LONG MEMORY ESTIMATION OF STOCHASTIC VOLATILITY FOR INDEX PRICES

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Dedicated to my beloved family and friends

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ABSTRACT

One of the typical ways of measuring risk associated with persistence in financial data set can be done through studies of long memory and volatility. Finance is a branch of economics concerned with resource allocation which deals with money, time and risk and their interrelation. The investors invest at risk over a period of time for the opportunity to gain profit. Since the last decade, the complex issues of long memory and short memory confounded with occasional structural break had received extensive attention. Structural breaks in time series can generate a strong persistence and showing a slower rate of decay in the autocorrelation function which is an observed behaviour of a long memory process. Besides that, the persistence in volatility cannot be captured easily because some of the mathematical models are not able to detect these properties. To overcome these drawbacks, this study developed a procedure to construct long memory stochastic volatility (LMSV) model by using fractional Ornstein-Uhlenbeck (fOU) process in financial time series to evaluate the degree of the persistence property of the data. The drift and volatility parameters of the fractional Ornstein-Unlenbeck model are estimated separately using least square estimator (LSE) and quadratic generalized variations (QGV) method respectively. Whereas, the long memory parameter namely Hurst parameter is estimated by using several heuristic methods and a semi-parametric method. The procedure of constructing LMSV model and the estimation methods are applied to the real daily index prices of FTSE Bursa Malaysia KLCI over a period of 20 years. The findings showed that the volatility of the index prices exhibit long memory process but the returns of the index prices do not show strong persistence properties. The root mean square errors (RMSE) obtained from various methods indicates that the performances of the model and estimators in describing returns of the index prices are good.

ABSTRAK

Salah satu cara tipikal untuk mengukur risiko yang berkaitan dengan keterusan dalam set data kewangan boleh dilakukan melalui kajian memori panjang dan turun naik. Kewangan adalah satu cabang ekonomi berkenaan dengan peruntukan sumber yang berkaitan dengan wang, masa dan risiko dan saling kaitan antara mereka. Para pelabur melabur dalam keadaan risiko sepanjang tempoh masa untuk peluang bagi mendapatkan keuntungan. Sejak sedekad yang lalu, isu-isu yang kompleks memori panjang dan memori pendek yang dikaburi dengan pemisahan struktur telah mendapat perhatian luas. Pemisahan struktur dalam siri masa boleh menjana keterusan yang kuat dan menunjukkan kadar susutan yang lebih perlahan dalam fungsi autokorelasi adalah satu tingkah laku yang diperhatikan dalam proses memori yang panjang. Selain itu, keterusan dalam turun naik tidak boleh diperhati dengan mudah kerana beberapa model matematik tidak dapat mengesan sifat-sifat ini. Untuk mengatasi kelemahan ini, kajian ini membina prosedur untuk membangunkan model memori panjang turun naik stokastik (LMSV) dengan menggunakan proses pecahan Ornstein-Uhlenbeck (fOU) dalam siri masa kewangan untuk menilai tahap sifat keterusan data. Parameter hanyutan dan turun naik model pecahan Ornstein-Unlenbeck dianggarkan secara berasingan masing-masing menggunakan kaedah kuasa dua terkecil (LSE) dan variasi teritlak kuadratik (QGV). Manakala, parameter memori panjang iaitu parameter Hurst dianggar menggunakan beberapa kaedah heuristik dan kaedah semi-parametrik. Prosedur membangunkan model LMSV dan kaedah anggaran digunakan kepada harga indeks harian FTSE Bursa Malaysia KLCI untuk tempoh 20 tahun. Hasil kajian ini menunjukkan bahawa turun naik harga indeks mengalami memori panjang tetapi pulangan harga indeks tidak menunjukkan sifat-sifat keterusan yang kuat. Min ralat kuasa dua (RMSE) yang diperolehi daripada pelbagai kaedah menunjukkan bahawa prestasi model dan penganggar berupaya menerangkan pulangan harga indeks dengan baik.

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LIST OF ABBREVIATIONS

ACF	Autocorrelation function
ARFIMA	Autoregressive fractionally integrated moving average
CUSUM	Cumulative sum
d	Fractional differencing parameter
DFA	Detrended fluctuation analysis
fOU	Fractional Ornstein-Uhlenbeck
fBm	Fractional Brownian motion
fGn	Fractional Gaussian noise
GPH	Geweke Porter-Hudak test
H	Hurst Parameter
H-sssi	Exponent self-similarity with stationary increments
LMSV	Long memory stochastic volatility
LSE	Least square estimator
OLS	Ordinary least squares
QGV	Quadratic generalized variations
OML	Quasi maximum likelihood
RMSE	Root mean square error
R/S	Rescaled range method
SDE	Stochastic differential equation

LIST OF SYMBOLS

$\gamma(k)$ - Autocovariance at time lag *k*

$\rho(k)$ - Autocorrelation at time lag *k*

$f(\lambda)$ - Spectral density

$\{X_t, t \in \mathbb{R}\}$ - Real-valued stochastic process

$B_H(t)$ - Fractional Brownian motion

$\{Z_i^H, i \in \mathbb{Z}\}$ - Fractional Gaussian noise

$\{\varepsilon_t\}$ - Gaussian white noise

$I_N(\lambda)$ - Periodogram

N - Number of samples

$(\Omega, \mathscr{F}, P)$ - Probability space

θ - Model parameters

λ - Drift parameter

σ_t - Volatility Process

β - Diffusion coefficient

S_t - Index Prices

$\{X_t\}$ - Returns

$\{X_t^2\}$ - Squared returns

$|X_t|$ - Absolute returns

$(I_+^\alpha f)(t)$ - Riemann-Liouville fractional integral

$\Gamma(.)$ - Gamma function

α - Scaling exponent

$W_n^0(t)$ - Empirical fluctuation process

$\hat{\lambda}_N$ - Least square estimator

$V_{N,a}$	-	Generalized quadratic variations
$\hat{H}_N$	-	Estimator of H using QGV
$\hat{\beta}_N$	-	Estimator of β using QGV
p	-	Number of sample paths

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CHAPTER 1

INTRODUCTION

1.1 Background of study

Market indices are inter-related to the performance of local and global economies and become an important guideline of investor confidence. Therefore, people have always searched for the skill to predict behavior of prices in financial market. The very first success and famous mathematical option pricing model namely the Black-Scholes option pricing model which assumed the constant volatility had been proven to have many flaws (Casas and Gao, 2008; Chronopoulou and Viens, 2012b). The constant volatility assumption is inconsistent with the empirical observation of varying volatility across varying time. In fact, a typical financial time series of returns had many common properties or so-called “stylized facts”, such as excess kurtosis, volatility clustering and almost no serial correlation in the level but with a persistent correlation in the squared returns and absolute returns. This “stylized facts” phenomenon can be explained by an appropriate volatility model which will consider the persistence properties of the data set. Volatility is an essential factor in measurement of the variability in price movements. The volatility of the prices has significant influence on the dynamics of the financial time series. Thus, an appropriate model for volatility will help to improve the measurement and provide useful information to the investors and economist.

The aim of this thesis is to develop a procedure to determine the characteristic of the FTSE Bursa Malaysia KLCI index prices intensively and comprehensively, in both returns and volatility. The modeling and parameters estimation on the relationship between the returns and volatility intend to help investors to have a clearer picture on the FTSE Bursa Malaysia KLCI index prices on decision making on their investments in Malaysia stock market. Besides that, the data analysis on the index prices can provide useful information to the Malaysia's government for successful development and implementation of policies on financial issues to improve our country's economy.

Constant volatility models have been proven in giving a poor fit on financial time series but the dynamic structures present a more realistic approach to volatility modeling. This is because volatility is affected by unpredictable changes such as the performance of the industry, political stability of particular country, news about new technology, natural disaster, product recalls and lawsuits that shall have positive and negative impact to the relevant company stocks, and therefore, the prices of the stock of a company are affected. Hence, many researches had been done in modeling volatility models in order to determine the dynamic fluctuation in the stock market. Among various volatility models, the autoregressive conditional heteroscedasticity (ARCH) model by Engle (1982) and generalised ARCH (GARCH) model by Bollerslev (1986) are very well known. The GARCH model assumes an ARMA-type structure for the volatility where the conditional volatility is a deterministic function of past returns. However, this assumption might be too restrictive in some of the problem and situations (Xie, 2008). As an example, the ARCH assumes that positive and negative shocks have same effects on volatility because it depends on the square of the previous shocks. In practice, it is well-known that price of a financial asset responds differently to positive and negative shocks. Therefore, another type of model, the stochastic volatility model which assumed to follow an autonomous and latent stochastic process will be more flexible.

Over the past two decades, many stochastic volatility (SV) models and estimation methods have been introduced to explain the market tendency. Stochastic volatility models have become popular for derivative pricing and hedging since the existence of a non-constant volatility surface has been classified. By assuming that the volatility of the underlying price is a stochastic process rather than a constant, it becomes possible to model derivatives more accurately. Stochastic volatility is a profound extension of the Black-Scholes model which describes a much more realistic trend in the financial world. There are several popular stochastic volatility models such as the Heston model, Orstein-Unlenbeck model and Cox-Ingersoll-Ross (CIR) model. These models have been widely used in the field of mathematical finance to evaluate the stock market. Once a particular stochastic volatility model is chosen, the calibration against the existing market data need to be carried out in order to identify the most likely set of model parameters given the observed data.

The main assumption of the SV model is that the volatility is a log-normal process. Taylor (1986) and Hull and White (1987) were among the first to study the logarithm of the stochastic volatility as an Ornstein-Uhlenbeck process. The statistical properties and probabilistic of a log-normal are well known. However, parameter estimation is a very challenging task due to the difficulty in finding the maximum likelihood (ML) function. The sampling methods for estimating the stochastic volatility are generally based on Bayesian approach or classical approach. Examples of Bayesian approach are Gaussian mixture sampling, single site Sampler and multi-move sampler, whereas the classical methods include quasi maximum likelihood, simulated method of moments and importance sampling among others.

Recent studies had showed that some of the financial data exhibit the properties of long-range dependence. However, these properties cannot be captured by the ordinary stochastic models. Since the pioneer work on detection in the presence of the long range dependence or long memory in minimum annual flow series of the Nile River by Hurst (1951), numerous studies have been carried out for testing and modeling long memory in various areas. In general, autocorrelation function is a measure of the dependence or persistence between the previous state and current state at various lags in a time series. A process is considered to exhibit long-memory or long-term persistence dependence if there is a significant autocorrelation at long lags.

In empirical modeling of long memory processes, Granger and Joyeux (1980) was the first to introduce a new model based on ARCH-type namely fractional integrated autoregressive moving average (ARFIMA) which had greatly improved the applicability of long memory in statistical practice. The model is characterized by hyperbolic decay rate of autocorrelation function. The term “fractional” is often used in the long memory context which usually refer to a model constructed using a generalized operation of non-integer order. In stochastic process, the models such as fractional Heston model and fractional Ornstein-Uhlenbeck (fOU) model had been modified to describe the long memory process in the sense that the present state of system is temporally dependent on all past states. Moreover, long memory is closely related with self-similar processes. Self-similar processes are stochastic models with the property that a scaling in time equivalent to an appropriate scaling in space. The connection between the two types scaling is determined by a constant which is known as Hurst exponent. Many of the empirical studies of long memory are based on the estimation method by Geweke and Porter-Hudak (1983). Besides that, there are some other estimation methods to detect the long memory based on the heuristic approaches such as rescaled range (R/S) statistics, detrended fluctuation analysis (DFA), periodogram method and aggregated variance method where neither of them needs any specific models assumptions.

The long memory in the volatility of the financial data had been discovered in the earlier of 1990’s. Ding *et al.* (1993) were among the first to investigate that there is strong correlation between absolute returns of the daily S&P 500 index prices. The fractional power transformations of the absolute returns showed high autocorrelations for high lags which provide the evidence of long-range dependence (Crato and de Lima, 1994; Deo and Hurvich, 2001; Ezzat, 2013). Besides that, the long term correlation is also found in the squared returns on various financial markets (Casas, 2008; Xie, 2008; Günay, 2014).

Since the last decade, the issue of confusing long memory and occasional structural breaks in mean had received great attention (see, (Diebold and Inoue, 2001; Granger and Hyung, 2004; Smith, 2005; Cappelli and Angela, 2006; Yusof *et al.*, 2013; Mensi *et al.*, 2014). Diebold and Inoue (2001) showed that there is a bias in favor of finding long memory processes in a time series when structural breaks are not accounted. Indeed, there is evidence that a stationary short memory process that encounters occasional structural breaks in the mean may show a slower rate of decay in the autocorrelation function and other properties of fractionally integrated processes (Cappelli and Angela, 2006). Thus, a time series with structural breaks can generate a strong persistence in the autocorrelation function which performs as the behaviour of a long memory process.

The early study of SV models was mainly focused on short memory volatility process. The long memory stochastic volatility (LMSV) model which is appropriate for describing series of financial returns at equally-spaced intervals of time had received extensive attention for last few years. Breidt *et al.* (1998) and Harvey (1998), simultaneously, was among the first who suggested a long memory stochastic volatility (LMSV) in discrete time where the log-volatility is modeled as an autoregressive fractional integrated moving average (ARFIMA) process. Comte and Renault (1998) proposed a continuous time fractional stochastic volatility model which adopted the fractional Brownian motion to replace the Brownian motion.

The LMSV models carry on many advantages of a general stochastic volatility model. However, unlike the usual short memory models, the LMSV model is neither a Markovian process nor can it be easily transformed into a Markovian process. This makes the likelihood evaluation and the parameter estimation for the LMSV model challenging tasks. Most of the previous research of LMSV model focused on the discrete time model which may due to the difficulty in constructing the computational work in continuous time model. In fact, the stock and index price process are only observed in discrete time, and the volatility itself cannot be directly observed, whether the underlying model is in discrete or continuous time or whether one believes that the underlying phenomena are discrete or continuous (Chronopoulou and Viens, 2012b). Therefore, from a financial modeling point of view, the statistical inference problem of estimating volatility under these conditions is then very crucial.

In terms of discrete-time models, Geweke and Porter-Hudak (1983) proposed a log-periodogram regression method which is also known as GPH method. In addition, Deo and Hurvich (2001) had presented the expressions for asymptotic bias and variance of the GPH estimators. Arteche (2004) proposed the Gaussian semiparametric or local Whittle estimator to estimate the long memory parameter.

There are very few papers that have developed the parameter estimations for long-memory stochastic volatility in continuous-time. Comte and Renault (1998) propose a discretization procedure to approximate the solution of their continuous-time fractional stochastic volatility (FSV) model and applied the log-periodogram regression approach to estimate the long memory parameter. Casas and Gao (2008) proposed the Whittle estimation method to estimate the parameters in a special class of FSV models. Chronopoulou and Viens (2012(a), 2012(b)) compared the performance of several long memory estimators and the implied value of H using real data of S&P 500 by calibrating it to option price.

Perhaps the most well-known approach of modelling long memory in continuous time stochastic volatility is to employ the fractional Brownian motion (fBm) as a long-memory driving source. The fractional Ornstein-Uhlenbeck (fOU) process is one of the popular model that contain the properties of long memory (see, (Cheridito *et al.*, 2003)). It would be optimal to estimate the parameters of fractional Ornstein-Uhlenbeck process and the long memory parameter jointly. But, most of the long memory models, still none provides a rigorous way for estimating a joint vector of parameters. Chronopoulou and Viens (2012(a), 2012(b)) suggested estimating the parameters separately and proposed to use calibration technique to fit models with various Hurst parameters. Many authors estimated the drift parameter and diffusion parameter separately with assumption of the Hurst parameter is known (see, (Hu and Nualart, 2010; Xiao *et al.*, 2011; Brouste and Iacus, 2013; Wang and Zhang, 2014)).

1.2 Statement of the Problem

One of the perplexing issues with regards to the detection of long memory is the confusion between long memory and non-linear effects such as parameter changes in time. There is evidence that a stationary short memory process that has occasional structural breaks in the mean can show a slower rate of decay in the autocorrelation function and other properties of long memory process. The structural change in the series camouflages the stationary short memory process. The long memory may be apparent for a certain sample, but a deeper investigation would be needed to show the long memory property as results from structural breaks or slow regime switching in the time series. Thus, long memory may be detected spuriously if structural break are not accounted for. Many authors had found that the financial time series exhibit the long memory properties. The question to ask is if it is really a long memory process or a short memory with structural break? One of our main concerns in this study is to investigate whether our financial data set exhibits a true long memory process.

Discovering the behaviour of market prices is not a simple task. The stock market prices tend to have complicated distributions with strong skewness and fat tails which commonly known as “stylized facts”. It is very essential to estimate the volatility in order to forecast the dynamic of the prices, i.e. what is the expected prices for tomorrow and how much it will differ from today’s price. Therefore, an appropriate modelling for the volatility becomes our second task in our study. There are a lot of evidences showing the existence of strong persistence in volatility of financial series. This study tends to propose a stochastic volatility model that will take into consideration of the volatility persistence.

The estimation of the volatility process is one of the most difficult and complicated problems in econometrics. There are neither volatility simulation techniques nor volatility data collections are completely ideal. The main difficulties are the fact that volatility itself is never directly observed. Therefore, in practice, one would be restricted to use the values of asset at discrete time even for the most liquid indexes or assets. Thus, we tend to propose a procedure to estimate the volatility process of the financial data series.

1.3 Objectives of the Study

The objectives of the study are:

1. To identify the true long memory from long memory process on the returns and volatility of the financial time series.
2. To propose continuous-time diffusion process in state space form for more flexible modeling of continuous dynamics in financial time series.
3. To establish a structured procedure in estimating the volatility process from the proxies of volatilities with the parameters estimation on the long memory stochastic volatility model.
4. To assess the long memory stochastic volatility model and estimation methods.

1.4 Scope of the Study

The scope of this research is given as follows:

1. This study establishes a general framework for analysing the closing index prices of FTSE Bursa Malaysia KLCI beginning from 3rd December 1993 until 31st December 2013. Here, the long memory properties of the closing index prices will be determined based on the returns and the volatilities of the data. The structural break analysis will be carried out to justify whether it is the true long memory or the spurious one.

2. This work presents the LMSV model with explanation on its basic properties. The LMSV state space model with fractional Ornstein-Uhlenbeck process including the long memory properties will be constructed. The models are developed to observe the volatilities persistence on the index prices.

3. The long memory parameter will be estimated with the Heuristic and semi-parametric methods. Whereas, the drift parameter and the diffusion coefficient will be estimated using least square method and quadratic generalised variation method respectively. This study performs the

numerical simulation of the model based on Monte Carlo method to illustrate the performance of the model and estimations methods.

4. The complete procedures are developed to analyse the characteristic of the closing index prices and constructing the LMSV model with estimation methods which will be assessed by their consistency. The comparison between the simulated and empirical returns is evaluated by the root mean square error and descriptive statistic.

1.5 Significance of the Study

The contribution of this research is in developing the procedures to analyse the index prices of FTSE Bursa Malaysia KLCI. As we all know, the volatility which measures the variability in price movement is at the centre of models for financial time series. This research considers a general class of stochastic volatility models either with long range dependence, intermediate range dependence or short range of dependence.

Long memory is one of the characteristic in financial time series. The return of the prices usually exhibits little or no autocorrelation, but volatility often has a strong autocorrelation structure. However, an argument exists saying that a short memory process with an occasional structural break can show the properties of long memory process. A spurious long memory will be detected if the structural break of the time series is not considered. Therefore, this study develops a strategy to detect true long memory of the returns and volatilities in financial data set.

Secondly, we propose a long memory stochastic volatility model in state space form using fractional Ornstein-Uhlenbeck process that can capture the characteristic observed in the financial time series. The LMSV model is more flexible assuming the volatility follows an autonomous and latent stochastic process. This model can provide a useful way of modeling the relationship between the returns and the volatility of the series exhibiting strong persistence in its level yet with varying time.

The LMSV model can help to identify the structure of the index prices in deriving the returns and volatility patterns. The estimated parameters on drift, diffusion coefficient and Hurst parameter in the model are practically useful for investor to have a clear picture on characteristics of the index prices. The goals of this research are to study the properties of FTSE Bursa Malaysia KLCI index prices via the long memory stochastic volatility models to solve the problem of excessive persistence in the composite linear and nonlinear models by introducing a probabilistic approach in allowing different volatility states in time series. The estimated returns show good result of the model in describing the dynamics of the FTSE Bursa Malaysia KLCI index prices. In this way, the accurate information can be provided for the forecasting in the future.

To the best of the author’s knowledge, there are no studies on applying long memory stochastic volatility models with parameters estimation to address the issues of the Malaysia economics. The proposed methods on estimating the volatility process from the proxies of volatilities using the modified LMSV model are the main contribution of this research. By establishing a complete procedure to identify the characteristic of the FTSE Bursa Malaysia KLCI index prices, the modified LMSV model manage to explain the Malaysia market tendency.

1.6 Organization of Thesis

The structure of this thesis can be summarized as follows. The thesis consists of introductory material which includes motivation, objectives, scope and significance of study in Chapter 1, literature review of the LMSV model and estimations methods in Chapter 2, the methodology and our novel contributions in Chapter 3, 4, 5, and the conclusion and future works in Chapter 6.

Chapter 2 presents literature review on long memory process with stochastic volatility and the estimations methods. The basic mathematical formulation for long memory process and stochastic volatility are defined. Then, the previous studies on LMSV models are introduced. The fractional Ornstein-Uhlenbeck model and its parameters estimation methods are also discussed. Furthermore, the literature on the methods of estimating the long memory parameter based on heristric and semi-parametric approaches are presented. The application of the long memory in financial time series will be evaluated from the point of view of modelling and the methods employed to estimate the parameters. Last but not least, the challenges of parameters estimation of LMSV model are included.

The long memory processes with definition and several aspects of their characteristics are introduced in Chapter 3. The long memory process will be defined in terms of its autocorrelations and spectral density. Then, the self-similar processes of long memory and fractional Brownian motion are described. Here, the modeling of long memory process in discrete time namely autoregressive fractional integrated moving average (ARFIMA) model and continuous time namely fractional Ornstein-Uhlenbeck (fOU) model are also discussed briefly. In this study, the focus mainly on fOU model in the modeling of long memory stochastic volatility model. The fractional calculus that is applied in this study will be explained as well.

In Chapter 4, a discussion on the modeling and parameters estimation of long memory stochastic volatility is given. First of all, the direction of volatility modeling in financial market is discussed. Then, some of the basic stochastic volatility model based on discrete and continuous time is presented. Next, this research modified a long memory stochastic volatility (LMSV) model in state space form. Methods for testing the existence of long memory are illustrated here. Moreover, the method for structural break analysis is studied in order to determine the true long memory. While, methods of parameters estimation included the drift and diffusion coefficient in the fOU volatility process of the LMSV model are also discussed. Last but not least, the methods to show the efficiency of the model and parameters estimation methods will be derived.

Chapter 5 presents the analysis of the index prices of FTSE Bursa Malaysia KLCI based on the proposed model and estimation methods of this research. Firstly, the description of the data and its transformation is presented. Then, how the long memory parameter estimation using heuristic and semiparametric approaches are employed on the data is shown. The results are given based on time domain and frequency domain according to the methods. Further, the results of structural break analysis are also presented. This is the chapter where the parameters estimation on the fractional Ornstein-Uhlenbeck (fOU) model for the long memory stochastic volatility is carried out based on the methodologies. Lastly, the contribution in this study is highlighted.

Chapter 6 is the final chapter which summaries the research findings. Besides, some suggestions for future works which might be potential and useful for further development or improvement of the proposed models and estimation methods are discussed.

REFERENCES

Aıt-Sahailia, Y. and Kimmel, R. (2007). Maximum likelihood estimation of stochastic volatility models. *Journal of Financial Economics*. 83(2), 413-452.

Alvarez-Ramirez, J., Alvarez, J., Rodriguez, E., and Fernandez-Anaya, G. (2008). Time-varying Hurst exponent for US stock markets. *Physica A: Statistical Mechanics and its Applications*. 387(24), 6159-6169.

Arteche, J. (2004). Gaussian semiparametric estimation in long memory in stochastic volatility and signal plus noise models. *Journal of Econometrics*. 119(1), 131-154.

Azmoodeh, E. and Morlanes, J.I. (2015). Drift parameter estimation for fractional Ornstein–Uhlenbeck process of the second kind. *Statistics*. 49(1), 1-18.

Azmoodeh, E. and Viitasaari, L. (2015). Parameter estimation based on discrete observations of fractional Ornstein–Uhlenbeck process of the second kind. *Statistical Inference for Stochastic Processes*. 18(3), 205-227.

Bai, J. and Perron, P. (2003). Computation and analysis of multiple structural change models. *Journal of applied econometrics*. 18(1), 1-22.

Bardet, J.-M. and Kammoun, I. (2008). Asymptotic properties of the detrended fluctuation analysis of long-range-dependent processes. *IEEE Transactions on information theory*. 54(5), 2041-2052.

Bassingthwaighte, J.B. and Raymond, G.M. (1994). Evaluating rescaled range analysis for time series. *Annals of biomedical engineering*. 22(4), 432-444.

Belfadli, R., Es-Sebaiy, K., and Ouknine, Y. (2011). Parameter estimation for fractional Ornstein-Uhlenbeck processes: non-ergodic case. *arXiv preprint arXiv:1102.5491*.

Beran, J. (1994). *Statistics for long-memory processes*. (CRC Press.

Bercu, B., Coutin, L., and Savy, N. (2011). Sharp large deviations for the fractional Ornstein-Uhlenbeck process. *Theory of Probability & Its Applications*. 55(4), 575-610.

Berkes, I., Horváth, L., Kokoszka, P., and Shao, Q.-M. (2006). On discriminating between long-range dependence and changes in mean. *The annals of statistics*, 1140-1165.

Bhattacharya, S.N. and Bhattacharya, M. (2012). Long memory in stock returns: A study of emerging markets. *Iranian Journal of Management Studies*. 5(5), 67-88.

Biagini, F., Hu, Y., Øksendal, B., and Zhang, T. (2008). *Stochastic calculus for fractional Brownian motion and applications*. (Springer Science & Business Media.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of econometrics*. 31(3), 307-327.

Bollerslev, T., Cai, J., and Song, F.M. (2000). Intraday periodicity, long memory volatility, and macroeconomic announcement effects in the US Treasury bond market. *Journal of empirical finance*. 7(1), 37-55.

Bollerslev, T., Engle, R.F., and Nelson, D.B. (1994). ARCH models. *Handbook of econometrics*. 4, 2959-3038.

Bollerslev, T. and Mikkelsen, H.O. (1996). Modeling and pricing long memory in stock market volatility. *Journal of econometrics*. 73(1), 151-184.

Bos, C.S. (2011). Relating stochastic volatility estimation methods.

Box, G.E.P. and Jenkins, G.M. (1976). *Time series analysis: forecasting and control*. (*2nd ed*). Holden-Day.

Breidt, F.J., Crato, N., and De Lima, P. (1998). The detection and estimation of long memory in stochastic volatility. *Journal of econometrics*. 83(1), 325-348.

Brockwell, P.J. and Davis, R.A. (1991). Time series: theory and methods. *Springer, New York*.

Broto, C. and Ruiz, E. (2004). Estimation methods for stochastic volatility models: a survey. *Journal of Economic Surveys*. 18(5), 613-649.

Brouste, A. and Iacus, S. (2013). Parameter estimation for the discretely observed fractional Ornstein–Uhlenbeck process and the Yuima R package. *Computational Statistics*. 28(4), 1529-1547.

Cappelli, C. and Angela, D. (2006). Long memory and structural break analysis of environmental time series.

Casas, I. (2008). Estimation of stochastic volatility with LRD. *Mathematics and Computers in Simulation*. 78(2–3), 335-340.

Casas, I. and Gao, J. (2008). Econometric estimation in long-range dependent volatility models: Theory and practice. *Journal of Econometrics*. 147(1), 72-83.

Chan, N.H. and Petris, G. (2000). Long memory stochastic volatility: a bayesian approach. *Communications in Statistics-Theory and Methods*. 29(5-6), 1367-1378.

Chavez-Demoulin, V. and Davison, A. (2012). Modelling time series extremes. *REVSTAT-Statistical Journal*. 10(EPFL-ARTICLE-180506), 109-133.

Cheridito, P., Kawaguchi, H., and Maejima, M. (2003). Fractional ornstein-uhlenbeck processes. *Electron. J. Probab*. 8(3), 14.

Chronopoulou, A. and Viens, F.G. (2012a). Estimation and pricing under long-memory stochastic volatility. *Annals of Finance*. 8(2-3), 379-403.

Chronopoulou, A. and Viens, F.G. (2012b). Stochastic volatility and option pricing with long-memory in discrete and continuous time. *Quantitative Finance*. 12(4), 635-649.

Clark, P.K. (1973). A subordinated stochastic process model with finite variance for speculative prices. *Econometrica: journal of the Econometric Society*, 135-155.

Comte, F. (1998). Long memory in continuous-time stochastic volatility models. *Mathematical finance*. 8(4), 291-323.

Comte, F., Coutin, L., and Renault, É. (2009). Affine fractional stochastic volatility models. *Annals of Finance*. 8(2-3), 337-378.

Comte, F. and Renault, E. (1998). Long memory in continuous-time stochastic volatility models. *Mathematical Finance*. 8(4), 291-323.

Cont, R. (2001). Empirical properties of asset returns: stylized facts and statistical issues.

Couillard, M. and Davison, M. (2005). A comment on measuring the Hurst exponent of financial time series. *Physica A: Statistical Mechanics and its Applications*. 348, 404-418.

Cox, J.C., Ingersoll Jr, J.E., and Ross, S.A. (1985). An intertemporal general equilibrium model of asset prices. *Econometrica: Journal of the Econometric Society*, 363-384.

Crato, N. and de Lima, P.J. (1994). Long-range dependence in the conditional variance of stock returns. *Economics Letters*. 45(3), 281-285.

Davidson, J. and Sibbertsen, P. (2009). Tests of bias in log-periodogram regression. *Economics Letters*. 102(2), 83-86.

De Groot, O. (2015). Solving asset pricing models with stochastic volatility. *Journal of Economic Dynamics and Control*. 52, 308-321.

Deo, R.S. and Hurvich, C.M. (2001). On the log periodogram regression estimator of the memory parameter in long memory stochastic volatility models. *Econometric Theory*. 17(04), 686-710.

Dickey, D.A. and Fuller, W.A. (1979). Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American statistical association*. 74(366a), 427-431.

Diebold, F.X. and Inoue, A. (2001). Long memory and regime switching. *Journal of econometrics*. 105(1), 131-159.

Dieker, T. (2004). Simulation of fractional Brownian motion. in: *MSc theses, University of Twente, Amsterdam, The Netherlands*.

Dietrich, C. and Newsam, G.N. (1997). Fast and exact simulation of stationary Gaussian processes through circulant embedding of the covariance matrix. *SIAM Journal on Scientific Computing*. 18(4), 1088-1107.

Ding, Z.X., Granger, C.W.J., and Engle, R.F. (1993). A long memory property of stock market returns and a new model. *Journal of Empirical Finance*. 1(1), 83-106.

Duffie, D., Pan, J., and Singleton, K. (2000). Transform analysis and asset pricing for affine jump diffusions. *Econometrica*. 68(6), 1343-1376.

Duncan, T.E., Hu, Y., and Pasik-Duncan, B. (2000). Stochastic calculus for fractional Brownian motion I. Theory. *SIAM Journal on Control and Optimization*. 38(2), 582-612.

Engle, R.F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica: Journal of the Econometric Society*, 987-1007.

Ezzat, H. (2013). Long Memory Processes and Structural Breaks in Stock Returns and Volatility: Evidence from the Egyptian Exchange. University Library of Munich, Germany.

Farma, E.F. (1965). The behavior of stock-market prices. *The journal of Business*. 38(1), 34-105.

Ferraz, R.O. and Hotta, L.K. (2007). Quasi-Maximum Likelihood Estimation of Long-Memory Stochastic Volatility Models. *Brazilian Review of Econometrics*. 27(2), 223-232.

Florescu, I. and Viens, F.G. (2008). Stochastic volatility: option pricing using a multinomial recombining tree. *Applied Mathematical Finance*. 15(2), 151-181.

Fouque, J.P. and Han, C.H. (2003). Pricing Asian options with stochastic volatility. *Quantitative Finance*. 3(5), 353-362.

Fouque, J.P., Papanicolaou, G., and Sircar, K. (2000). Stochastic volatility correction to black-scholes. *Risk*. 13(2), 89-92.

Geweke, J. and Porter-Hudak, S. (1983). The estimation and application of long-memory times series models, Vol. 4, Journal of Time Series Analysis, pp. 221-238.

Granger, C.W. (1966). The typical spectral shape of an economic variable. *Econometrica: Journal of the Econometric Society*, 150-161.

Granger, C.W. and Hyung, N. (2004). Occasional structural breaks and long memory with an application to the S&P 500 absolute stock returns. *Journal of empirical finance*. 11(3), 399-421.

Granger, C.W.J. and Joyeux, R. (1980). An Introduction to Long-Memory Time Series Models and Fractional Differencing. *Journal of Time Series Analysis*. 1(1), 15-30.

Grech, D. and Mazur, Z. (2004). Can one make any crash prediction in finance using the local Hurst exponent idea? *Physica A: Statistical Mechanics and its Applications*. 336(1), 133-145.

Günay, S. (2014). Long Memory Property and Structural Breaks in Volatility: Evidence from Turkey and Brazil. *International Journal of Economics and Finance*. 6(12), 119.

Harvey, A., Ruiz, E., and Shephard, N. (1994). Multivariate stochastic variance models. *The Review of Economic Studies*. 61(2), 247-264.

Harvey, A.C. (1998). *Long Memory in Stochastic Volatility*. (Oxford ,Boston: Butterworth Heinemann.

He, L.Y. and Qian, W.B. (2012). A Monte Carlo simulation to the performance of the R/S and V/S methods—statistical revisit and real world application. *Physica A: Statistical Mechanics and its Applications*. 391(14), 3770-3782.

Heston, S.L. (1993). A closed-form solution for options with stochastic volatility with applications to bond and currency options. *Review of financial studies*. 6(2), 327-343.

Heston, S.L., Loewenstein, M., and Willard, G.A. (2007). Options and bubbles. *Review of Financial Studies*. 20(2), 359-390.

Hosking, J.R. (1981). Fractional differencing. *Biometrika*. 68(1), 165-176.

Hu, Y. and Nualart, D. (2010). Parameter estimation for fractional Ornstein–Uhlenbeck processes. *Statistics & Probability Letters*. 80(11), 1030-1038.

Hull, J. and White, A. (1987). The pricing of options on assets with stochastic volatilities. *The journal of finance*. 42(2), 281-300.

Hurst, H.E. (1951). Long-term storage capacity of reservoirs. *Transactions of American Society of Civil Engineering*. 116, 770-799.

Hurvich, C.M., Deo, R., and Brodsky, J. (1998). The mean squared error of Geweke and Porter‐Hudak's estimator of the memory parameter of a long‐memory time series. *Journal of Time Series Analysis*. 19(1), 19-46.

Hurvich, C.M. and Ray, B.K. (2003). The local Whittle estimator of long-memory stochastic volatility. *Journal of Financial Econometrics*. 1(3), 445-470.

Hurvich, C.M. and Soulier, P. (2009). Stochastic volatility models with long memory. in: *Handbook of Financial Time Series*, pp. 345-354. Springer

Istas, J. and Lang, G. (1997). Quadratic variations and estimation of the local Hölder index of a Gaussian process. *Annales de l'IHP Probabilités et statistiques*. pp. 407-436.

Jacquier, E., Polson, N.G., and Rossi, P.E. (1994). Bayesian analysis of stochastic volatility models. *Journal of Business & Economic Statistics*. 12(4), 371-389.

Jacquier, E., Polson, N.G., and Rossi, P.E. (2004). Bayesian analysis of stochastic volatility models with fat-tails and correlated errors. *Journal of Econometrics*. 122(1), 185-212.

Jean-François, C. (2000). Simulation and Identification of The Fractional Brownian Motion: Abibliographical and Comparative Study. *Journal of statistical software*. 5, 1-53.

Jefferis, K. and Thupayagale, P. (2008). Long memory in Southern African stock markets. *South African Journal of Economics*. 76(3), 384-398.

Kaarakka, T. and Salminen, P. (2011). On fractional ornstein-uhlenbeck processes. *Communications on Stochastic Analysis* 5(1), 121-133.

Kang, S.H., Cheong, C., and Yoon, S.-M. (2011). Structural changes and volatility transmission in crude oil markets. *Physica A: Statistical Mechanics and its Applications*. 390(23), 4317-4324.

Kleptsyna, M. and Le Breton, A. (2002). Statistical analysis of the fractional Ornstein–Uhlenbeck type process. *Statistical Inference for stochastic processes*. 5(3), 229-248.

Krištoufek, L. (2010). Rescaled range analysis and detrended fluctuation analysis: Finite sample properties and confidence intervals. *Czech Economic Review*(3), 315-329.

Kroese, D.P. and Botev, Z.I. (2013). Spatial Process Generation. *V. Schmidt (Ed.). Lectures on Stochastic Geometry, Spatial Statistics and Random Fields*. II.

Kwiatkowski, D., Phillips, P.C., Schmidt, P., and Shin, Y. (1992). Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? *Journal of econometrics*. 54(1-3), 159-178.

Lamperti, J.W. (1962). Semi-stable stochastic processes. *Transactions of American Mathematical Society*. 104(62-78).

Levy, P. (1953). Random Functions: General Theory with Special Reference in Laplacian Random Functions. *University of California Publications in Statistics*. 1, 331-390.

Liew, V.K.-S., Lau, S.-H., and Ling, S.-E. (2005). A complementary test for ADF test with an application to the exchange rates returns.

Lo, A. (2002). Fat Tails, Long Memory, and the Stock Market Since the 1960's', Economic Notes, 26 (2), 213-46. *International Library of Critical Writings in Economics*. 146(2), 221-254.

Lo, A.W. (1991). Long-term memory in stock market prices. *Econometrica*. 59, 1279-1313.

Lobato, I.N. and Savin, N.E. (1998). Real and spurious long-memory properties of stock-market data. *Journal of Business & Economic Statistics*. 16(3), 261-268.

Løvsletten, O. (2016). Consistency of detrended fluctuation analysis. *arXiv preprint arXiv:1609.09331*.

Lux, T. (1996). Long-term stochastic dependence in financial prices: evidence from the German stock market. *Applied Economics Letters*. 3(11), 701-706.

Maheshchandra, J.P. (2012). Long Memory Property In Return and Volatility: Evidence from the Indian Stock Markets. *Asian Journal of Finance & Accounting*. 4(2), 218-230.

Mandelbrot, B.B. (1975). Limit theorems on the self-normalized range for weakly and strongly dependent processes. *Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete*. 31(4), 271-285.

Mandelbrot, B.B. and Taqqu, M.S. (1979). Robust R/S analysis of long-run serial correlation. *Bulletin of the International Statistical Institute*. 48(2), 59-104.

Mandelbrot, B.B. and Van Ness, J.W. (1968). Fractional Brownian motions, fractional noises and applications. *SIAM review*. 10(4), 422-437.

Mandelbrot, B.B. and Wallis, J.R. (1968). Noah, Joseph, and operational hydrology. *Water resources research*. 4(5), 909-918.

Mandelbrot, B.B. and Wallis, J.R. (1969). Robustness of the rescaled range R/S in the measurement of noncyclic long run statistical dependence. *Water Resources Research*. 5(5), 967-988.

Marinucci, D. and Robinson, P.M. (1999). Alternative forms of fractional Brownian motion. *Journal of statistical planning and inference*. 80(1), 111-122.

McGonegal, B. (2014). Fractional Brownian Motion and the Fractional Stochastic Calculus. *arXiv preprint arXiv:1401.2752*.

McLeod, A.I. and Hipel, K.W. (1978). Preservation of the rescaled adjusted range, 1: a reassessment of the Hurst phenomenon. *Water Resources Research*. 14(3), 491-508.

Melino, A. and Turnbull, S.M. (1990). Pricing foreign currency options with stochastic volatility. *Journal of Econometrics*. 45(1), 239-265.

Mensi, W., Hammoudeh, S., and Yoon, S.-M. (2014). Structural breaks and long memory in modeling and forecasting volatility of foreign exchange markets of oil exporters: The importance of scheduled and unscheduled news announcements. *International Review of Economics & Finance*. 30(0), 101-119.

Mishura, Y.S. (2008). *Stochastic calculus for fractional Brownian motion and related processes*. (Springer.

Molina, G., Han, C.-H., and Fouque, J.-P. (2010). MCMC estimation of multiscale stochastic volatility models. in: *Handbook of quantitative finance and risk management*, pp. 1109-1120. Springer

Palma, W. (2007). *Long-memory time series: theory and methods*. (John Wiley & Sons.

Peng, C.-K., Havlin, S., Hausdorff, J., Mietus, J., Stanley, H., and Goldberger, A. (1995). Fractal mechanisms and heart rate dynamics: long-range correlations and their breakdown with disease. *Journal of electrocardiology*. 28, 59-65.

Peng, C.K., Havlin, S., Stanley, H.E., and Goldberger, A.L. (1995). Quantification of scaling exponents and crossover phenomena in nonstationary heartbeat time series. *Chaos: An Interdisciplinary Journal of Nonlinear Science*. 5(1), 82-87.

Peng, C.K., Buldyrev, S.V., Havlin, S., Simons, M., Stanley, H.E., and Goldberger, A.L. (1994). Mosaic organization of DNA nucleotides. *Physical review.E, Statistical physics, plasmas, fluids, and related interdisciplinary topics*. 49(2), 1685.

Perron, P. (1989). The great crash, the oil price shock, and the unit root hypothesis. *Econometrica: Journal of the Econometric Society*, 1361-1401.

Ploberger, W. and Krämer, W. (1992). The CUSUM test with OLS residuals. *Econometrica: Journal of the Econometric Society*, 271-285.

Priestley, M.B. (1981). *Spectral Analysis and Time Series*. (Academic Press.

Rangarajan, G. and Ding, M. (2000). Integrated approach to the assessment of long range correlation in time series data. *Physical Review E*. 61(5), 4991.

Robinson, P.M. (1995). Log-periodogram regression of time series with long range dependence. *The annals of Statistics*, 1048-1072.

Rogers, L.C.G. (1997). Arbitrage with fractional Brownian motion. *Mathematical Finance*. 7(1), 95-105.

Shao, X. (2011). A simple test of changes in mean in the possible presence of long ‐ range dependence. *Journal of Time Series Analysis*. 32(6), 598-606.

Smith, A. (2005). Level shifts and the illusion of long memory in economic time series. *Journal of Business & Economic Statistics*. 23(321-335).

Sowell, F. (1992). Maximum likelihood estimation of stationary univariate fractionally integrated time series models. *Journal of econometrics*. 53(1), 165-188.

Sprott, J.C. (2003). *Chaos and time-series analysis*. (Oxford University Press

Suhaila, J. and Jemain, A. (2008). Fitting the statistical distribution for daily rainfall in Peninsular Malaysia based on AIC criterion. *J. Appl. Sci. Res*. 4(12), 1846-1857.

Sun, Y. and Phillips, P.C. (2003). Nonlinear log-periodogram regression for perturbed fractional processes. *Journal of Econometrics*. 115(2), 355-389.

Taylor, S.J. (1986). *Modeling financial time series*. (Chichester,UK:

Teverovsky, V., Taqqu, M.S., and Willinger, W. (1999). A critical look at Lo's modified R/S statistic. *Journal of statistical Planning and Inference*. 80(1), 211-227.

Tong, Z. (2012). Option Pricing with Long Memory Stochastic Volatility Models, Université d'Ottawa/University of Ottawa.

Trolle, A.B. and Schwartz, E.S. (2009). A general stochastic volatility model for the pricing of interest rate derivatives. *Review of Financial Studies*. 22(5), 2007-2057.

Wang, X. and Zhang, W. (2014). Parameter Estimation for Long-Memory Stochastic Volatility at Discrete Observation. *Abstract and Applied Analysis*. Hindawi Publishing Corporation.

Weron, R. (2002). Estimating long-range dependence: finite sample properties and confidence intervals. *Physica A: Statistical Mechanics and its Applications*. 312(1), 285-299.

Weron, R. and Przybyłowicz, B. (2000). Hurst analysis of electricity price dynamics. *Physica A: Statistical Mechanics and its Applications*. 283(3), 462-468.

Xiao, W., Zhang, W., and Xu, W. (2011). Parameter estimation for fractional Ornstein–Uhlenbeck processes at discrete observation. *Applied Mathematical Modelling*. 35(9), 4196-4207.

Xie, L. (2008). Long-Memory Stochastic Volatility Models: A New Method for Inference and Applications in Option Pricing. in: *Department of Statistics*, Vol. Ph.D, Carnegie Mellon University.

Young, L.C. (1936). An inequality of the Hölder type, connected with Stieltjes integration. *Acta Mathematica*. 67(1), 251-282.

Yusof, F., Kane, I.L., and Yusop, Z. (2013). Structural break or long memory: an empirical survey on daily rainfall data sets across Malaysia. *Hydrology and Earth System Sciences*. 17(4), 1311-1318.

Zähle, M. (1998). Integration with respect to fractal functions and stochastic calculus. I. *Probability theory and related fields*. 111(3), 333-374.

Zeileis, A. (2000). p values and alternative boundaries for CUSUM tests.