

MODELLING OF TRANSVERSELY ISOTROPIC NONLINEAR
INCOMPRESSIBLE SOFT TISSUES USING SPECTRAL INVARIANTS

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To my beloved mother and father, my wife Noraini Adnan, my sons Fairuz Hazwan, Fairuz Safwan, Fairuz Hazman, Mohd Hasrat , my daughters Farah Diana, Farah Nadia, Farah Liana, Farah Dayana, Zamilah Mat Noor and Noor Shahida Mohamad.

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ABSTRACT

In isotropic elasticity, numerous strain energy functions with different types of invariants are developed to serve certain purposes. This wealth of functions has partly contributed to the knowledge of the mechanical behaviour of isotropic elastic solids. In general, soft tissues are not isotropic but can be modelled as transversely isotropic solid. The knowledge of the mechanical behaviour of transversely isotropic elastic solids is not as profound as isotropic solid. Hence, the need to develop accurate strain energy functions to understand the mechanical behaviour of transversely isotropic soft tissues. In isotropic elasticity, phenomenological strain energy functions with principal stretches have certain attractive features from both the mathematical and physical viewpoints. These forms of strain energy have been widely and successfully used in prediction of elastic deformations. This research is an extension from classical invariants of isotropic models to characterize transversely isotropic soft tissues with spectral invariants. In order to obtain a specific form of the strain energy function from an experiment, it is convenient to have explicit and analytic expressions for the derivatives of the strain energy function with respect to its invariants. Three of the invariants are the principal extension ratios and the other two are the cosines of the angles between the principal directions of the right stretch tensor and the material preferred direction. These direct physical interpretations of the invariants shows that the model has an experimental advantage where a triaxial test can vary a single invariant while keeping the remaining invariants fixed. The symmetrical and orthogonal properties developed here are similar to that possessed by a strain energy function of an isotropic elastic solid written in terms of principal stretches. A specific constitutive model was applied to biological soft tissues and the model compares well with existing experimental data.

ABSTRAK

Dalam keanjalan berisotropi, pelbagai fungsi tenaga terikan dengan pelbagai jenis tak varian dibangunkan untuk mencapai matlamat tertentu. Kekayaan fungsi-fungsi ini sebahagiannya menyumbang kepada pengetahuan tentang tabiat bermekanik pepejal anjal berisotropi. Secara umum, tisu lembut tidak berisotropi tetapi boleh dimodelkan sebagai pepejal melintang berisotropi. Pengetahuan tentang tabiat bermekanik pepejal melintang anjal berisotropi tidak begitu mendalam seperti pepejal berisotropi. Oleh itu, keperluan untuk membangunkan fungsi tenaga terikan yang tepat untuk memahami tabiat mekanikal tisu lembut melintang berisotropi. Dalam keanjalan berisotropi, fenomenologi fungsi tenaga terikan dengan regangan utama mempunyai ciri-ciri menarik tertentu dari kedua-dua sudut pandangan matematik dan fizikal. Bentuk-bentuk tenaga terikan telah berjaya digunakan secara meluas dalam ramalan ubah bentuk anjal. Penyelidikan ini adalah lanjutan daripada model klasik tak varian berisotropi untuk mencirikan pepejal melintang berisotropi dengan spektrum tak varian. Dalam usaha untuk mendapatkan satu bentuk tertentu fungsi tenaga terikan daripada eksperimen, ia mudah untuk mempunyai ungkapan yang jelas dan analisis bagi terbitan fungsi tenaga terikan terhadap tak variannya. Tiga daripada tak varian adalah nisbah lanjutan utama dan dua yang lain adalah kosinus sudut antara arah utama tensor regangan yang betul dan arah pilihan bahan. Pentafsiran fizikal langsung tak varian ini menunjukkan bahawa model ini mempunyai kelebihan eksperimen di mana suatu ujian tiga paksi boleh mengubah satu tak varian tunggal sementara mengekalkan tak varian yang selebihnya. Sifat-sifat simetri dan ortogon yang dikembangkan di sini adalah sama dengan fungsi tenaga terikan dari pepejal anjal berisotropi ditulis dari segi regangan utama. Model juzukan tertentu digunakan kepada biologi tisu lembut dan model dibandingkan dengan data eksperimen yang sedia ada.

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LIST OF ABBREVIATIONS

MF	-	Muscle Fiber
IVP	-	Initial Value Problem
IEEE	-	Institute of Electrical and Electronics Engineers
ASME	-	American Society of Mechanical Engineers
SBR	-	Styrene Butadiene Rubber
FEM	-	Finite Element Method

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CHAPTER 1

INTRODUCTION

1.1 Introduction

In the literature there have been several different studies in which the macroscopic response of fiber-reinforced materials has been analysed in the context of anisotropic non-linear elasticity. Fiber-reinforced materials often exhibit non-linear stress-strain behaviour. This behavior is associated both with the properties of the material and with the interaction between them. In non-linear elasticity, the macroscopic description of the material response is given in terms of a strain-energy function, which is dependent on certain strain invariants. The presence of fiber reinforcement introduces specific invariants into the strain energy that affect stretches in the reinforcing direction. Several different phenomena related to fiber-reinforced materials have been captured within this framework.

A unified treatment that enables prediction of fiber instability or fiber failure in fiber-reinforced composite materials was provided by Merodio and Ogden in (Merodio and Ogden, 2002; Merodio and Ogden, 2003), for incompressible and compressible materials, respectively. The fiber failure was associated with the loss of ellipticity of the governing differential equations. Fiber instabilities have also been studied previously by Triantafyllidis and Abeyaratne (1983), Kurashige (1981) and Danescu (1991) in the context of bifurcation away from simple deformations in the fiber direction or

transverse to the fiber direction. Fiber kink broadening was studied by Merodio and Pence (Merodio and Pence, 2001a; Merodio and Pence, 2001b). Other phenomena related to the behavior of fiber-reinforced materials, such as the response to shear deformations in off-fiber directions, the existence of residual stress and cavitation instabilities have been analysed in England *et al.* (1992), Rogers (1975), Hoger (1996), Qiu and Pence (1997), and Polignone and Horgan (1993). However, in this thesis we are not concerned with stability or loss of ellipticity.

The analyses mentioned above have involved different strain-energy functions. For fiber-reinforced materials it is common to work with a strain energy that has two terms, one associated with the isotropic base material and the other with the transversely isotropic character of the material, i.e. an isotropic base material is augmented by a uniaxial reinforcement in what is referred to as the *fiber direction*. In each case the same reinforcing model was used to characterize the anisotropy of the constitutive equation, namely the *standard reinforcing model*. Here, we follow the same procedure and define the strain energy in terms of an augmented isotropic base material but we use a somewhat different reinforcing model.

In general (in three dimensions), two independent invariants are generally used to characterize the anisotropic nature of a transversely isotropic material model, one of which is related directly to the fiber stretch and is denoted by I_4 . The standard reinforcing model is a quadratic function that depends only on this invariant. The other invariant, denoted I_5 , is also related to the fiber stretch but introduces an additional effect that relates to the behavior of the reinforcement under shear deformations. When the deformation is restricted to plane strain with the fiber direction in the considered plane these two invariants are no longer independent (Merodio and Ogden, 2002; Merodio and Ogden, 2003).

1.2 Research Background

1.2.1 Phenomenology of Biomechanics

Biomechanics is often defined as ‘mechanics applied to biology’ (Fung, 1990), but biomechanics is better defined as the development, extension and application of mechanics for the purposes of understanding better physiology and pathophysiology as well as the diagnosis and treatment of disease and injury. The birth of the modern field of biomechanics had to await the development of an appropriate theoretical foundation, an enabling technology, mathematical methods and heightened motivation.

With regard to biomechanics, the Journal of Biomechanics was founded in 1968, the ASME Journal of Biomechanical Engineering in 1977, Computer Methods in Biomechanics and Biomedical Engineering in 1998, and most recently Biomechanics and Modeling in Mechanobiology in 2002. These journals, and others such as the Annals of Biomedical Engineering and the IEEE Transactions for Biomedical Engineering, continue to promote the growth of biomechanics.

Biomechanics is part of a larger, multidisciplinary activity whose goal is to understand better the conditions of health as well as those of disease and injury. Consequently, biomechanics has and will continue to benefit greatly from developments in the basic of life sciences, medical sciences, mathematics and materials science.

Histology is defined as the study of the fine structure of tissues; it is thus fundamental to biomechanics. Similarly, cell biology is the study of how cells grow, move, function and communicate with their surroundings; it, too, is fundamental to biomechanics, particularly many of the open problems that face us today.

Soft biological tissues exist in many different forms, each specialized to perform a specific function and each having a unique microstructure. Nonetheless, soft tissues are composed of the same basic constituents: cells and extracellular matrix. Cells are the fundamental structural and functional unit of tissues and organs.

The formulation of appropriate constitutive relations has long been central importance in biomechanics as highlighted in Fung (1993): “the greatest need lies in the direction of collecting data in multiaxial loading conditions and formulating a theory for the biological of living tissues when stresses and strains vary with time in an arbitrary manner. The general characteristic behaviours exhibited by soft tissues been known that biological soft tissues behave very differently from traditional engineering materials such as metals, wood and concrete.”

For the material modelling of biological soft tissues a variety of interesting works have been published in the last three decades. Constitutive model of soft tissues has been derived from constitutive relations which is described on the response of a material to applied loads, which depends of course on the internal constitution of the material. The emphasize of constitutive relations describe the behaviour of a material under conditions of interest, not the material itself. That is, although the equations that describes the behaviour of a particular material under all conditions (eg. water in its solid, liquid and gaseous phases depending on the local temperature and pressure), we can generally expect to identify relations that hold only under specific conditions of interest. Regarding to technical literature, as e.g. Holzapfel and Ogden (2003), Humphrey (1995), Humphrey (2002) and Cowin and Humphrey (2002), for an overview of the models for biological tissues. In Vaishnav *et al.* (1973) a two dimensional model for a canine ortha is proposed based on three polynomial expressions. Due to the fact that biological soft tissues are characterized by exponential stress-strain response, in Fung *et al.* (1979) a first model is introduced for the two dimensional mathematical description of such arteries reflecting the exponential material behaviour in the physiological domain. An extension to this model is given in Fung and Liu (1989), where residual stress occurring in unloaded configuration of

arteries are considered.

Although tissues may be best classified as mixture-composites that exhibit inelastic behaviours, under particular conditions of interest it may be sufficient to model their behaviour within the context of an elasticity or viscoelasticity theory.

1.2.2 Strain Energy Function with Spectral Invariants

Strain energy functions with spectral invariants in isotropic elasticity have certain attractive features physically and mathematically (Ogden, 1972). This kind of strain energy function have been used in many research and successfully used in predicting properties of deformation (Shariff, 2000). The Valanis and Landel (1967) strain energy function for isotropic materials has a simple form and very successful in modelling many types of isotropic solids (Shariff, 2000), and their model only used a single variable function. The normally used strain energy function for transversely isotropic elastic materials is written in classical invariants (Spencer, 1984),

$$W(C, D) = W(I_1, I_2, I_3, I_4, I_5) \quad (1.1)$$

$$I_1 = \text{tr } \mathbf{C}, \quad I_2 = \frac{((\text{tr } \mathbf{C})^2 - \text{tr } \mathbf{C}^2)}{2}, \quad I_3 = \det \mathbf{C}, \quad I_4 = \mathbf{a} \cdot \mathbf{C} \mathbf{a}, \quad I_5 = \mathbf{a} \cdot \mathbf{C}^2 \mathbf{a}.$$

where \mathbf{a} and \mathbf{C} are the preferred direction in the reference configuration and the right Cauchy-Green deformation tensor respectively.

Motivated by the principal stretch successes and the model proposed in simple form of Valanis and Landel (1967), we construct a strain energy function which contains only a general single variable function. We propose a constitutive equation based on the recent principal axis formulation of Shariff (2008) for transversely isotropic materials.

The proposed strain energy function for the constitutive equation depends on four simple spectral invariants that have physical meaning . Two of the invariants are the principal stretches α_i ($i = 1, 2$) and $1 \geq \beta_i = (\mathbf{a} \bullet \mathbf{e}_i)^2 \geq 0$, where \mathbf{e}_1 and \mathbf{e}_2 are principal directions where \mathbf{U} is the right stretch tensor and, \mathbf{a} is the preferred direction of the transversely isotropic solid. The square of the cosine of the angle between the principal direction \mathbf{e}_i and the preferred direction \mathbf{a} is β_i . A strain energy formulation using non-immediate-physical-interpretation invariants is, in general, not experimentally friendly. For example, an isochoric uniaxial stretch in one of the preferred direction will perturb all the classical invariants given in Equation (1.1), hence they are not experimentally friendly unlike the immediate-physical-interpretation invariants used here which are experimentally friendly as described in Shariff (2008).

When a nonlinear incompressible transversely isotropic strain energy function is specialized to classical (infinitesimal) elasticity, it should contain three independent classical ground state constants (Spencer, 1984) to fully characterize an arbitrary material in infinitesimal strain deformations. Some strain energy functions proposed in the past, however, have ground state constants that are numerically less than three which indicate that, in their models, either some of the three classical ground state constants are assumed to be zero or the three classical ground state constants are dependent. Generally, it is good practice, at the onset, to assume three independent constants in the constitutive equation unless (sensible) experimental data suggest otherwise. Simplicity is one of the reasons why some authors proposed strain energy functions with less than three ground state constants. In this thesis a constitutive model is proposed; it contains only a general single variable function and the three independent classical ground state constants appear explicitly. A specific form of strain energy function is proposed for soft tissues. One advantage of having the ground state constants appear explicitly in the model is that we could easily put restrictions on their values (for physically reasonable responses) (Shariff, 2008).

We propose a strain energy function written in terms of principal stretches have

a symmetrical property which similar to the symmetry properties by a strain energy function of an isotropic elastic solid written in classical invariants. A strain energy functions written in terms of the invariants proposed in references (Chui *et al.*, 2007; Shariff, 2011; Shariff, 2013) are not symmetrical with respect to their invariants. By applying this model to a biaxial deformation such as extension and inflation of a thick-walled tube and a simple shear deformation. Through these application using principal axes expansion technique shows that the proposed model which has symmetrical properties can be written as a combination of the Valanis and Landel form (Valanis and Landel, 1967) and a symmetric function. The Valanis and Landel model also form can be easily incorporated into the transversely isotropic constitutive equation through an augmented form.

The proposed model with these advantages, would lead to our goal which is to express a strain energy function of a transversely isotropic elastic material in a different form. This model can benefit to other researchers to expand a bigger class of strain energy function and open alternative methods in transversely isotropic studies. We do not intend to discuss the performance and the range of validity of specific forms of the proposed strain energy function. However, in this thesis, we will discuss a proposed specific form which is based on spectral invariants.

1.3 Problem Statement

- (i) The existing strain energy function in terms of classical invariant do not have physical meanings in the sense that there are not experimental friendly.
- (ii) Most of the existing constitutive models may be accurate in curve fitting but not accurate in predicting mechanical behaviour of various types of soft tissues.
- (iii) Although some of the invariants in the literature have physical interpretation but it is difficult to perform an experiment based on these invariants since not all of them have a physical meaning and it is difficult to design a rational experiment.
- (iv) Existing strain energy function do not possess symmetry properties that may

facilitate the analysis of the biological soft tissues.

1.4 Research Objectives

This study embarks on the following objectives:

- (i) To develop specific constitutive equation to characterise the mechanical behaviour of biological soft tissues using spectral invariants.
- (ii) To propose an alternative constitutive model that has an advantage in experiment which is easy to analyse.
- (iii) To develop new constitutive equation that may be better than existing constitutive equation.
- (iv) To develop a constitutive equation in a simpler form and has experimentally friendly features.

1.5 Scope of the Study

This study is intended to develop a non-linear constitutive equation of transversely isotropic materials to provide adequate representation of the mechanical response of transversely isotropic materials. Various experimental data will be collected from previous works to apply to our constitutive model and compare the result to the other methods.

1.6 Significance of the study

- (i) The proposed constitutive model written in terms of spectral invariants have immediate physical interpretation and experimentally friendly because the stress-

strain formulation can be easily translated to an experiment.

- (ii) The proposed constitutive model possessed the symmetric and the orthogonal properties that can facilitate to analysis of the properties of biological soft tissues.
- (iii) The proposed constitutive model is an alternative method in predicting mechanical behaviour of biological soft tissues and the model is not very complicated as the existing model in literature.

1.7 Research Methodology

The understanding of the subject of continuum mechanics is extremely important to provide the knowledge in derivation of constitutive equation and strain energy function in both isotropic and transversely isotropic materials. The first topic to be discussed is the rigid body motion and the deformation theory. The next topic is the stress of the solid materials and the discussion emphasize on the acting in the interior of the continuous body. The another important topic to be discussed are biot stress, nominal stress and Cauchy stress before we discussed on some of the linear theories of continuum mechanics.

Constitutive equation is important for describing the mechanical behaviour and characteristic of materials such as biological soft tissues. Strain energy function is a part of constitutive equation, therefore the strain energy function must be derived to obtained the constitutive equation. Basically constitutive equation of transversely isotropic materials based on classical invariants of isotropic material widely used in rubberlike materials. The symmetric and rotation of isotropic and transversely isotropic materials will be discussed and continue to the strain energy function of isotropic and transversely isotropic materials. The introduction concept of hyperelasticity with strain energy function used the spectral invariants and applied to homogeneous biaxial deformation to show that the constitutive equation with spectral invariants is

mathematical simplicity.

The strain energy function contained five spectral invariants and the model has orthogonal properties. The strain energy function for transversely isotropic incompressible materials reduced to four spectral invariants have the correlation between the theory and experiment. The specific strain energy function with spectral invariants for soft biological soft tissues and the strain energy function possessed the unique properties any direction of deformation. Finally the specific constitutive equation for biological soft tissues will be derived.

The curve fitting technique is plotted against available experimental data from the literature to test the performance of the proposed constitutive model and shown that the theory compared well to the experimental data.

1.8 Thesis Outlines

In Chapter 2, literature review; we discussed previous models of isotropic material such as rubberlike materials that have been successfully used in the experiments. We show that all authors except for Shariff (2008) used classical invariant in their constitutive equation of transversely isotropic materials proposed by Spencer (1984). Finally we proposed spectral invariants in transversely isotropic materials that have physical meaning and experimental friendly.

In Chapter 3, research methodology; we discussed on kinematics and stresses. In kinematics, discussion will be focused on theory of deformation tensor, rigid body motion including deformation gradient tensor, left and right CauchyGreen deformation tensor left and right deformation stretch tensor and their relation. End of kinematic we discussed on example of some finite deformation. In stress, first we discussed on

surface traction and the derivation of first and second Piola-Kirchhoff stress and the Cauchy stress. At the end of the chapter we discussed a linear stress to show and their relation to non-linear stress.

In Chapter 4, constitutive equation; the constitutive equation is expressed in terms of strain energy function. Stress can be determined if we know the constitutive equation in the first place. First, we discussed the strain energy function of isotropic material which used the classical invariants and their orthogonal and symmetric properties. In the final section of the chapter, we derive the strain energy function of transversely isotropic of incompressible material in-term of principal stretches.

In Chapter 5, a model using spectral invariants of a transversely isotropic material is proposed based on the model of an augmented form of isotropic materials. Our model is shown to have good orthogonal properties. Here we showed good correlation between theory and experiment can be showed. The model has an an experimental advantage, where in a simple triaxial test we can vary a single invariant while keeping the remaining invariants fixed. A specific strain energy function for biological soft tissues is proposed.

In Chapter 6, derivation of non-linear spectral strain energy function from infinitesimal strain energy function is given. The function contained two terms, isotropic and transversely isotropic. The strain energy functions have six parameters $\alpha_i = 1, 2, 3$ and $\beta_i, i = 1, 2, 3$ and the material constants are μ_L, μ_T and ζ . We showed that the proposed spectral strain energy function has the unique value property, called the P-property. Finally we propose a specific form of constitutive equation for biological soft tissues.

In Chapter 7, data extracted from stress-strain experimental data of anterior and posterior mitral valve leaflet and excised epicardium of heart using Corel-Draw X5. Curve fitting from experimental data to determined the material constants $\mu_L,$

μ_T and ζ of constitutive models using software Maple 13 and Mathematica 9. We analyzed all the result and verified the the performance of the constitutive models to the experimental data. We have shown that all the curve fit identically to the experimental data. Finally in the discussion we concluded that the theory compare well to the experimental data and the proposed constitutive model predicted the mechanical behaviour of the biological soft tissues accurately and efficiently.

In Chapter 8, the summary on this thesis will be outlined, then the conclusion is given on the performance of the proposed constitutive model applied to the experimental data. We also stated the contribution of this thesis to the development of the research on nonlinear transversely isotropic incompressible materials or similar biological soft tissues and application to the real life such as to the medical and health problem.

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