

FLUTTER ANALYSIS OF A FLEXIBLY SUPPORTED WING

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Specially dedicated to my soulmate

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ABSTRACT

Flutter is a dynamic aeroelastic phenomenon. Current aeroelastic theoretical models have some issues with the parameters related to the outcomes of wing flutter speed analysis. The typical trend in applying static derivatives in estimating wing flutter speed is one of the factors for the inconsistency. This study aimed to establish dynamically measured derivatives with comparisons to conventional static derivatives in predicting the wing flutter speed by using aeroelastic stiffness and damping equation. A free oscillation wind tunnel test rig was designed to measure the dynamic derivatives of rigid wings with flexible mounting at root simulated within a reduced frequency, K_m range from 0.04 to 0.40 under subsonic incompressible flow. The dynamically measured aerodynamic stability derivatives were determined from oscillation frequencies and amplitude decay of the wind-off and wind-on time response history. Four rectangular 3D wing models with NACA 0010, NACA 0012, NACA 0014 and NACA 0018 aerofoil configurations were tested. Each wing model has a wingspan of 0.36m and chord length of 0.16m with an aspect ratio of 4.5. The aerodynamic loads model with the dynamic derivatives was applied into the aeroelastic equation of motion to solve the flutter speed via eigenvalue solution. It was found that the $(C_{L_\alpha})_{Dynamic}$ and $(C_{M_\alpha})_{Dynamic}$ were 10%-40% higher than $(C_{L_\alpha})_{Static}$ and $(C_{M_\alpha})_{Static}$ for all the wing models. However, the differences between the dynamically and statically measured derivatives reduced by 12% for C_{L_α} and 7% for C_{M_α} as the thickness-to-chord ratio of the wing model increased. The measured $(C_{L_q} + C_{L_{\dot{\alpha}}})_{Dynamic}$ increases when aerofoil thickness-to-chord ratio increases. Besides, less fluctuations in $(C_{L_q} + C_{L_{\dot{\alpha}}})_{Dynamic}$ was seen for $K_m < 0.10$ and the measured $(C_{M_q} + C_{M_{\dot{\alpha}}})_{Dynamic}$ was reduced as K_m increased, with insignificant differences for all wing models. The predicted wing flutter speeds with dynamic derivatives are two times less than the estimations with Theodorsen model and 20% higher than the estimations with Scanlan model. These show that the dynamically measured derivatives have improved the wing flutter speed analysis for optimisation.

ABSTRAK

Kibaran merupakan satu fenomena aeroanjalan dinamik. Model teori aeroanjalan kini mempunyai beberapa masalah dengan parameter yang berkaitan dengan hasil analisis kelajuan kibaran sayap. Kecenderungan tipikal dalam menggunakan derivatif statik dalam menganggarkan kelajuan kibaran sayap merupakan salah satu faktor ketidaktentuan tersebut. Kajian ini bertujuan untuk mengenalkan penggunaan derivatif yang diukur secara dinamik dengan perbandingan terhadap derivatif statik lazim bagi meramalkan kelajuan kibaran menggunakan persamaan kekakuan dan redaman aeroelastik. Rig ujian terowong angin ayunan bebas direka bentuk untuk mengukur derivatif dinamik sayap tegar dengan pemasangan fleksibel di pangkal yang disimulasikan dalam julat frekuensi pemudah, K_m dari 0.04 ke 0.40 di bawah aliran angin subsonik tak boleh mampat. Derivatif kestabilan aerodinamik yang diukur secara dinamik ditentukan daripada frekuensi ayunan dan reputan amplitud sejarah respon masa tanpa angin dan dengan angin. Empat model sayap 3D segiempat tepat dengan konfigurasi aerofoil NACA 0010, NACA 0012, NACA 0014 dan NACA 0018 telah diuji. Setiap model sayap mempunyai rentang sebesar 0.36m, rerentas 0.16m dan nisbah bidang 4.5. Model beban aerodinamik dengan derivatif dinamik digunakan dalam persamaan pergerakan aeroanjalan untuk menyelesaikan kelajuan kibarannya melalui penyelesaian nilai eigen. Didapati bahawa $(C_{L_\alpha})_{Dinamik}$ dan $(C_{M_\alpha})_{Dinamik}$ adalah 10%-40% lebih tinggi daripada $(C_{L_\alpha})_{Statik}$ dan $(C_{M_\alpha})_{Statik}$ bagi semua model sayap. Walau bagaimanapun, perbezaan di antara derivatif yang diukur secara dinamik dan statik berkurang sebanyak 12% untuk C_{L_α} dan 7% untuk C_{M_α} apabila nisbah ketebalan kepada rerentas bagi model sayap bertambah. $(C_{L_q} + C_{L_{\dot{\alpha}}})_{Dinamik}$ yang diukur bertambah apabila nisbah ketebalan kepada rerentas bertambah. Di samping itu, kurang perubahan dalam $(C_{L_q} + C_{L_{\dot{\alpha}}})_{Dinamik}$ dilihat bagi $K_m < 0.10$ dan $(C_{M_q} + C_{M_{\dot{\alpha}}})_{Dinamik}$ yang diukur berkurang apabila K_m bertambah, dengan perbezaan yang tidak signifikan untuk semua model sayap. Kelajuan kibaran sayap yang diramalkan dengan derivatif dinamik adalah dua kali ganda kurang daripada anggaran dengan model *Theodorsen* dan 20% lebih tinggi daripada anggaran dengan model *Scanlan*. Ini menunjukkan bahawa derivatif dinamik menambahbaik analisis kibaran sayap bagi pengoptimuman.

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LIST OF SYMBOLS

AC	-	Aerodynamic center
$A_{i=1,2,3}^*$	-	Scanlan's flutter derivatives for pitch motion
$A_{n=1,2,3}$	-	Structural inertia matrix
a	-	Distance ratio of b w.r.t h
a_A	-	Aerofoil lift curve slope
a_w	-	Wing lift curve slope
$B_{n=1,2,3}$	-	Aerodynamic damping matrix
b	-	Distance between the leading edge and the reference axis, h
bh	-	Half-wing span (m)
C	-	Chord length (m)
\bar{c}	-	Mean aerodynamic chord
CG	-	Center of gravity
$C(K_m)$	-	Theodorsen's circulation function
C_A	-	Aerodynamic damping
$C_{A,U}$	-	Plunge aerodynamic damping
$C_{A,\theta}$	-	Pitch aerodynamic damping
$C_{A,\delta}$	-	Control surfaces aerodynamic damping
C_{l_α}, a	-	Aerofoil aerodynamic lift curve slope (rad^{-1})
C_L	-	Wing lift coefficient (deg^{-1})
C_{L_0}	-	Wing aerodynamic lift at zero angle of attack (rad^{-1})
C_{L_α}	-	Wing aerodynamic lift stiffness derivative (Wing lift curve slope) (rad^{-1})
$C_{L_{\dot{\alpha}}}$	-	Wing aerodynamic lift damping derivative due to angle of attack rate
C_{L_q}	-	Wing aerodynamic lift damping derivative due to pitch Rate

$C_{L_{\delta}}$	-	Control surface aerodynamic lift curve slope (rad^{-1})
$C_{L_{\dot{\delta}}}$	-	Control surface aerodynamic lift derivative with respect to time
$C_{m_{\alpha}}$	-	Aerofoil pitching moment derivative (rad^{-1})
C_M	-	Wing pitching moment coefficient (deg^{-1})
C_{M_0}	-	Wing aerodynamic pitching moment at zero angle of Attack (rad^{-1})
$C_{M_{\alpha}}$	-	Wing aerodynamic pitching moment stiffness Derivative (rad^{-1})
$C_{M_{\dot{\alpha}}}$	-	Wing aerodynamic pitching moment damping derivative due to angle of attack rate
$C_{M_{\dot{q}}}$	-	Wing aerodynamic pitching moment damping derivative due to pitch rate
$\mathbf{C}_{n=1,2,3}$	-	Aerodynamic stiffness matrix
C_s	-	Structural damping
$C_{S,U}$	-	Plunge structural damping
$C_{S,\theta}$	-	Pitch structural damping
$C_{S,\delta}$	-	Control surfaces structural damping
C_T	-	Equivalent system damping
$\mathbf{D}_{n=1,2,3}$	-	Structural damping matrix
e_1	-	distance from origin of body fixed reference to center of gravity of the BACT wing by excluding control surfaces
e_2	-	distance from origin of body fixed reference to center of gravity of the trailing edge control surface
$\mathbf{E}_{n=1,2,3}$	-	Structural stiffness matrix
F	-	Linear displacement force
F	-	Forcing function
$F(k)$	-	Real value of Theodorsen's circulation function
$f_{d,U}$	-	Plunge damped frequency (Hz)
$f_{d,\theta}$	-	Pitch damped frequency (Hz)

f_f	-	Flutter frequency (Hz)
$f_{n,U}$	-	Plunge natural frequency (Hz)
$f_{n,\theta}$	-	Pitch natural frequency (Hz)
$G(k)$	-	Imaginary value of Theodorsen's circulation function
$H_{i=1,2,3}^*$	-	Scanlan's flutter derivatives for plunge motion
I_1	-	angular moment of inertia of BACT/PAPA body (excluding control surfaces) about the origin of the body fixed reference
I_2	-	angular moment of inertia of trailing edge control surface about its hinge line
I_u	-	Plunge moment of inertia)
$I_{u\theta}$	-	Coupling of plunge and pitch moment of inertia
I_{ZZ}	-	Moment of inertia at Z-axis
I_θ	-	Pitch moment of inertia
I_δ	-	Control surface moment of inertia
$I_{\theta\delta}$	-	Coupling of pitch and control surface moment of inertia
K_m	-	Reduced frequency
K_r	-	Pitch equivalent system stiffness
K_A	-	Aerodynamic stiffness
$K_{A,U}$	-	Plunge aerodynamic stiffness
$K_{A,\theta}$	-	Pitch aerodynamic stiffness
$K_{A,\delta}$	-	Control surfaces aerodynamic stiffness
K_r	-	Pitch equivalent system stiffness
K_s	-	Structural stiffness
$K_{S,U}$	-	Plunge structural stiffness
$K_{S,\theta}$	-	Pitch structural stiffness
$K_{S,\delta}$	-	Control surface structural stiffness
K_T	-	Equivalent system stiffness
L	-	Aerodynamic lift force (N)
L_U	-	Partial derivative with respect to aerodynamic lift force for plunge motion
$L_{\dot{U}}$	-	Partial derivative with respect to secondary aerodynamic lift force derivative with respect to time

		for plunge motion
L_{θ}	-	Partial derivative with respect to aerodynamic lift force for pitch motion
$L_{\dot{\theta}}$	-	Partial derivative with respect to secondary aerodynamic lift force derivative with respect to time for pitch motion
ℓ_b	-	Half torsional moment arm length (m)
m_1	-	mass of the BACT wing by excluding control surfaces
m_2	-	mass of the control surfaces
M	-	Aerodynamic pitching moment force
M_s	-	Model mass (kg)
M_U	-	Partial derivative with respect to aerodynamic pitching moment for plunge motion
$M_{\dot{U}}$	-	Partial derivative with respect to secondary aerodynamic pitching moment derivative with respect to time for plunge motion
M_w	-	Mass of the wing model (kg)
M_Z	-	Pitching moment force
$M_{\dot{\theta}}$	-	Unsteady aerodynamic term
M_{θ}	-	Partial derivative with respect to aerodynamic pitching moment for pitch motion
$M_{\dot{\theta}}$	-	Partial derivative with respect to secondary aerodynamic pitching moment derivative with respect to time for pitch motion
q	-	Movement of the wing components
\dot{q}	-	Movement of the wing components derivative with respect to time
\ddot{q}	-	Secondary movement of the wing components derivative with respect to time
\bar{q}	-	Dynamic pressure
Q	-	Generalized aerodynamic lift and pitching moment

Q_0	-	Generalized aerodynamic forces at zero angle of attack
Q_T	-	Generalized aerodynamic forces with gravitational effect
S	-	Planform wing area (m^2)
S_{wet}	-	Wetted wing area (m^2)
t_m	-	Aerofoil thickness at $AC=0.25 \bar{c}$
T	-	Kinetic energy
t	-	Time (s)
U	-	Plunge motion or horizontal deflection of the wing (Positive out)
\dot{U}	-	Plunge motion derivative with respect to time
\ddot{U}	-	Secondary plunge motion derivative with respect to time
U	-	Potential energy
U_g	-	Gravitational potential energy
V	-	Freestream airspeed (ms^{-1})
V_{DF}	-	Maximum dive speed (ms^{-1})
V_f	-	Flutter speed (ms^{-1})
V_{NE}	-	Never exceed speed (ms^{-1})
x	-	Linear displacement in x-direction
\dot{x}	-	Linear displacement derivative with respect to time
\ddot{x}	-	Secondary linear displacement derivative with respect to time
X_f	-	Flexural axis (m)
α	-	Angle of attack (degree)
$\dot{\alpha}$	-	Angle of attack derivative with respect to time
ζ_U	-	Damping ratio of plunge motion
ζ_θ	-	Damping ratio of pitch motion
θ	-	Pitch angle (degree)
$\dot{\theta}$	-	Pitch angle derivative with respect to time
$\ddot{\theta}$	-	Secondary pitch angle derivative with respect to time
θ_T	-	Turnable angle
ρ	-	Air density (kgm^{-3})
ω_d	-	Damped frequency ($rads^{-1}$)

ω_n	-	Natural frequency (rads^{-1})
$\omega_{n,U}$	-	Natural frequency of plunge motion (rads^{-1})
$\omega_{n,\theta}$	-	Natural frequency of pitch motion (rads^{-1})

LIST OF ABBREVIATIONS

ASD	-	Aerodynamic stiffness and damping
ASE	-	Aeroservoelasticity
CFD	-	Computational Fluid Dynamics
CSD	-	Computational Structure Dynamics
DOF	-	Degree of Freedom
FEM	-	Finite Element Modelling
LCO	-	Limit Cycle Oscillations
LST	-	Low Speed Tunnel
MSC	-	MacNeal-Schwendler Corporation
NASTRAN	-	NASA Structure Analysis
PSD	-	Power Spectral Density
ROM	-	Reduced Order Model
UAV	-	Unmanned Aerial Vehicle
NACA	-	National Advisory Committee for Aeronautics

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Flutter is the most challenging structural dynamic problem which is defined as an aeroelastic vibration that causes instability to a structure. The occurrence of flutter is due to the coupling of aerodynamic force, inertial force and elastic force as illustrated in Fig. 1.1. This is known as the Collar Aeroelastic diagram, which was created for a better understanding of the overall aeroelastic problems. The combination of aeroelastic forces and elastic forces causes the aeroelastic effects on the structure.

For instance, the occurrence of wing flutter depends on the elasticity of the wing structure itself rather than the influence of vibrational forces from other aircraft components compared to buffeting and dynamic response. Buffeting is caused by the aerodynamic impulses that are generated by the wake behind the wing, nacelle or other parts of aircraft structure while dynamic response occurs when sudden aerodynamic loads input is detected due to gusts, hard landing or vigorous manoeuvre of control surfaces. Thus, this leads to extra precautionary steps being required during the structural design process to avoid flutter as comparing buffeting and dynamic response.

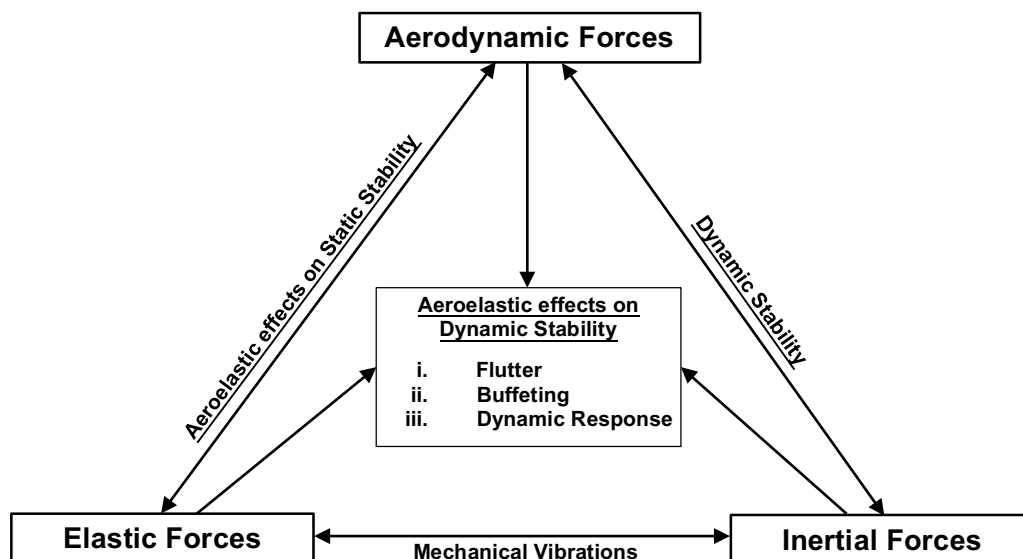


Figure 1.1 Collar Aeroelastic Diagram (Bisplinghoff and Ashley, 1962)

Since the flutter phenomenon is disastrous, ground vibration testing and Flutter Flight Tests (FFT) are compulsory requirements for an aircraft to be certified free from flutter. It is stated in CFR, section 23.2245 that an aircraft should be able to demonstrate that flutter did not occur within the designed flight envelope of a 20% safety margin or the flutter speed, V_{FL} should be higher than the dive speed, V_D (Sedaghat *et al.*, 2000; Shokrieh and Behrooz, 2001). However, the actual FFT can be risky and time consuming when the predicted aircraft flutter speed is overestimated during the aircraft design process (Lind and Brenner, 1997). It can cause flutter early during the flight test. This is dangerous for the test pilots as the flight speed increases where prominent vibrations might occur before the predicted wing flutter speed during the flight test. Furthermore, flutter phenomenon is due to the dynamic instability of the aircraft which can be shifted drastically by a few knots increase in flying speed. Hence, this situation shows the importance in estimating the aircraft flutter speed with accuracy before the FFT.

The flutter speed estimation can be carried out through computational, numerical or experimental analysis. The three analysis methods for wing flutter speed solutions have been investigated individually or coupled among each other for better flutter speed estimations over time. Firstly, improvements on the computational capability of the computer lead to the fast growing of aeroelasticity analysis software to solve complicated aeroelastic problems. Currently, ZAERO/ZONA is the most

commonly used software by the industries especially in the field of aeronautical engineering for aeroelastic related problems (De Leon *et al.*, 2012). It is a complete software package that can solve fluid-structure interaction problems. The aeroelastic problems can also be solved by coupling the solution from both computational fluid dynamic (CFD) analysis and computational structural dynamic (CSD) analysis. The CSD analysis can be carried out in any FEM programs such as MSC/NASTRAN for most of the cases.

The wing flutter phenomenon can also be modelled mathematically into an aeroelastic equation of motion and solved for its flutter speed through eigenvalue solution. Estimation of wing flutter speed through the solution of the mathematical model is the most popular method among the three due to its simplicity and relative ease of understanding to estimate the flutter speed.

The equation is constructed by modelling the inertial forces, elastic forces and aerodynamic loads of the aeroelastic structure. The elastic forces carry the structural stiffness and damping of the wing, while the aerodynamic loads carry the aerodynamic stiffness and damping of the wing. In this study, the aerodynamic loads are referred to the aerodynamic lift and pitching moment of the wing under flying conditions. Ashley, Zartarian and Neilson (1951) stated that aerodynamic loads were the important input parameters in the flutter speed estimation process as they can alter the total system stiffness and damping of an aeroelastic structure.

The mathematical model for wing flutter speed estimation of a 2D rectangular wing model under subsonic and incompressible flow regime has been established as derived by Theodorsen (1935). Uncertainties and applied assumptions for the input aerodynamic stability derivatives of the aerodynamic loads model in the established mathematical model are found to be one of the contributing factors that can cause inaccurate estimation of the wing flutter speed as illustrated in Fig. 1.2 (Dowell and Tang, 2002). The three most commonly used aerodynamic loads modelling are discussed in the following paragraphs.

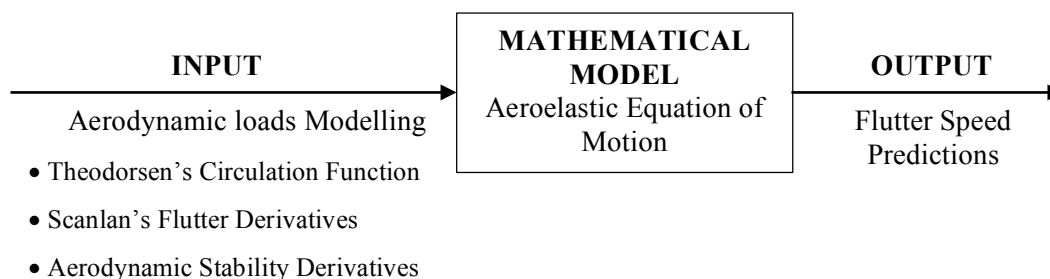


Figure 1.2 Flutter speed prediction procedure through solution with mathematical modelling

Both Theodorsen's Function and Scanlan's flutter derivatives modelled the aerodynamic loads with frequency domain by generalizing the motions into reduced frequency as $K_m = \pi f C / V$ where f is the oscillation frequency (Hz), C is the chord length (m) and V is the freestream velocity (ms^{-1}). Both aerodynamic lift and pitching moment that were modelled with Theodorsen's function and Scanlan's flutter derivatives includes the Theodorsen's circulation function, $C(K_m) = F(K_m) + iG(K_m)$ in the function of reduced frequency. The application of both methods were claimed to result in reasonable flutter speed estimations for aircraft wings. However, the main drawback of the two methods is the assumption of thin aerofoil theory for the aerodynamic loads modelling where the aerofoil thickness to chord ratio of the wing was neglected in both methods. These shortcomings will be carried forward to the aeroelastic equation of motion which causes uncertainties on the estimated wing flutter speed.

In this study, the dynamically measured aerodynamic stability derivatives were chosen as input parameters for the aerodynamic loads modelling. The aerodynamic loads were modelled with the dynamically measured aerodynamic stability derivatives. These were determined from dynamic wind tunnel tests to simulate the transient case conditions. In this case, the accuracy of the estimated wing flutter speed was improved as the inclusion of dynamically measured aerodynamic stability derivatives in the aeroelastic equation is able to resemble the actual flow structure for the wing rather than the statically measured aerodynamic stability derivatives. Furthermore, it was strengthened by taking the aerofoil thickness to chord ratio into consideration for each wing models. Both the dynamically and statically measured

aerodynamic stability derivatives were obtained through the wind tunnel test by using a free oscillation testing technique.

The design of the dynamic free oscillatory test rig was adapted and modified to measure the aerodynamic stability derivatives transiently. The dynamically measured aerodynamic stability derivatives were determined from the recorded time response histories of the oscillating wing model. The test rig is a 2-DOFs free oscillating system for the wing model where only pitch motion was able to be simulated in this study due to the limitation of the test rig in plunge motion. The plunge motion of the wing model is analysed theoretically by using the experimentally measured data of the pitch motion. The relative differences between the measured time response plots of the wind-on and wind-off condition for the tested wing model in the wind tunnel was assumed to be the effects solely caused by the aerodynamic lift and pitching moment. It is identical to the “Aerodynamics is Aeroelasticity Minus Structure (AAEMS)” identification method as mentioned by Song, Kim and Song (2012).

1.2 Problem Statement

The process to identify the flutter speed for an aircraft wing is complex due to non-linearities and time varying nature of the aeroelastic structure (Dansberry *et al.*, 1993; Zhao and Hu, 2004; Ertveldt *et al.*, 2013). The aerodynamic loads where they may either dampen, stiffen, deaden or soften the total wing structural stiffness and damping during flying conditions (STRGANAC, 1988; Song, Kim and Song, 2012). Thus, the estimated wing flutter speed is inaccurate and becoming more complicated in transient conditions. The prediction of aerodynamic loads of a wing in the form of stiffness and damping in the aeroelastic model are not well modelled especially in transient case and it is influenced by a primary configuration such as aerofoil thickness to chord ratio (Hoa, 2004; Florance, Chwalowski and Wieseman, 2010).

Three commonly used methods to model the aerodynamic loads for aeroelastic equation in predicting wing flutter speed are Theodorsen’s circulation function by Theodorsen (1935), Scanlan’s flutter derivatives by Scanlan and Tomo (1971) and

aerodynamic stability derivatives by Waszak (1998). However, Theodorsen's circulation function assumes the generated wake at the trailing edge of the aerofoil to be flat. The assumption is only true for thin aerofoil sections and very large aspect ratio wings. Furthermore, the quasi-steady aerodynamic model from Theodorsen's function by assuming $C(K_m)=1$ has been proven to produce an underestimated flutter speed (Haddadpour and Firouz-Abadi, 2006). Meanwhile, Scanlan's flutter derivatives are obtained with reference to Theodorsen's function. Both Theodorsen's and Scanlan's method apply linear steady-state lift curve slope, C_{l_α} in the formulated aeroelastic equation of motion for flutter speed solution which limit the accuracy of the predicted flutter speed.

In this study, the dynamically measured aerodynamic stability derivatives were proposed to model the transient aerodynamic loads and will be applied into the aeroelastic equation rather than using conventional statically measured aerodynamic stability derivatives. It is proven by experimental evidence that the existence of transient growth of energy in the time response data of an oscillating wing model up to the critical flutter speed in reality and worth to be studied (Hémon, De Langre and Schmid, 2006).

1.3 Research Objectives

The objectives of this study are to:

- (i) Determine the mathematical modelling of aerodynamic loads model in the form of aerodynamic stability derivatives for better prediction of wing flutter speed with the input from dynamically measured aerodynamic stability derivatives.
- (ii) Determine the aerodynamic stability derivatives with respect to time, reduced frequency and aerofoil thickness to chord ratio through dynamic wind-tunnel test.
- (iii) Prediction and analysis of the wing flutter speed by using the theoretical, statically and dynamically determined aerodynamic

stability derivatives through the established aeroelastic mathematical model for both thin and thick aerofoil configurations.

1.4 Scopes of the Research

After reviewing and analysing the previous methods for wing flutter speed prediction, this study focuses on the application of aeroelastic equation of motion in determining the flutter speed. The aerodynamic lift and pitching moment model in the equation were modelled in the form of aerodynamic stability derivatives. In this study, the dynamically measured aerodynamic stability derivatives are to be transient case derivatives as they were determined from dynamic wind tunnel testing. Furthermore, the dynamically measured aerodynamic stability derivatives are highly dependent on its oscillating frequency, referred time and wind speed.

A dynamic test rig was designed to perform a free oscillation test for plunge and pitch motion in order to measure C_{L_α} , $C_{L_q} + C_{L_{\dot{\alpha}}}$, C_{M_α} and $C_{M_q} + C_{M_{\dot{\alpha}}}$ dynamically. The oscillation frequency was control by a set of linear springs with different stiffness in order to match the range of reduced frequency that were used to sense the aerodynamic loads during wind-on condition inside the wind tunnel. The task was carried out by the free oscillation test rig where the derivatives of the wing model were obtained based on the relative difference between the measured wind-on and wind-off time response data of the oscillation amplitudes.

The measurement of steady-state aerodynamic stability derivatives were also taken from the static wind-tunnel test. This was to quantify the difference by comparing the statically measured aerodynamic stability derivatives to the dynamically measured derivatives in the form of magnification factor. A parametric study on the effect of aerofoil thickness to chord ratio on the estimated wing flutter speed was also performed in this study as the parameters were not considered by the other two methods (i.e. Theodorsen's and Scanlan's methods). A set of NACA series symmetrical aerofoil configurations which are NACA 0010, NACA 0012, NACA 0014 and NACA 0018 were selected as the tested wing model for the wind tunnel test. The prediction of wing flutter speed was executed by solving the aeroelastic equation

with inclusion of theoretically, statically and dynamically measured aerodynamic stability derivatives through eigenvalue solution in Matlab program.

1.5 Significance of Study

In this study, a dynamic wind tunnel test rig had been developed to measure aerodynamic stability derivatives to replicate the transient case of a 3D rectangular wing. The lift and pitching moment derivatives are formulated in the form of aerodynamic stiffness and damping instead of the Theodorsen's function and Scanlan flutter derivatives. This method studies the effect of thickness to chord ratio of the wing aerofoil where the aspect ratio of the wing is fixed at $AR=4.50$ in predicting the wing flutter speed by using aeroelastic equation of motion. The dynamically measured aerodynamic stability derivatives are utilised in the aerodynamic loads model instead of statically measured derivatives which is not emphasize by both Theodorsen's and Scanlan's method. In this method, the flutter speed is determined by considering the damping term of the aeroelastic equation of motion is zero.

Thus, the input data of the damping term is crucial for the flutter speed predictions. The advantage of this method is that the formulated aeroelastic equation of motion not only includes the dynamically measured aerodynamic stiffness derivatives (i.e. C_{L_α} and C_{M_α}) but also the dynamically measured aerodynamic damping derivatives (i.e. $C_{L_q} + C_{L_{\dot{\alpha}}}$ and $C_{M_q} + C_{M_{\dot{\alpha}}}$). Therefore, the accuracy of the predicted wing flutter speed can be improved. By comparing the results from the three methods, the correlations among those methods can be used as a guideline for realistic flutter speed predictions.

Although the proposed method in this study is limited to an un-tapered, un-swept and un-cambered wing with symmetrical NACA aerofoil under subsonic incompressible flow condition, a good prediction of wing flutter speed with a better aerodynamic loads model will assist during the preliminary aircraft design phase by preventing the wing structure from being over-designed or under-designed. Lastly, as the aeroelastic equation of motion was formulated in the form of state-space equation

which allows it to be analysed computationally and made it compatible for control system design in the future study.

1.6 Thesis Outline

An introduction to the problem statements of aircraft wing flutter phenomena that lead to the research objectives of this study are stated in Chapter 1. It is followed by the scopes and significance of this study.

In Chapter 2, the Theodorsen's circulation function method, Scanlan's flutter derivatives method and aerodynamic stability derivatives method are further discussed. The assumptions and limitations of the three methods in modelling the aerodynamic loads model for flutter speed estimation are clarified in the chapter. Some oscillatory test rig designs for the wind tunnel test from previous researchers are reviewed as their experimental technique is utilised in determining the aerodynamic stability derivatives in this study.

Chapter 3 highlights the research methodology of this study and is presented in a flow chart for better understanding. The emphasis of the chapter is the derivation of the aeroelastic equation of motion with aerodynamic stability derivatives in solving for flutter speed. Both wind-on and wind-off conditions are included in the derivations in order to determine the dynamically (transient case) measured aerodynamic stability derivatives from the dynamic wind tunnel test.

The development on the mechanism of oscillatory test rig for wind-tunnel test will be discussed in Chapter 4. The experimental setup, instrumentation techniques and the calibration of the sensor are explained in this chapter. The validation process of the Matlab coding in extracting the aerodynamic stability derivatives from the recorded time response data are also discussed in the chapter.

In Chapter 5, preliminary results of the static and dynamic wind tunnel test of NACA 0012 wing model are discussed as a baseline wing model for wing flutter analysis. The discussion is focused on the statically (steady-state case) and dynamically (transient case) measured aerodynamic stability derivatives. Comparisons among the predicted wing flutter speeds and flutter frequency that are solved from the Theodorsen's method, Scanlan's method, theoretical, static method and dynamic method for the four tested wing models (i.e. NACA 0010, NACA 0012, NACA 0014 and NACA 0018) are explained in the chapter.

In Chapter 6, the study draws conclusion on the differences of the predicted wing flutter speed by using the dynamically measured aerodynamic stability derivatives compared to the Theodorsen's method, Scanlan's method, theoretical method and static method. The parametric study on the effect of aerofoil thickness to chord ratio, t_m/C to the dynamically measured aerodynamic stability derivatives is concluded.

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