

SURVIVAL ANALYSIS OF TIME-TO-FIRST BIRTH AFTER MARRIAGE IN
NIGERIA USING THREE PARAMETER INVERSE GAUSSIAN
DISTRIBUTION

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UNIVERSITI TEKNOLOGI MALAYSIA

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DISTRIBUTION

AMUSAN AJITONI SIMEON

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requirements for the award of the degree of
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To God be the glory

When my back was against the wall, and it seemed as if it was all over, He made a way. He moved mountains, He caused walls to fall, with Your power, performed miracles, there's nothing that's impossible, I'm standing here only because He made a way. I don't know how, but He did it. To God be the Glory.

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ABSTRACT

Overpopulation is a huge problem for the people of sub-Saharan African countries, including Nigeria. In order to tackle this problem, demographers have in the last five decades made immense efforts towards understanding the causes and mechanisms of fertility, and to explain the variations inherent in it. Several theoretical approaches that have been proposed mainly focus on cumulative or completed fertility as the main dependent variable. Moreover, it is equally important to examine variations in fertility in such entities such as timing and spacing of births. Data on birth interval offers rich and more detailed information for the analysis of reproductive behavior than does the data on number of births. And more importantly, timing of first birth after marriage which also corresponds to first-birth-interval (FBI) is strongly correlated to the number of children a woman would have by the end of her reproductive life. In this research, FBI is modeled stochastically as a survival response variable. Suitability of a class of some standard parametric models is investigated and the study comes up with a novel three-parameter inverse Gaussian model through the exponentiation of probability distribution approach. Estimations of parameters are obtained by maximizing the log-likelihood functions of the data in R software programming. Different approaches employed to discriminate between the distribution and the existing ones considered, suggests that the new model, Lehmann type II inverse Gaussian, provides a better fit for the FBI and other lifetime data. Akaike Information Criterion (AIC) which is a resemblance of coefficient of determination (R^2) in linear regression, also shows a significant reduction in value as covariates are introduced into it in comparison with regressions from other distributions. This research finds that age of women at marriage, education attainment and region where women reside are significant prognostic factors that influence the chance of first birth after marriage. The thesis concludes by calling on agencies and organizations working on population control, especially in Nigeria, to initiate programs that will encourage girl-child education to higher level.

ABSTRAK

Lebih penduduk adalah suatu masalah besar kepada rakyat negara-negara Afrika sub-Sahara, termasuk Nigeria. Untuk menangani masalah ini, ahli demografi telah melakukan pelbagai usaha dalam tempoh lima dekad yang lalu ke arah memahami punca-punca dan mekanisme kesuburan, dan untuk menerangkan variasi yang wujud di dalamnya. Beberapa pendekatan teori yang telah dicadangkan hanya tertumpu kepada kesuburan kumulatif atau keseluruhan sebagai pembolehubah bersandar utama. Tambahan pula, ia adalah sama penting untuk meneliti variasi kesuburan serta entiti tersebut seperti masa dan jarak kelahiran. Data mengenai jarak kelahiran menawarkan banyak maklumat yang lebih terperinci untuk analisis tingkah laku pembiakan berbanding dengan data mengenai jumlah kelahiran. Lebih penting lagi, masa kelahiran pertama selepas perkahwinan, yang juga sepadan dengan selang kelahiran pertama (FBI), amat berkait rapat dengan bilangan anak seseorang wanita akan melahirkan pada akhir tempoh usia kesuburan beliau. Dalam penyelidikan ini, FBI dimodelkan secara stokastik sebagai pembolehubah sambutan mandiri. Kesesuaian beberapa model berparameter yang piawai disiasat dan kajian ini telah menghasilkan suatu model yang novel iaitu model Gaussian songsang tiga parameter melalui pendekatan pengeksponenan taburan kebarangkalian. Anggaran parameter diperolehi dengan memaksimumkan fungsi log-kemungkinan bagi data dengan menggunakan pengaturcaraan perisian R. Pendekatan berbeza telah diguna untuk mendiskriminasikan antara taburan baharu dengan taburan sedia ada, menyarankan bahawa model baharu, Lehmann Jenis II Gaussian songsang, mempunyai penyuaian yang lebih baik untuk data FBI dan data masa hayat yang lain. Maklumat Kriteria Akaike (AIC), yang menyerupai pekali penentu (R^2) dalam regresi linear, juga menunjukkan pengurangan nilai yang ketara apabila kovariat diperkenalkan ke dalam model baharu ini berbanding dengan model regresi daripada taburan yang lain. Penyelidikan ini mendapati bahawa umur wanita ketika perkahwinan, pencapaian pendidikan dan rantau di mana wanita tinggal adalah faktor-faktor prognostik penting yang mempengaruhi peluang untuk kelahiran pertama selepas perkahwinan. Tesis ini diakhiri dengan menyeru agensi-agensi dan organisasi yang bertanggungjawab terhadap kawalan populasi penduduk, khususnya di Nigeria, supaya memulakan program-program yang akan menggalakkan pendidikan kanak-kanak perempuan ke tahap yang lebih tinggi.

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LIST OF ABBREVIATIONS

AIC	-	Akaike Information Criteria
CEA	-	Census Enumeration Area
CDF	-	Cumulative Density Function
CIA	-	Central Intelligence Agency
EF	-	Exponentiated Frechet
EG	-	Exponentiated Gamma
EG _u	-	Exponentiated Gumbel
EGF	-	Exponentiated Generalized Function
Exp~F	-	Exponentiated cdf
FBI	-	First Birth Interval -
FCT	-	Federal Capital Territory
gof	-	Goodness of fit
HR	-	Hazard Ratio
IG	-	Inverse Gaussian
K-L	-	Kullback-Leibler
KM	-	Kaplan-Meier
LGA	-	Local Government Area
LL	-	Log-Likelihood
LIIG	-	Lehmann type II inverse Gaussian
MLE	-	Maximum Likelihood Estimate
MSE	-	Mean Squared Error
NDHS	-	National Demographic and Health Survey
<i>NPC</i>	-	National Population Commission
PDF	-	Probability Density Function
PHM	-	Proportional Hazard Model
PL	-	Product-Limit

PPA	-	Post-Partum Amenorrhea
qq	-	quartile-quartile
UN	-	<i>United Nations</i>
UNDP	-	United Nations Development Programme
UNICEF	-	United Nations Children Emergency Funds
UNDP	-	United Nations Population Funds

LIST OF SYMBOLS

R^2	-	Coefficient of determination
β_0	-	Intercept in the regressions
β_i	-	Regression coefficients
μ	-	Mu - Mean parameter
α	-	Alpha- shape parameter in LIIG
λ	-	Scale parameter for LIIG
Λ	-	Likelihood Ratio
χ^2	-	Chi – squared
N_c	-	Number censored
N_f	-	Number failed in lifetime data
(.]	-	Open-closed interval
[.)	-	Close-opened interval
(..)	-	Open-open
{..]	-	Close-close
\int	-	Integral
$\frac{\partial y}{\partial x}$	-	First order partial differential of y with respect to x
Δ	-	Small change
Σ	-	Sigma - Summation
Π	-	Pi – Product

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CHAPTER 1

INTRODUCTION

1.1 Overview

The birth of a first child after marriage of a woman is the first visible outcome of fertility process in her life. This event is important and significant in every human society as it marks the proof of womanhood of a married woman. Therefore, the waiting time to the occurrence of first birth after marriage which is often referred to first-birth-interval (FBI), is not only vital to a successful and harmonious marital life of the couple, but of great importance to demographers and social scientists for its implications on the completed family size and consequently on the population size dynamics. FBI has been recognized as one of the strongest and most consistent factors affecting populations, with longer interval usually associates with lower fertility while shorter interval corresponds to higher fertility. Nevertheless, there has been very little documentation on FBI after marriage in many of the sub-region of Sahara African countries including Nigeria.

FBI falls into the category of survival data, which corresponds to the time an individual enters a study until the occurrence of a particular event of interest or study ends. It is characterized by skewing and censoring thereby rendering the use of Gaussian distribution assumptions inappropriate for its analysis and modeling. Here

in the research described in this thesis, this interval among married women is modeled stochastically as a survival response variable.

This chapter contains the background of the study, motivation behind the research, statement of the problem, objectives, scope and limitation of the study, and its significance. Later in the chapter, outline of the thesis organization is presented.

1.2 Problem background

Overpopulation is a huge problem for the peoples of the sub-Sahara African countries, including Nigeria. More importantly, Nigeria has been identified as one of the major countries responsible for the high population in the region. It has, by far, the largest population with 184 million inhabitants accounting for 16 per cent of African population in 2015 as projected by Unicef-Africa 2030 (unicef, 2014). It is also projected that in just 35 years, Nigeria population will be 2.5 times its current size reaching 440 million. The major factor responsible for the increase in the population of the country is the relatively high fertility level. The current total fertility rate (TFR) of 5.7 births per woman compared to the overall TFR of 5.2 births in Africa, is considered relatively high (Commission, 2009a). If the current pace of population growth in Nigeria is not however put in check, there is bound to be catastrophes such as mass starvation, tension among the people for the limited resources, high crime rates and so on. For instance, a typical day in Lagos, one of many cities in Nigeria is as shown in Figure 1, where both the human and vehicular traffics are chaotic. Against this backdrop, it is therefore, imperative to study and understand the mechanisms of fertility changes in the population.



Figure 1.1: Nigeria's ticking population bomb (A street in Lagos)

While acknowledging the fact that demographers have in the last five decades made immense progresses towards understanding the causes and mechanisms of fertility, and to elucidate the variations inherent in it; several theoretical approaches (Davis and Blake, 1956; Becker, 1960; Bongaarts, 1978; Easterlin, 1975) have been proposed, developed and refined. Although our perspectives on differentials in fertility have been undoubtedly broadened by these approaches, but they mostly focus on cumulative or completed fertility as the main dependent variable. However, the study of variations in fertility in such entities such as timing and spacing of births is equally important. Data on birth interval offers rich and more detailed information for the analysis of reproductive behaviour than does the data on number of births (Henry, 1961). And also from the theoretical point of view, timing of the first birth and subsequent births are very important in fertility studies since family building involves a series of stages; where women move from marriage to the first birth, then to the second birth and so on until the desired fertility size is achieved (Rodriguez and Hobcraft, 1980)

More specifically, data on time-to-first birth forms better data in terms of quality than other birth intervals in the study of fertility behavior in a population of

women of reproductive age group. First birth, being the earliest and first event of her marital life remains memorable to the woman. The date of the event hardly suffers memory lapse during survey unlike other births whose dates may be somewhat affected by memory loss. More so the length from marriage to first birth, that is, first birth interval (FBI), is also free from the influence of the period of lactation of the nursing mother, called post-partum amenorrhea period (ppa) which is associated with other birth intervals. As a result, statistics obtained from these affected data from other birth intervals would be somewhat lacking in asymptotic properties such as efficiency and reliability (Amin and Bajracharya, 2011). FBI is recognized as one of the strongest and most consistent factors affecting populations, with longer interval usually associates with lower fertility while shorter interval corresponds to higher fertility (Kazembe, 2009; Singh *et al.*, 2006). Nonetheless, there has been very little documentation on FBI after marriage in many of the sub-region of Sahara African countries including Nigeria.

The world fertility patterns have changed over the last two decades after the International Conference on Population and Development (ICPD) in 1994, giving rise to a diverse patterns of child bearing (Nations, 2015). Many countries in Asia have been able to reduce their fertility rates through various government policies. For instance, Vietnam and China have experienced declining fertility as a response to government policies discouraging arranged and early marriage (Löfstedt *et al.*, 2005). Also, some of the Sub-Sahara African countries have been observed to be currently undergoing fertility transition from high to low level; and this process is caused by long birth intervals among the women of reproductive age group irrespective of their age and parity. In essence, in the countries where fertility transition has begun, these long intervals are being driven by birth postponement, and the phenomenon is following a fundamentally different path from earlier transitions elsewhere in the world as it occurs at every parity of women, as speculated by Caldwell and Moultrie. This has reflected in the South Africa recent total fertility rate (TFR) of 2.3 where all birth intervals including FBI have been reportedly lengthened irrespective of the age of the woman at marriage (Moultrie *et al.*, 2012). Also in Ghana, birth intervals have increased, resulting into the TFR of 4.7 (C.I.A, 2011) and so on. However, the situation on birth intervals especially on

FBI in Nigeria remains elusive. Nevertheless, all the resultant effects of this speculation about this important index of population change during fertility transition, birth intervals, do not seem to manifest in the Nigerian population dynamics as the population still remains the highest in the sub-region, placing tenth in the world despite the current fertility decline in most of the countries as reported in the National Demographic and health Survey 2008 (Commission, 2009a). The trend of TFR decline in Nigeria has been slow and seems to have stalled according to the trend of the following figures: 6.4 live births per woman during 1981/82 survey, 6.0 in 1990 survey, 5.7 in 2003 and also 5.7 in the 2008 demographic and health survey. (Commission, 2009a; C.I.A, 2011). Hence, in order for the government to formulate an effective policy that would motivate people for longer first birth interval after marriage, it is extremely important to study the effects of different socio-economic and demographic factors that apparently affect the interval.

Albeit, a handful available studies show that they are carried out using either non-parametric or semi-parametric method which will not give actual description of this interval. Therefore as evident in this study, we explore the available data on FBI and come up with a parametric survival model with better estimators of various characteristics of this interval. Literatures also reveal that certain natural and socio-cultural factors affect this interval. Effects of some variables are examined on this interval using the distribution identified as baseline. For instance, age at marriage of a woman should be expected to play out differently in early and late marriage regimes as earlier mentioned. Other covariates such as religion, region of residence, place of residence, whether rural or urban and level of education of respondents are examined. Investigation into the effects of these concomitant variables gives rise to prognostic regression model in this research study.

1.3 Research motivation

Nigeria population still remains the highest in the continent of Africa in spite of the current fertility decline that is being experienced in most of the countries where fertility transition has begun. In these countries, birth intervals have been observed to be long irrespective of the parity and age of the woman. For example, in South Africa TFR has rapidly declined to 2.3 births per woman which is a significant drop from 6.5 in the 1960s as a result of birth postponements, an attitudinal change which is largely independent of ages and parities of the women (Bremner *et al.*, 2010; Moultrie *et al.*, 2012). However in Nigeria, fertility transition appears to have stalled as observed in the 2008 demographic and health survey having total fertility rate (TFR) of 5.7 live births per woman. This figure is not different from what was observed in the 2003 of similar survey in the country and is clearly higher than the overall TFR of 5.2 live births in Africa. However, FBI which has been recognized as one of the strongest and most consistent factors affecting fertility levels in a population has not received the attention it deserves in the literatures. More so, in developing countries and where the use of contraceptive use is low, analysis of birth intervals is often preferred to the total children ever born to women and again when the results of population dynamics are urgently needed FBI is easily computed (Baldwin and Amato, 2012).

The importance of efforts to achieve significant fertility decline in Nigeria cannot be overemphasized, as it has been recognized that poor socioeconomic development in the sub-region is attributable to high fertility and as such, all hands must be on deck to understand the phenomena that could accelerate the pace of fertility decline in the country. More so, no meaningful success can be achieved along this line without understanding the characteristics of the interval between marriage and first birth which has not been researched into exhaustively unlike other birth intervals. Available studies for some countries in Africa are carried out using distribution free methods whose results may be somewhat deficient in terms of accuracy. However, the study carried out by Shayan, 2011 on the FBI for a small community in Iran identified Log-logistic as the distribution of the interval in the area studied. The distribution was subsequently employed as the baseline distribution

to model cumulative incidence of first birth in the face of some covariates. This is the main idea that inspires the current research study. Therefore, obtaining a parametric model for FBI would go a long way to understand how the interval is influenced by certain factors and as a result assists policy makers on fertility regulation in Nigeria.

Therefore, in order for the government to formulate an effective policy that would motivate people for longer first birth interval after marriage, it is crucial to study the effects of different socio-economic and demographic factors that apparently affect the interval.

1.4 Problem statement

The fact that first births mostly take place within marriage has made demographers to devote little attention to the interval between these vital events than it actually deserves. Even though some available studies have recognized the interval as one of the strongest and most consistent factors affecting populations, with longer interval which is usually associated with lower fertility and vice-versa (Singh *et al.*, 2006; Kazembe, 2009), yet there has been very little documentation on this interval after marriage in many developing countries, including Nigeria. Moreover, the causal nature of the relationship between marriage and first birth needs to be explicitly addressed if researchers are interested in policy-related issues. For instance, what happens to the interval if the marriage is experienced at later age of the woman? More importantly, this interval would likely play out differently under different conditions of women whose FBI is under investigation. It is therefore crucial to employ adequate statistical modeling and analytic methods in order to understand complex relations that may exist between social-demographic processes and the interval.

In this research, a parametric model is proposed for the survival time to first birth after marriage in Nigeria. The main problem to be addressed can be succinctly specified as:

“How can we model survival time to first birth among the population of married women in Nigeria using parametric method: and also determine prognostic factors of the interval?” That is, the factors affecting the interval.

In order to address the main research problem given above, we need to provide answers to the following research questions as pre-requisites.

- i. What is the distribution of the time to first birth to a married woman in Nigeria?
- ii. And how do we carry out survival modeling of time to first birth in Nigeria using parametric method?
- iii. What are the factors influencing this interval in Nigeria? That is, how do we incorporate covariates into this survival model?
- iv. How do we evaluate this model?

1.5 Research objectives

The main focus of this research is to explore survival analysis techniques to study the time between marriage of a woman and occurrence of first child. Survival analysis is applied to interpret the survival time, failure time and effect or efficiency of different variables on the survival time variable. In the light of the aforementioned issues raised in Section 1.4, this research study is to develop a parametric survival model for the time from marriage to first birth to a woman in Nigeria, using recognized prognostic factors. In order to achieve this aim, the following specific objectives have been identified:

- i. To identify the model amongst Weibull, Log-logistic, Gamma and Inverse Gaussian that appears to describe the waiting time to first birth of a woman after marriage in Nigeria.
- ii. To construct a new model, from the identified model among Weibull, Log-logistic, Gamma and Inverse Gaussian that appears to fit better the data on survival time to first birth after marriage of a woman.
- iii. To determine the effects of concomitant variables, such as age of the woman, her religion, educational status, zone of respondents, place of residence – rural or urban.
- iv. To evaluate the models constructed.
- v. To identify the properties of the new model.

1.6 Significance of the study

Importance of efforts to achieve significant fertility decline in Nigeria cannot be overemphasized, as it has been observed that poor socioeconomic development in the country is attributed to high fertility. And first birth after marriage is strongly correlated with the pace of subsequent fertility and, often rapid first birth leads to rapid transition to higher parities and higher fertility (Amin and Bajracharya, 2011). Therefore, as part of contributions to gain understanding of the phenomena that could accelerate the pace of fertility decline in the country it is crucial to identify the distribution of FBI. Furthermore, no meaningful success can be achieved along this line without understanding the determinants of FBI which have not been researched into exhaustively unlike for other birth intervals.

Therefore, modeling of FBI will go a long way to provide direction to government and non-government agencies working on fertility decline in Nigeria on the strategy to embark upon. More so, in developing countries and where the use of contraceptive use is low, analysis of birth intervals is often preferred to the total children ever born to women whenever urgent results on populations are required

(Baldwin and Amato, 2012). And also, in order for the government to formulate an effective policy that would motivate people for longer first birth interval after marriage, it is crucial to study the effects of different socio-economic and demographic factors that apparently affect the interval.

1.7 Scope of the study

In this thesis, survival analysis technique is employed to model time-to-first birth to a woman after marriage in Nigeria. This survival time response variable has been designated as FBI. The distribution of FBI identified from exploratory data analysis is inverse Gaussian and is subsequently adopted to construct the survival model for the interval. Moreover, a new distribution is developed for lifetime data alongside the inverse Gaussian called Lehmann type II inverse Gaussian model. And finally, prognostic analyses of factors such as age of the woman at marriage, religion, region of residence and educational level attained by the respondents are performed.

All the parameters are estimated by the method of maximum likelihood carried out in R-project statistical software and Matlab. The study is also limited to the data on women of reproductive ages in Nigeria collected in the most recent Nigeria Demographic and Health Survey of 2008; published in 2009. Like all other studies for the award of degree, the study is constrained by some factors such as time of duration of the program, dearth of data on first birth to a woman in usable format for previous periods.

1.8 Expected outcome

The development of survival models based on a parametric approach is expected to estimate the survival rates and hazards rates of time to first birth after

marriage in Nigeria. Therefore, the models can assist demographers and state actors working on population control in the country.

1.9 Organisation of thesis

The thesis of this research is structured into five chapters. Following the introductory part in Chapter One, Chapter Two presents the review of available literatures relevant to the research. Importance of birth intervals in the study of fertility changes is highlighted. We also discuss the determinants of FBI and the concept of fertility transition. Chapter Three describes the methodology employed in this research. Theories of statistical tools used are presented here. Estimation and inference on the parameters including model accuracy and selection are presented. Discussion about the research area, Nigeria and the data source for the research are given here.

While Chapter Four presents the results of various analyses carried out in this research, including the results of the simulation studies of proposed model. Discussions of these results are given in this chapter. Chapter Five is the final chapter that gives recommendation and conclusion. Direction of future work is also highlighted here.

thesis that women's education could influence both the timing of marriage and the first birth interval. Educated women typically delay marriage. This delay of entry into marriage by educated women may be motivated by a wish to delay childbearing, especially in societies where it is normative to have children quickly after marriage like Nigerian's.

Since short FBI corresponds to high fertility, and this condition is associated with poverty, maternal and child mortality, Nigerian governments have been trying to reduce the prevalence of diseases and alleviate poverty since post-independence. These efforts have remained a mirage till present. Therefore, the following recommendations are offered to the policy makers who are working towards fertility reduction in Nigeria – be it, government or non-governmental organizations.

Girls-child should be mandated to stay in school for long years. This is necessary especially in the Muslim dominated communities. As we have seen that delay in marriage cannot effectively control population increase if not followed by attitudinal change to delay first birth. Couples should be encouraged to imbibe the culture of contraceptive use to delay first birth, and not necessarily for spacing birth alone.

Job opportunity for women will go a longer way to engage them in order to lengthen all the birth intervals, including first birth generally.

Infrastructural development in the rural arrears should be embarked upon, as rural women tend to have a long FBI, which indicates that they do not pursue education as they may have been married off at a tender age while they are not fecund yet to have babies. Nigeria stands to gain enormously well in her effort to reduce population if adequate investment is made on girls education and infrastructure.

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