectivity and interpretation by the researcher. The interpretation is subjective and depends on the researcher's skill. Thus, the researcher must be very cautious and impartial in interpreting verbal reports. The protocol data must be integrated with more objective measures to achieve credibility (or validity) of the data. In this study, participants checked the transcripts of their task-based interview and triangulation was used to achieve credibility of data. In addition, experts were requested to verify part of the data analysis to ensure objectivity of the analysis.

1.12 Operational Definitions

The following sections define the terminology used in this study.

1.12.1 Algebra

Algebra is the abstract study of the properties of numbers, using letters to stand for the numbers; these letters are called variables (Sidebotham, 2002). It involves using arithmetic to find the answer to an undefined quantity (Choike, 2000). It is also a language of generalization, relationships between quantities and solving certain kinds of numerical problems. Topics related to algebra include linear equations, slope, permutations and combinations, exponents, quadratics and logarithms (Usiskin, 1995). Algebra includes algebraic expressions such as operational signs, constants and variables (Seng, 2010). Algebra consists of various domains of mathematics and is related to other topical domains (Eddy *et al*., 2015). In the context of the Malaysia curriculum, algebra consists of Indices and Logarithms, functions, quadratic equations, quadratic functions, simultaneous equations, progressions, and linear laws, referred to within additional mathematics. Additional mathematics is the subject extension of mathematics for students aged 16 and 17. In this study, the researcher focused on the Linear Equations topic from the syllabus for Form Two students in Malaysia's Integrated Curriculum for Secondary Schools.

1.12.2 Algebraic thinking

Algebraic thinking is a type of mathematical thinking. It is used to solve complex problems which consist of all mathematics strands and is formed based on conceptual understanding of computational fluency and numbers, reasoning using geometry and processes of measurement and statistics (Windsor and Norton, 2011). In this study, algebraic thinking consists of manipulation of symbols and procedures, exploring relationships, generalizing and formalizing, using algebra as a tool, reasoning about and with representations and connecting representations (Walkoe, 2013).

1.12.3 Conventional Approach (CA)

In this approach, the teacher taught the students using a marker pen, a whiteboard, and textbook. The teacher taught Linear Equations using Daily Lesson Plan *(RPH)* based on Curriculum Specifications for Mathematics Form 2 (Curriculum Development Centre, Ministry of Education, Malaysia, 2002).

1.12.4 Integration of Algebraic Thinking (AT)

In this study, AT refers to learning algebra with the integration of algebraic thinking, which emphasizes six key characteristics of algebraic thinking, namely the manipulation of symbols and procedures, exploring relationships, generalizing and formalizing, using algebra as a tool, reasoning about and with representations and connecting representations. Learning algebra also encourages the teacher to pose and discuss questions in the classroom to encourage students' algebraic thinking in terms of these characteristics. In this study, the teacher and students were provided with learning tasks that integrated algebraic thinking (*AThinking*) during the teaching and learning process.

1.12.5 Learning Task Integration of Algebraic Thinking (*AThinking*)

The AT learning task (*AThinking*) was employed in the learning process as guidance for teacher and students. It consisted of eleven Daily Lesson Plans (*RPH 1* to *RPH 11*). Each *RPH* involved two worksheets with reflection and evaluation questions respectively. Details of the learning tasks are provided in Chapter 3.

1.12.6 Problem-Based Learning (PBL)

PBL is an instructional approach where students learn by themselves through their own experience (Hmelo-Silver and Barrows, 2006). The implementation of PBL will be a success if it is based on open-ended, real life, challenging and well designed 'problems' or 'triggers' (Barron, Lambert, Conlon and Harrington, 2008). The focus of this study was to develop algebraic thinking through PBL. Algebraic thinking was measured through the learning process of implementing PBL in mathematics. In this study, the implementation of the PBL learning process is based on the model developed by Tan (2003).

1.12.7 Problem-Based Learning (PBL) Approach with the Integration of Algebraic Thinking (ATPBL)

Learning algebra through PBL with the integration of algebraic thinking emphasizes the manipulation of symbols and procedures, exploring relationships, generalizing and formalizing, using algebra as a tool, reasoning about and with representations and connecting representations. Algebra learning implemented the PBL process of learning. Learning algebra also encourages the teacher to pose and discuss questions in the classroom to encourage students' algebraic thinking. In this study, the teacher and students were provided with learning tasks that allowed the integration of algebraic thinking through PBL (*PBMAThinking*) during the teaching and learning process.

1.12.8 Scenario Problem

Problems is an important element in problem-based learning (PBL) role as to boost reasoning/thinking, able to organize the content and knowledge as well as motivation for learning process (Hung, 2006). According to Azer, Peterson, Guerrero and Edgren (2012), case scenarios are used to relate students' learning with real life, integrate the knowledge with clinical issues, stimulate reasoning and drive students' retention in long-term memory. Problems in PBL are referred as " triggers", "scenario" or "cases" aimed to trigger students in the learning process whether in textual format, computer simulations or pictures (Sockalingam and Schmidt, 2011; Sockalingam, 2010). In this study, scenario problems are related with their real life. Scenario problems are presented in situational based where students were assigned roles in solving problem. Two scenario problems are involved in this study, whereby in scenario problem 1, the students were required to make a decision of choosing the supplier and handling with an outsider in negotiating price of t-shirt. Furthermore, in Scenario Problem 2, the students were required to obtain an advice from the nutritionists for healthy dietary for female and male students.

1.12.9 Learning Task through Problem-Based Learning (PBL) Approach with the Integration of Algebraic Thinking (ATPBL) (*PBMAThinking*)

The ATPBL (*PBMAThinking*) learning task was employed in the learning process for PBL as guidance for teacher and students. It consists of daily lesson plans (*RPH*), two scenario problems, the process for the implementation of ATPBL, forms of teaching notes 1 and 2, forms of self-reading 1 and 2, forms of reflection 1 and 2, forms of self-evaluation 1 and 2, forms of evaluation Scenario Problem 1 and 2, forms of action plan 1 and 2, forms of KNL 1 and 2. However, only forms of teaching notes 1 and 2, forms of self-reading 1 and 2, forms of reflection 1 and 2, forms of self-evaluation 1 and 2, forms of evaluation of Scenario Problem 1 and 2 were used for data analysis. Details of the learning tasks are provided in Chapter 3.

1.12.10 Boarding School (*Sekolah Berasrama Penuh*) (*SBP*)

Boarding schools (*Sekolah Berasrama Penuh: SBP)* in Malaysia are schools at which the students are fully residential in hostels. They have fixed schedules for students' activities from early morning until bedtime. Boarding school entry is under the management of the Ministry of Education (*Kementerian Pelajaran Malaysia: KPM).* It is based on excellent academic achievement in the *Ujian Penilaian Sekolah Rendah (UPSR)* to enter at Form One level and in the *Penilaian Menengah Rendah (PMR)* or *Pentaksiran Tingkatan 3* (*PT3*) to enter at Form Four level and is supported by co-academic or co-curriculum (Ilias, 2012; Khalidah *et al*., 2014; Muhriz *et al*., 2011). Many researchers use the term 'boarding school' (*Sekolah Berasrama Penuh)* (Ghani *et al*., 2010; Ghani, Siraj, Mohd and Elham, 2011; Ghani, Siraj, Kassim, Kenayathulla, Marzuki and Elham, 2013; Abdullah, 2009). In other studies, boarding schools are referred to as fully residential schools (Jamil, Petras and Mohamed, 2013; Mahfar *et al*., 2014; Suleiman *et al*., 2010; Talif and Jayakaran, 1994). In this study, the researcher will used the term 'boarding school' because this is the term used by the Ministry of Education (*Kementerian Pelajaran Malaysia: KPM).*

1.12.11 Lower Secondary School Students

In this study, lower secondary school students are Form Two students in boarding school (*SBP*) with an age of 14 years old.

1.12.12 Characteristics of Algebraic Thinking

The abilities that constitute the characteristics of algebraic thinking, according to Walkoe (2013), are examined below;

a) Manipulation of Symbols and Procedures
The ability to state or use a known definition such as *x* or *y*, or another letter, or a symbol such as a picture of a fruit, and to manipulate symbols and procedures, such as an equals sign or known definition such as *x* or *y* or another letter or another symbol such as a picture of a fruit.

b) Exploring Relationships
The ability to identify the structure of a relationship between variables, to find patterns and to know the changes of relationships between variables.

c) Generalizing and Formalizing
The ability to make generalizations towards patterns, such as noticing that every time you multiply an even number by an odd number, the product is even, and to describe generalization towards a pattern formally; for example, "an even times an odd is even" or "if *X* or *Y* is even, *XY* is even."

d) Using Algebra as a Tool

The ability to state an example for comparison, the relationship between equation and object, to solve problems, justify and prove mathematically.

e) Reasoning about and with Representations

The ability to relate variables with equations, graphs, tables and real situations.

f) Connecting Representations

The ability to form equations, connect two or more variables and use a variable in reasoning another variable.

This algebraic thinking framework was proposed by Walkoe (2013) to identify students' algebraic thinking. It can be used to help teachers to identify lower secondary students' algebraic thinking.

1.13 Summary

Algebra is an important domain in mathematics. Learning algebra is pivotal in order to proceed to the next level of education. However, algebra is widely considered to be difficult, abstracted, and disconnected from real situations. One of the reasons for this is the teaching approach. There still exist some teachers who teach algebra through the conventional approach, which emphasizes procedural and step-by-step solutions (Abidin and Zamri, 2014; Hossain, Tarmizi and Ayud, 2012; Wong, 2005). Many students also fail to connect arithmetic and algebra. However, algebra is a way of thinking. Prior study suggests that to succeed in algebra, algebraic thinking should be introduced in earlier grades. In Malaysia, the mathematics curriculum focuses on thinking and reasoning in learning mathematics in general.

In this study, the integration of algebra thinking (AT) was tested to identify students' algebraic thinking using the framework developed by Walkoe (2013). However, there are other issues that hinder the application of algebra in real situations. Many students think that it is meaningless to learn algebra and to think algebraically. Therefore, this study applied the PBL approach for the intervention, based on the learning process designed by Tan (2003).

Constructivism theory is at the core of this study. It consists of cognitive development theory, developed by Piaget (1970), and social constructivism theory, developed by Vygotsky (1978). The conceptual framework for this study has led to the integration of Walkoe's (2013) framework of algebraic thinking into the learning process of the PBL model of Tan (2003) to form a framework for integrating algebraic thinking through PBL. This chapter has set out the importance, scope, and limitations of the study and provided the operational definitions of terms. The next chapter will present a literature review of the whole area of study.

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