MATHEMATICAL MODELLING OF BIOLOGICAL WASTEWATER TREATMENT OF OXIDATION POND AND CONSTRUCTED WETLAND SYSTEMS

AMIR SYAFIQ SYAMIN SYAH B AMIR HAMZAH

UNIVERSITI TEKNOLOGI MALAYSIA

MATHEMATICAL MODELLING OF BIOLOGICAL WASTEWATER TREATMENT OF OXIDATION POND AND CONSTRUCTED WETLAND SYSTEMS

AMIR SYAFIQ SYAMIN SYAH B AMIR HAMZAH

A thesis submitted in fulfilment of the
requirements for the award of the degree of
Doctor of Philosophy (Mathematics)

Faculty of Science
Universiti Teknologi Malaysia

JULY 2017

To my beloved mother and father

ACKNOWLEDGEMENT

In preparing this thesis, I was in contact with many people, researchers, academician, and practitioners. They have contributed towards my understanding and thought. In particular, I wish to express my sincere appreciation to my main supervisor Assoc. Prof. Dr. Ali Hassan B. Mohamed Murid for his continuous support during my Ph.D study and related research, his patience, motivation, and immense knowledge. His guidance has helped me throughout my research and the writing of this thesis.

I would also like to give my biggest appreciation to my beloved parents Amir Hamzah B. Nordin and Hamidah Bt. Hassan for their pray and support since i was born until this moment.

In addition, I would also like to thank my co-supervisor Prof. Dr. Razman B. Salim for his support and assistance in strengthening my knowledge in this field. My sincere thanks also goes to Dr. Akbar Banitalebi from UTM Centre for Industrial and Applied Mathematics (UTM-CIAM) who has provided me an opportunity to learn many things especially in the field of optimization.

I would like to express my profound gratitude to the Director of UTM-CIAM, Prof. Dr. Zainal B. Abdul Aziz and the Head of Mathematical Sciences Department, Assoc. Prof. Dr. Rohanin B. Ahmad who had gave me the access to laboratory and research facilities.

I would also like to thank the Ministry of Higher Education Malaysia (MOHE) for the financial support through the research grant R.J130000.7809.4F637. Part of the research was undertaken while visiting the Mathematical Institute, University of Oxford, UK. I wish to thank the members of the institute for the pleasant mathematical atmosphere they offered. I would also gratefully acknowledge the help of Prof. Dr. Graham Sander on clarifying certain aspects of mathematical modeling of wetland system. This visit was supported by UTMLead, UTM and Graduate Employability Grants (STEM), UTM.

I wish to express my appreciation and thanks to all lecturers, laboratory technicians, and all administrative staffs of UTM-CIAM and the Department of Mathematical Sciences especially Shahliza, Siti Amirah, Norafidah, Farid, Liyana, and Tuan Mariam for their help and support. I would like to pass my thanks to all lecturers, Ph.D and Master students, and laboratory technicians of the UTM for their assistance.

Not to be forgotten, all my family members, Amir Syakir, Amir Syahreza, Nurul Izza Syakirah, Amir Syahir, Nurul Ain Syafiqah, Amir Syarifuddin, Amir Takiyuddin Saifullah, Nurul Anis Amirah, Amir Anas Iskandar and Nurul Hakimah Syamimi. All my friends, Basri Badyalina, Ahmad Izul, Mohd Aidil, Ahmad Hanis, Mohamed Amerul, Mior Uzair, Saiful Iezam, Azam Mohamad for supporting me spiritually and encouraging me to be more patience throughout my study.

Without their pray and support, i could never completed this thesis.

ABSTRACT

Wastewater treatment methods are intended to improve the quality of wastewater to prevent many health problems stemming from water sources. Among popular treatment methods are oxidation pond and constructed wetland (CW) treatment. There are some mathematical models for simulating oxidation pond process where some important parameters are considered such as bacteria (cleansing agent), pollutants and dissolved oxygen (DO). However, previous results did not provide good approximation of the required parameters. Meanwhile, for constructed wetland models, the stability analysis was rarely considered. However, the steady-state and bifurcation analyses are usually crucial in determining the reliability of the models that is under study. In this thesis, dynamic mathematical models are developed to allow simulation and prediction of the wastewater treatment process for both oxidation pond and CW case studies. The nonlinear system of ordinary differential equations (ODE) using multiple substrate limiting factors with interactive reactions and partial differential equations (PDE) using advection-diffusion-reaction equations are implemented for CW and oxidation pond, respectively. Water quality indexes considered in this study are chemical oxygen demand (COD), biochemical oxygen demand (BOD), ammonium nitrogen (NH_4^+), nitrate (NO_3), and DO. For oxidation pond system, the input of microbe-based product (mPHO) is added to the model, whereas the effect of living plants (*Typha Angustifolia*) is introduced in the CW treatment system to mimic the natural behaviour of the wetland system. Since the models are nonlinear, coupled, and dynamic, computational algorithms with specific numerical methods are employed to simulate the dynamical behaviour of the system. Implicit Runge-Kutta method is selected for solving the ODE model. Whereas, for the PDE, the implicit Crank-Nicolson method is used. The process model built is then optimised using gradient-free optimisation method (least squares) algorithms `NonlinearModelFit` in $Mathematica$ to identify the optimal solution for improving the efficiency of the simulation process. Stability, bifurcation, and numerical analyses are presented to illustrate the dynamical behaviour of the proposed model. Numerical results also revealed that the proposed models have good accuracy when compared to the experimental data. The two separate mathematical models for oxidation pond and constructed wetland, both are then applied to simulate a wastewater treatment site with pond-constructed wetland system. The combined mathematical model results in a further removal of COD as well as an increase of DO up to 94.1% and 97.4% respectively when compared to a single oxidation pond model.

ABSTRAK

Kaedah rawatan air sisa adalah bertujuan untuk meningkatkan kualiti air sisa bagi mengelakkan banyak masalah kesihatan yang berpunca daripada sumber air. Antara kaedah rawatan yang popular adalah kolam pengoksidaan dan tanah bencah yang dibina (CW). Terdapat beberapa model matematik untuk mensimulasikan proses kolam pengoksidaan di mana beberapa parameter penting diambilkira seperti bakteria (ejen pembersihan), pencemaran dan oksigen terlarut (DO). Walau bagaimanapun, keputusan yang sedia ada tidak memberikan aggaran yang baik bagi parameter yang diperlukan. Sementara itu, bagi model tanah bencah yang dibina, analisis kestabilan jarang dipertimbangkan. Walau bagaimanapun, analisis kestabilan dan pencabangan biasanya penting dalam menentukan kebolehpercayaan model yang dikaji. Dalam tesis ini, model matematik dinamik dibangunkan untuk membolehkan simulasi dan ramalan proses rawatan air sisa untuk kedua-dua kajian kes kolam pengoksidaan dan CW. Sistem tak linear persamaan pembezaan biasa (ODE) menggunakan faktor terhad pelbagai substrat dengan reaksi interaktif dan persamaan pembezaan separa (PDE) menggunakan persamaan alir lintang-penyebaran-reaksi diterapkan masing-masing untuk CW dan kolam pengoksidaan. Indeks kualiti air yang diukur dalam kajian ini ialah permintaan oksigen kimia (COD), permintaan oksigen biokimia (BOD), ammonia nitrogen (NH_4^+), nitrat (NO_3), dan DO. Untuk sistem kolam pengoksidaan, fungsi input produk berasaskan biologi (mPHO) ditambah pada model, manakala faktor tumbuhan hidup (*Typha Angustifolia*) diperkenalkan bagi sistem rawatan CW untuk menggambarkan sistem semula jadi tanah bencah. Memandangkan model yang dibina tak linear, terkait, dan dinamik, algoritma pengiraan dengan kaedah berangka yang khusus digunakan untuk mensimulasikan sifat dinamik sistem. Kaedah Runge-Kutta tersirat dipilih untuk menyelesaikan model ODE. Bagi model PDE, kaedah Crank-Nicolson tersirat digunakan. Model proses yang dibina kemudiannya dioptimumkan dengan algoritma `NonlinearModelFit` dalam $Mathematica$ iaitu kaedah pengoptimuman bebas kecerunan (kuasa dua terkecil) untuk mengenalpasti penyelesaian yang optimum bagi meningkatkan kecekapan proses simulasi. Kestabilan, pencabangan, dan analisis berangka dibentangkan untuk menggambarkan keadaan dinamik model yang dicadangkan. Keputusan berangka juga menunjukkan ketepatan yang baik apabila dibandingkan dengan data eksperimen. Dua model matematik yang berasingan untuk kolam pengoksidaan dan tanah bencah yang dibina kemudiannya kedua-duanya digunakan sekali untuk mensimulasikan proses rawatan air sisa dengan sistem kolam-tanah bencah. Model matematik gabungan menghasilkan penyingkiran COD serta peningkatan DO masing-masing sehingga 94.1 % dan 97.4 % berbanding model kolam pengoksidaan tunggal.

TABLE OF CONTENTS

CHAPTER	TITLE	PAGE
	DECLARATION	ii
	DEDICATION	iii
	ACKNOWLEDGEMENT	iv
	ABSTRACT	vi
	ABSTRAK	vii
	TABLE OF CONTENTS	viii
	LIST OF TABLES	xi
	LIST OF FIGURES	xiv
	LIST OF ABBREVIATIONS	xxi
	LIST OF SYMBOLS	xxiii
	LIST OF APPENDICES	xxvii
1	**INTRODUCTION**	**1**
	1.1 Motivation	1
	1.1.1 Wastewater	2
	1.1.2 Oxidation Pond	2
	1.1.3 Constructed Wetland	6
	1.2 Background of the Problem	8
	1.3 Problem Statement	12
	1.4 Objectives of the Research	12
	1.5 Scope of the Research	13
	1.6 Significance of the Research	13

1.7 Thesis Organization 14

2 LITERATURE REVIEW 16

2.1 Introduction 16

2.2 Wastewater Treatment Process Modelling 16

2.2.1 Mathematical Models of River and Pond 17

2.2.2 Mathematical Models of Constructed Wetland 23

2.3 Activated Sludge Process 26

3 METHODOLOGY 30

3.1 Introduction 30

3.2 Mathematical Modelling 30

3.3 Nondimensionalisation 41

3.4 Steady State Solution 45

3.5 Parameter Estimation 51

3.6 One-Dimensional Conservation Equation 54

3.7 Coefficient of Determination (R^2) 56

3.8 Numerical Implementation (Implicit Runge-Kutta Method) 57

3.9 Numerical Implementation (Implicit Crank-Nicolson Method) 59

3.10 Application of mPHO at Taman Timur Oxidation Pond 64

3.11 Horizontal Subsurface Flow Constructed Wetland 68

4 OXIDATION POND SYSTEM 71

4.1 Introduction 71

4.2 Mathematical Model 1: Three Competing Species Model 71

4.3 Mathematical Model 2: Coupled-Reaction Equations Model 86

4.4 Partial Differential Equation Models 95

4.4.1 Mathematical Model 3: Advection-Reaction Equation Model 97

4.4.2 Nondimensionalisation of PDE Model 102

4.4.3 Mathematical Model 4: Advection-Diffusion-Reaction Equation Model (Transport of Pollutant) 105

5 CONSTRUCTED WETLAND SYSTEM 118

5.1 Introduction 118

5.2 Mathematical Model 1: Nonlinear Ordinary Differential Equations Model 118

5.3 Mathematical Model 2: The Simplified Model 131

5.4 Mathematical Model 3: Dimensionless Model 133

5.5 Steady State Analysis 136

6 POND-CONSTRUCTED WETLAND SYSTEM 144

6.1 Introduction 144

6.2 Pond-Constructed Wetland Model 147

7 CONCLUSION 150

7.1 Summary of the Research 150

7.2 Recommendation for Further Study 152

REFERENCES 154

Appendices A - C 163-175

LIST OF TABLES

TABLE NO.	TITLE	PAGE
2.1	Chronology of Major Conclusions Made about WWTP Between 2011-Present Year	21
2.2	Chronology of Major Conclusions Made about WWTP Between 2011-Present Year	22
3.1	Constitutive Mechanisms for Transport Phenomena [65]	34
3.2	Initial Layout for Routh Table	51
3.3	Conditions of State Variable (C)	61
3.4	Treatment Schedule for mPHO Application at Taman Timor Oxidation Pond (Phase 1) [3]	66
3.5	Treatment Schedule for mPHO Application at Taman Timor Oxidation Pond (Phase 2) [3]	67
4.1	Variables and Parameters Used in Mathematical Model 1	72
4.2	Process Rates Involved in Mathematical Model 1	73
4.3	Population of *E. coli* from Experimental Data at CP1 and CP2 (MISG 2014 Report [3])	73
4.4	Population of *Coliform* from Experimental Data at CP1 and CP2 (MISG 2014 Report [3])	74
4.5	Population of PSB from Experimental Data at CP1 and CP2 (MISG 2014 Report [3])	74
4.6	List of Parameters in Mathematical Model 1	79
4.7	Calculation of Cost and Amount of mPHO that can be Saved for Considered Schedule	85

4.8 The Variables and Parameters Used in Mathematical Model 2 87
4.9 Process Rates Involved in Mathematical Model 2 88
4.10 The Amount of COD from Experimental Data at CP1 and CP2 (MISG 2014 Report [3]) 88
4.11 The Amount of Dissolved Oxygen from Experimental Data at CP1 and CP2 (MISG 2014 Report [3]) 89
4.12 List of Parameters in Mathematical Model 2 93
4.13 The Variables and Parameters Used in Mathematical Model 3 97
4.14 Description of State Variables Used in Mathematical Model 4 106
4.15 Process Rates 106
4.16 Parameter Values Used in the Mathematical Model 4 107
4.17 Values of Calibrated Constants and Values Specified in Literature 112

5.1 Description of State Variables in Model 1 120
5.2 State Variables and Parameter Values Used in Model 1 121
5.3 All Process Rates Involved in Model 1 122
5.4 The Amount of COD from Experimental Data at CP1 and CP2 (Chew [15]) 125
5.5 The Amount of BOD from Experimental Data at CP1 and CP2 (Chew [15]) 125
5.6 The Amount of NH_4^+ from Experimental Data at CP1 and CP2 (Chew [15]) 126
5.7 The Amount of NO_3 from Experimental Data at CP1 and CP2 for Planted Case (Chew [15]) 126
5.8 Values of Calibrated Constant and Values Specified in Literature 126
5.9 Description of Parameters Used in the Mathematical Model 3 135

6.1 The Variables and Parameters Used in Pond-Constructed Wetland Model 145

LIST OF FIGURES

FIGURE NO.	TITLE	PAGE
1.1	Relationship between the Organic Carbon in Sewage [2]	4
1.2	Aerial View of Oxidation Pond, Tampoi, Johor Bahru [3]	4
1.3	Physical Condition of Oxidation Pond, Tampoi, Johor Bahru [3]	5
1.4	Beneficial Microbe-Based Product (mPHO) Produced by J-Biotech Company [3]	5
1.5	(a) Location of CP1 (influent) (b) Location of CP2 (effluent) [3]	5
1.6	(a) Phragmites Australis (b) Glyceria Maxima (c) Phalaris Arundinacea (d) Cattails (Typha Angustifolia) [8]	7
1.7	Putrajaya Lake and Wetland System, Putrajaya, Malaysia [12]	7
2.1	Mathematical Models of Wastewater Treatment Process from the Beginning until 2010. [19, 24–37]	18
2.2	Michaelis-Manten Kinetics [39]	20
2.3	Nitrogen Cycle in Constructed Wetland [8]	24
2.4	Activated Sludge Process for Treating Wastewater [41]	27
2.5	Schematic Diagram of Eutrophication Process	28
2.6	Schematic Diagram of Nitrogen Cycle	28
3.1	Overview of the Mathematical Modelling Process [65]	33
3.2	Model Validation Methodology [65]	34

3.3 Phase Portrait Diagram of Equation (3.24) with c_1 =1 and c_2 =1 47
3.4 Illustration of the Parameter Estimation Process 54
3.5 One Dimensional Pipe of Uniform Diameter 54
3.6 Dimensional Pipe of Cross Section Area of Length ($\triangle x$) 55
3.7 Sampling and Study Location at Taman Timor Oxidation Pond [3] 65
3.8 Layout of The Horizontal Flow Subsurface Constructed Wetland Units 69
3.9 Actual Image of Reactor 69
3.10 Horizontal Sections of Subsurface Constructed Wetlands 69

4.1 The Graphs of *E. coli* in the Oxidation Pond at CP1 and CP2 (refer Table 4.3) 75
4.2 The Graphs of *Coliform* in the Oxidation Pond at CP1 and CP2 (refer Table 4.4) 75
4.3 The Graphs of PSB in the Oxidation Pond at CP1 and CP2 (refer Table 4.5) 75
4.4 Graphical Comparison on the Population of *E. coli* at CP2. The Blue Curve Represents the Numerical Results and Red Dots are the Experimental Data (R^2 = 0.7954) 79
4.5 Graphical Comparison on the Population of *Coliform* at CP2 (R^2 = 0.7913) 80
4.6 Graphical Comparison on the Population of PSB at CP2 (R^2 = 0.6541) 80
4.7 Graphical Comparison on the Population of *E. coli* at CP2. The Blue Curve is Based on the Simulation of Applying 10 Litres Per Day of mPHO 81
4.8 Graphical Comparison on the Population of *Coliform* at CP2 (10 Litres Per Day of mPHO) 81

4.9 Graphical Comparison on the Population of PSB at CP2 (10 Litres Per Day of mPHO) 82

4.10 Graphical Comparison on the Population of *E. coli* at CP2 (30 Litres Per Day of mPHO) 82

4.11 Graphical Comparison on the Population of *E. coli* at CP2 (30 Litres Per Day of mPHO) 82

4.12 Graphical Comparison on the Population of PSB at CP2 (30 Litres Per Day of mPHO) 83

4.13 Graphical Comparison on the Population of *E. coli* at CP2 (50 Litres Per Day of mPHO) 83

4.14 Graphical Comparison on the Population of *Coliform* at CP2 (50 Litres Per Day of mPHO) 83

4.15 Graphical Comparison on the Population of PSB at CP2 (50 Litres Per Day of mPHO) 84

4.16 Graphical Comparison on the Population of *E. coli* at CP2 (without mPHO) 84

4.17 Graphical Comparison on the Population of *Coliform* at CP2 (without mPHO) 84

4.18 (a) Graph of Relative Error between Applying 30 Liters and 50 Liters of mPHO for *E. coli* Relative to $X_{2i}(30)$ (b) Graph of Relative Error between Applying 30 Liters and 50 Liters of mPHO for *E. coli* Relative to $X_{2i}(50)$ 85

4.19 (a) Graph of Relative Error between Applying 30 Liters and 50 Liters of mPHO for *Coliform* Relative to $X_{3i}(30)$ (b) Graph of Relative Error between Applying 30 Liters and 50 Liters of mPHO for *Coliform* Relative to $X_{3i}(50)$ 85

4.20 The Graphs of COD from CP1 and CP2 for 70 Days (refer Table 4.10) 87

4.21 The Graphs of DO from CP1 and CP2 for 70 Days (refer Table 4.11) 89

4.22 Graphical Comparison between Experimental Data and Simulated Model 2 for PSB (R^2 = 0.6531) 94

4.23 Graphical Comparison between Experimental Data and Simulated Model 2 for Coliform (R^2 = 0.7754) 94

4.24 Graphical Comparison between Experimental Data and Simulated Model 2 for COD (R^2 = 0.8472) 94

4.25 Graphical Comparison between Experimental Data and Simulated Model 2 for DO (R^2 = 0.7127) 95

4.26 3D Graph of PSB from Mathematical Model 3 99

4.27 Contour Plot of PSB from Mathematical Model 3 99

4.28 3D Graph of COD from Mathematical Model 3 100

4.29 Contour Plot of COD from Mathematical Model 3 100

4.30 Graphical Comparison between Experimental Data and Simulated Model 3 for PSB. The Blue Curve Represents the Numerical Results and Red Dots are the Experimental Data (R^2= 0.62) 101

4.31 Graphical Comparison between Experimental Data and Simulated Model 3 for COD. The Blue Curve Represents the Numerical Results and Red Dots are the Experimental Data (R^2= 0.85) 101

4.32 3D Graph of PSB Based on Mathematical Model 4 113

4.33 Contour Plot of PSB Based on Mathematical Model 4 113

4.34 3D Graph of COD Based on Mathematical Model 4 114

4.35 Contour Plot of COD Based on Mathematical Model 4 114

4.36 3D Graph of DO Based on Mathematical Model 4 115

4.37 Contour Plot of DO Based on Mathematical Model 4 115

4.38 Graphical Comparison between Experimental Data and Simulated Model 4 for PSB (R^2= 0.90) 116

4.39 Graphical Comparison between Experimental Data and Simulated Model 4 for COD (R^2= 0.86) 116

4.40 Graphical Comparison between Experimental Data and Simulated Model 4 for DO (R^2= 0.92) 117

5.1 Conceptual Diagram of HSSF Model Illustrating the Coupling of Eco-Physiological Process Through Carbon and Nitrogen Pathways 119

5.2 (a) Graph of M_B from Simulated Model 1 (Planted) (b) Graph of M_B from Simulated Model 1 (Unplanted) 124

5.3 (a) Graphical Comparison between Experimental Data and Simulated Model 1 for COD (Planted, R^2 = 0.7945) (b) Graphical Comparison between Experimental Data and Simulated Model 1 for COD (Unplanted, R^2 = 0.9964) 127

5.4 (a) Graphical Comparison between Experimental Data and Simulated Model 1 for BOD (Planted, R^2 = 0.8574) (b) Graphical Comparison between Experimental Data and Simulated Model 1 for BOD (Unplanted, R^2 = 0.6306) 127

5.5 (a) Graphical Comparison between Experimental Data and Simulated Model 1 for NH_4^+ (Planted, R^2 = 0.6954) (b) Graphical Comparison between Experimental Data and Simulated Model 1 for NH_4^+ (Unplanted, R^2 = 0.8137) 127

5.6 (a) Graphical Comparison between Experimental Data and Simulated Model 1 for NO_3 (Planted, R^2 = 0.8993) (b) Graphical Comparison between Experimental Data and Simulated Model 1 for NO_3 (Unplanted, R^2 = 0.9591) 128

5.7 (a) Graph of DO from Simulated Model 1 (Planted) (b) Graph of DO from Simulated Model 1 (Unplanted) 128

5.8 (a) Graph of Relative Error between Planted and Unplanted Cases for COD Relative to Planted Case (b) Graph of Relative Error between Planted and Unplanted Cases for COD Relative to Unplanted Case 128

5.9 (a) Graph of Relative Error between Planted and Unplanted Cases for BOD Relative to Planted Case (b) Graph of Relative Error between Planted and Unplanted Cases for BOD Relative to Unplanted Case 129

5.10 (a) Graph of Relative Error between Planted and Unplanted Cases for NH_4^+ Relative to Planted Case (b) Graph of Relative Error between Planted and Unplanted Cases for NH_4^+ Relative to Unplanted Case 129

5.11 (a) Graph of Relative Error between Planted and Unplanted Cases for NO_3 Relative to Planted Case (b) Graph of Relative Error between Planted and Unplanted Cases for NO_3 Relative to Unplanted Case 129

5.12 (a) Plot of the Error Magnitudes for Model 1 as Iterations Progress for Planted Case (20th iteration, 20.3891) (b) Plot of the Error Magnitudes for Model 1 as Iterations Progress for Unplanted Case (20th iteration, 8.0729) 130

5.13 Graph of Typha Angustifolia from Simulated Model 1 130

5.14 (a) Graphs of M_B, DO, and Typha Angustifolia for Extended Days up to One Year (planted) (b) Graphs of M_B and DO for Extended Days up to One Year (Unplanted) 130

5.15 (a) Graphs of COD, BOD, NH_4^+, and NO_3 for Extended Days up to One Year (planted) (b) Graphs of COD, BOD, NH_4^+, and NO_3 for Extended Days up to One Year (Unplanted) 131

5.16 (a) Graph of M_B from Simulated Model 2 (Planted) (b) Graph of M_B from Simulated Model 2 (Unplanted) 132

5.17 (a) Graphical Comparison between Experimental Data and Simulated Model 2 for COD (Planted, $R^2 = 0.7952$) (b) Graphical Comparison between Experimental Data and Simulated Model 2 for COD (Unplanted, $R^2 = 0.9967$) 133

5.18 (a) Graph of DO from Simulated Model 2 (Planted) (b) Graph of DO from Simulated Model 2 (Unplanted) 133

5.19 Steady-State Diagram of Model 3 Against c_3 with c_1 =1, c_2 =4, c_5 = 1, c_4 =2 for M_B 141

5.20 Steady-State Diagram of Model 3 Against c_3 for COD 142

5.21 Steady-State Diagram of Model 3 Against c_3 for DO 142

5.22 Time Plots of m, s, and x with Initial Conditions (1, 1, 1) for c_1 = 1, c_2 = 4, c_3 = 0.5, c_4 = 2, and c_5 = 1 142

5.23 Time Plots of m, s, and x for c_1 = 1, c_2 = 4, c_3 = 1.5, c_4 = 2, and c_5 = 1 143

6.1 Framework of the Pond-Constructed Wetland Model 146

6.2 (a) Graphical Comparison between ADR Model of Oxidation Pond and PCW Model for COD at Discharged Point (b) Graph Percentage Removal of COD by PCW Model at Discharged Point 148

6.3 (a) Graphical Comparison between ADR Model of Oxidation Pond and PCW Model for DO at Discharged Point (b) Graph of DO Increase Percentage from PCW Model at Discharged Point 149

6.4 Graphical Comparison between ADR Model of Oxidation Pond and PCW Model for M_B at Discharged Point 149

LIST OF ABBREVIATIONS

UTM	-	Universiti Teknologi Malaysia
COD	-	Chemical Oxygen Demand
BOD_5	-	5-day Biochemical Oxygen Demand
DO	-	Dissolved Oxygen
NH_4^+	-	Ammoniacal Nitrogen
NO_3	-	Nitrate
E. coli	-	Esrichia Coli
PSB	-	Phototrophic Bacteria
ODEs	-	Ordinary Differential Equations
PDEs	-	Partial Differential Equations
CW	-	Constructed Wetland
HSSF	-	Horizontal Sub-surface Flow
ADR	-	Advection-Diffusion-Reaction
TOC	-	Total Organic Carbon
H_2SO_4	-	Sulphuric Acid
mPHO	-	Microbe-Based Product prodeced by J-Biotech
WEPA	-	Water Environment Partnership in Asia
PO_4^{-3}	-	Phosphates
NO_3	-	Nitrates
CO_2	-	Carbon dioxide
$K_2Cr_2O_7$	-	Potassium Dichromate
C_6H_6	-	Benzene
CH_3	-	Toluene
J-Biotech	-	Johor Biotechnology & Biodiversity Corporation

nm - nanometer

μm - micrometer

LIST OF SYMBOLS

$\Re$ - The set of real numbers

$\Re^n$ - Real coordinate space of n dimensions

J - Jacobian Matrix

M_B - Mixed Culture bacteria

α - Saturated oxygen concentration

ϵ - Material porosity for constructed wetland

A - Cross section area of oxidation pond

v_p - Volume of the oxidation pond (2,864,139 litres of wastewater)

m_0 - Amount of PSB in mPHO (1.91×10^{-9} CFU / 100 ml)

$X_1(t)$ - Amount of PSB in the pond where t varies from initial time up to 35 days

$X_2(t)$ - Amount of *E. coli* in the pond (MPN / 100 ml)

$X_3(t)$ - Amount of *Coliform* in the pond (MPN / 100 ml)

$X_p(t)$ - Input wastewater containing pathogens (E. coli and Coliform) from CP1 (290,304 litres day^{-1})

$X_m(t)$ - Scheduled amount of mPHO applied at the oxidation pond (litre / day)

$X_{1i}(t)$ - Input of PSB added in the oxidation pond at CP1 (MPN / 100 ml)

$X_{1e}(t)$ - Existing of PSB in the oxidation pond at CP1 (MPN / 100 ml)

$X_{2i}(t)$ - Input of *E. coli* added in the oxidation pond at CP1 (MPN / 100 ml)

$X_{3i}(t)$ - Input of *Coliform* added in the oxidation pond at CP1 (MPN / 100 ml)

k_1 - Growth rate of PSB (1.000045 day^{-1})
k_2 - Mortality rate of PSB (4.8$\times$10^{-5} day^{-1})
k_3 - Predation rate on *E. coli* that increases the number of PSB
k_4 - Predation rate on *Coliform* that increases the number of PSB (3.2$\times$10^{-6} day^{-1})
k_5 - Input rate of PSB at the influent (CP1) (0.0152 day^{-1})
k_6 - Growth rate of *E. coli*
k_7 - Mortality rate of *E. coli*
k_8 - Predation rate that decreases the number of *E. coli*
k_9 - Input rate of *E. coli* at the influent (CP1)
k_{10} - Growth rate of *Coliform* (0.0081 day^{-1})
k_{11} - Mortality rate of *Coliform* (1.6$\times$10^{-7} day^{-1})
k_{12} - Predation rate that decreases the number of *Coliform* (2.7$\times$10^{-5} day^{-1})
k_{13} - Input rate of *Coliform* at the influent (CP1) (0.0275 day^{-1})
c_1 - Degradation rate coefficient for COD
c_2 - Half saturation coefficient concentration for COD decay
c_3 - Input rate of COD at the influent (CP1)
c_4 - Mass transfer rate for oxygen from air to water
c_5 - Input rate of oxygen at the influent (CP1)
$M(t)$ - Concentration of PSB in the pond (mg/litre) where t varies from initial time up to 70 days
$C(t)$ - Concentration of *Coliform* in the pond (mg/litre)
$P(t)$ - Concentration of COD in the pond (mg/litre)
$X(t)$ - Concentration of DO in the pond (mg/litre)
$U(t)$ - Amount of mPHO applied according to J-Biotech schedule (litre/day)

$M_i(t)$ - Input of PSB added in the pond at CP1 (mg/litre)
$C_i(t)$ - Input of *Coliform* added in the pond at CP1 (mg/litre)
$P_i(t)$ - Input of COD added in the pond at CP1 (mg/litre)
$X_i(t)$ - Input of DO added in the pond at CP1 (mg/litre)
m_0 - Concentration of PSB in one litre of mPHO (19.1 mg/litre)
S - Saturated oxygen concentration (10 mg/litre)
v_s - Average amount of incoming sewage (290,304 litres /day)
v_p - Volume of the pond (2,864,139 litres of wastewater)
p_0 - Concentration of COD in one m^3 of sewage from input CP1 (50.0 g m^{-3})
μ_b - Maximum growth rate for PSB (31 m^2 day^{-1})
D_M - Diffusion coefficient of PSB in the x direction (50 m^2 day^{-1})
D_P - Diffusion coefficient of COD in the x direction (50 m^2 day^{-1})
D_X - Diffusion coefficient of DO in the x direction (50 m^2 day^{-1})
k_L - Mass transfer rate for oxygen from air to water (4 m^2 day^{-1})
k_p - Half saturation oxygen demand concentration for COD decay (560 g m^{-3})
k_m - Half-saturation coefficient for growth of PSB (1.7×10^{-4} g m^{-3})
L - Length of the HSSF constructed wetland
W - Width of the constructed wetland
D - Depth of reed bed of the constructed wetland
w - Pore water volume
v - Constructed wetland holding volume
ϵ - Material porosity
β - Decay coefficient for mixed
b_P - Decay coefficient for living plants
α - Saturated oxygen concentration
μ - Maximum growth rate for mixed

q_i - Input flow rate of wastewater

q_o - Output flow rate of wastewater

k_l - Mass transfer rate of oxygen from atmosphere to soil

k_C - Half-saturation coefficient for $\mathrm{M_B}$ growth on COD

k_B - Half-saturation coefficient for $\mathrm{M_B}$ growth on BOD

k_A - Half-saturation coefficient for $\mathrm{M_B}$ growth on $\mathrm{NH_4^+}$

k_N - Half-saturation coefficient for $\mathrm{M_B}$ growth on $\mathrm{NO_3}$

k_{pua} - Rate of plant uptake on $\mathrm{NH_4^+}$ (0.0036 m $\mathrm{day^{-1}}$)

k_{pun} - Rate of plant uptake on $\mathrm{NO_3}$ (0.0044 m $\mathrm{day^{-1}}$)

k_{pa} - Half-saturation coefficient for plant uptake on $\mathrm{NH_4^+}$ 0.0620 g $\mathrm{m^{-3}}$)

k_{pn} - Half-saturation coefficient for plant uptake on $\mathrm{NO_3}$ 0.0440 g $\mathrm{m^{-3}}$)

$S_{Ci}(t)$ - Input of COD from the inlet source that varies with time (CP1)

$S_{Bi}(t)$ - Input of BOD from the inlet source that varies with time (CP1)

$S_{Ai}(t)$ - Input of $\mathrm{NH_4^+}$ from the inlet source that varies with time (CP1)

$S_{Ni}(t)$ - Input of $\mathrm{NO_3}$ from the inlet source that varies with time (CP1)

w_d - Calculated as $\dfrac{w}{L \cdot W \cdot \epsilon}$

c_1 - Ratio of decay rate for $\mathrm{M_B}$ to the water volume

c_2 - Ratio of maximum specific growth for $\mathrm{M_B}$ and saturated oxygen to the water volume

c_3 - Input of pollutant source (COD) from CP1

c_4 - Ratio of saturated oxygen to the half-saturation coefficient for growth of $\mathrm{M_B}$

c_5 - Ratio of mass transfer rate for oxygen to the water volume

LIST OF APPENDICES

APPENDIX	TITLE	PAGE
A	List of Publications	163
B	Oxidation Pond System	165
C	Constructed Wetland System	170

CHAPTER 1

INTRODUCTION

1.1 Motivation

Mathematical modelling is an important and well-known field of study which has led to the enrichment of science and technology. Fields that require mathematical modelling include medicine, ecology, biology, finance, and economics. This has further encouraged many researchers to develop new models to fulfill the demand arose from these fields. These new mathematical models are expected to aid in analyzing and solving the problems encountered by the fields mentioned. A mathematical model is a simplified version of the real world process employing the tools of mathematics such as statistics, probability theory, graph theory, and differential equations. These mathematical methods help in understanding the nature of problems that cannot be clearly interpreted through phenomenological observation. Sometimes, one needs to develop a new method to solve the problems or modify the standard previous methods that have been successful. Challenges should be taken as a motivation for researchers to mathematically model the current problems and facilitate it to be understood by the public.

1.1.1 Wastewater

One of the main problems widely concerned is the issue of environmental pollution. Development without a systematic planning is like ignoring the sustainability of environment. There are a huge number of wastes being produced daily as the result of human activities; for instance, solid waste, hazardous waste, wastewater (sewage and surface runoff), and radioactive waste. Wastewater can be classified into several types including industrial waste, municipal waste, food waste, and sewage from houses. This kind of wastes has to be carefully treated to ensure that there would be no harm to human and the environment. If untreated wastewater is allowed to accumulate in the river followed by the processing of decomposed organic material, it can lead to water pollution. Additionally, untreated wastewater usually contains numerous pathogens or diseases caused by microorganisms.

Severe pollution has become our main concern that leads to the production of a mathematical model that is able to preserve and conserve the environment to run smoothly, thus helping the development of human capital. The execution of wastewater treatment process depends on symbiotic relationships of biological organism found in a system. Therefore, understanding the ecological system is very crucial to construct the so-called symbiotic relationship and function related to wastewater treatment processes.

1.1.2 Oxidation Pond

Oxidation pond techniques have become very popular among small communities due to their low construction and operating costs [1]. The construction and maintenance costs of this treatment are inexpensive compared to other recognized treatment systems including microbial fuel cell (MFC), membrane bioreactor (MBR), and rotating biological contactor (RBC). The core procedure of an oxidation pond treatment process is the degradation of contaminants and organic matter in two conditions; where oxygen is present (aerobic) or absence (anaerobic). At each stage, existing microorganisms are used to breakdown either organic or inorganic substances of influent and to reduce organic material into more concise forms, which are carbon dioxide, water, and cell biomass.

Oxidation pond chosen for the pilot scale study is an exposed oxidation pond located at Taman Timor Oxidation Pond, Tampoi, Johor. This pond was chosen because it has been experimentally studied by J-Biotech for three months period to observe the effects of microbe-based product (mPHO) in treating sewage. Nonetheless, there is no specific study done on this product until now. Briefly, the size of this pond is estimated about 1,909 square metres with a depth of 1.5 metres and total water volume of 2,864.13 cubic metres or 2,864,125.13 litres (refer to Figure 1.2 and Figure 1.3). However, the input and output flow rate of wastewater as well as the volume of rain may not change the volume of water in the pond as the wastewater is discharged due to overflow. In order to intensify the effectiveness of oxidation pond technique and to accelerate the population of *Phototrophic* bacteria (PSB) in the pond, mPHO containing mainly PSB have been added regularly within three months period of study between 13 November 2013 to 12 February 2014.

The product mPHO is made from selected species of PSB (refer Figure 1.4) manufactured by J-Biotech. About 1,375 litres of mPHO were applied to the pond throughout three months of treatment. Samples were collected at two points, which are CP1 (influent and application of mPHO) and CP2 (effluent) (refer to Figure 1.5). Comparison of data taken at both points CP1 and CP2 demonstrated that mPHO has a good effect in reducing the concentration of pathogenic bacteria (*E. coli* and *Coliform*), BOD, COD and other pollutants as the PSB and dissolved oxygen (DO) concentration increases.

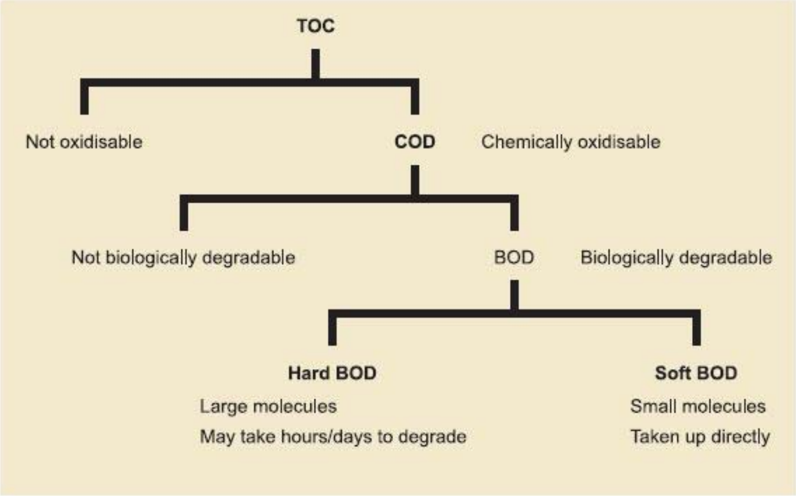


Figure 1.1 Relationship between the Organic Carbon in Sewage [2]

Figure 1.2 Aerial View of Oxidation Pond, Tampoi, Johor Bahru [3]

Figure 1.3 Physical Condition of Oxidation Pond, Tampoi, Johor Bahru [3]

Figure 1.4 Beneficial Microbe-Based Product (mPHO) Produced by J-Biotech Company [3]

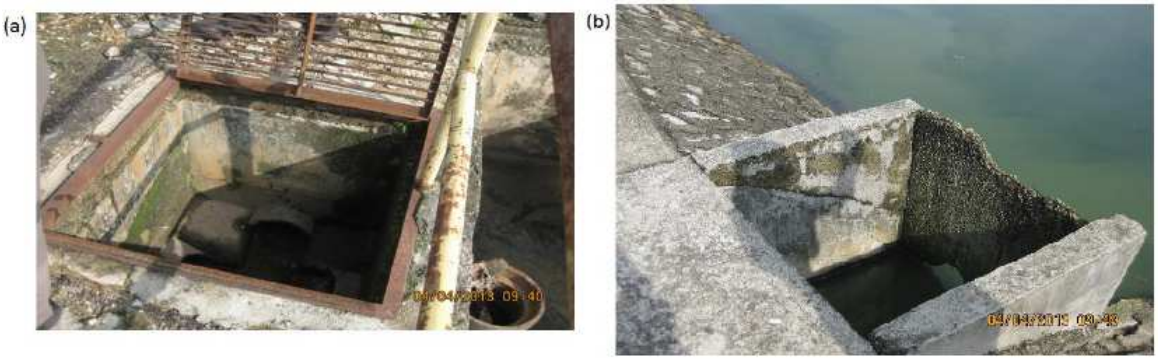


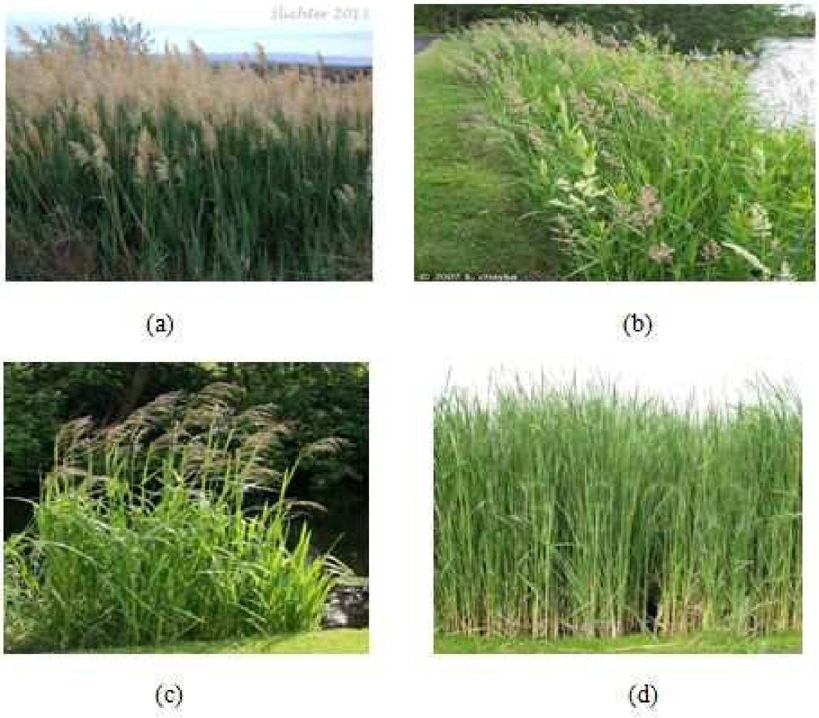
Figure 1.5 (a) Location of CP1 (influent) (b) Location of CP2 (effluent) [3]

1.1.3 Constructed Wetland

Constructed wetland system can be considered as a secondary or tertiary treatment facility for treating wastewater originated from the residential, municipal and industrial areas [4]. Besides playing an important role in wastewater treatment process to remove contaminants including organic matter and inorganic matter (based on COD removal and BOD removal), it is also helpful in maintaining the landscape that preserve the natural habitats of flora and fauna [5–7]. Wetlands treatment is defined as a treatment system using the aquatic root system of cattails, reeds, and similar plants to treat wastewater applied to either above or below the soil surface [8–10].

This treatment system acts as a filter to remove excess nutrients in the form of carbon and nitrogen from its source. The top layer of constructed wetland is planted with various types of plant, while the roots are allowed to develop deep and extensive roots that can penetrate the filter media. In fact, it can also help to develop porous throughout the land, allowing the wastewater to seep below the soil surface. At the root of the plant, there are fixed surfaces on which bacteria can attach and perform the breakdown of organic matter [11]. The vegetation provides an air flow to the root zone transporting an amount of oxygen. This environment will help aerobic bacteria to grow while maximizing the degradation process.

However, the primary role of vegetation is to maintain permeability in the filter and to provide habitats for microorganisms. Nutrients and organic material are absorbed and degraded by the dense microbial population. Unlike oxidation pond, constructed wetland system usually treats some sort of wastewater known as leachate. Leachate can be identified as any contaminated liquid generated from water permeating through a solid waste disposal site moving into subsurface regions. As these wastes are compacted or chemically react, bound water is discharged as leachate [10]. Therefore, landfill leachate treatment has been perceived as an essential part of solid waste management.



(a) (b)

(c) (d)

Figure 1.6 (a) Phragmites Australis (b) Glyceria Maxima (c) Phalaris Arundinacea (d) Cattails (Typha Angustifolia) [8]

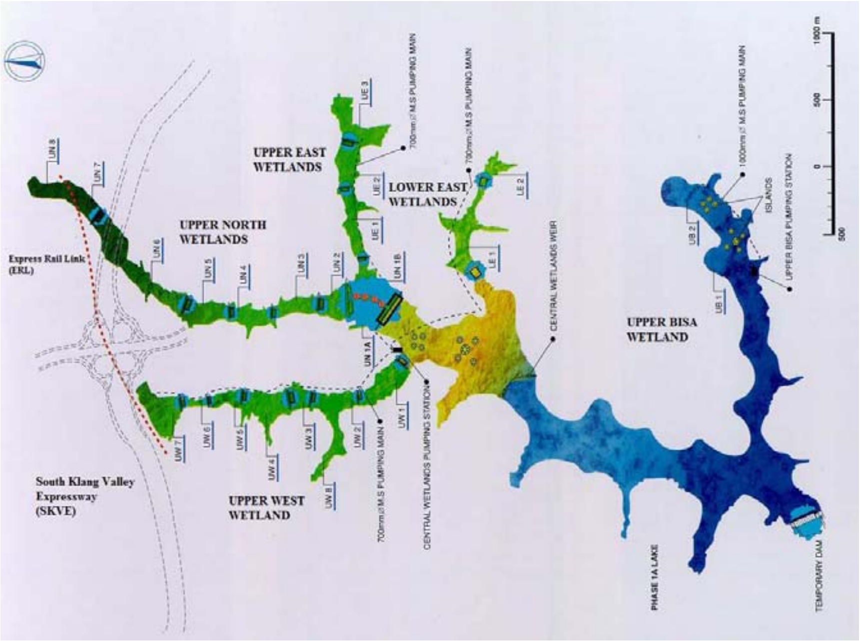


Figure 1.7 Putrajaya Lake and Wetland System, Putrajaya, Malaysia [12]

Constructed wetlands can be planted with a number of adapted, emergent wetlands plant species. Typha Angustifolia, which belongs to the Typhaceae family, was selected as the subject of this study (refer Figure 1.6). It is an erect, perennial freshwater aquatic herb that can grow three or more meters in height. The linear cattail leaves are thick, ribbon-like structures with a spongy cross-section exhibiting air channels. The subterranean steam rises from thick rhizomes [13]. This plant has been selected for various reasons. One of those is that it is among the most common wetland plants available in the region (refer Figure 1.7). Besides, typha types of plant have been extensively studied in the Europe as suitable species of vegetation in constructed wetlands [14].

According to the study by Chew [15], the removal efficiency of nutrient from landfill leachate in the form of ammoniacal nitrogen and nitrate by *Typha Angustifolia* ranges from 42.6%–88.9%. Meanwhile, the removal of BOD and COD ranging from 62.6%–72.8% and 64.5 %–85.7%, respectively.

1.2 Background of the Problem

This study aims to explain the biological processes that underpin the wastewater treatment system by showing how the bacteria deal with the pollutant in the sewage. Basically, there are three major processes involved in the treatment plant, which are biodegradation of pollutant, the decreased of oxygen levels, and the cleansing of wastewater. Wastewater can be divided into two types, which are the one produced by human and the other resulting from industrial activities. According to Fakhrul-Razi *et al.* [16], sewage is considered as the largest contributor of organic pollution to water resources around the world. In addition, the largest proportion (64.4%) of total waste in Malaysia is also contributed by sewage, followed by animal husbandry wastes (32.6%), agricultural resources (1.7%) and lastly industrial waste (1.3%) in terms of biochemical oxygen demand (BOD) load. If the wastewater is not well treated and directly discharged into the environment, water-borne diseases will be spread.

It all started around the early twentieth when many researchers are trying to design an environmental friendly system utilizing biological treatment. This biological treatment was constructed to preserve the environment and to treat the wastewater. Since then, the treatment system has become the foundation of many wastewater treatment systems worldwide. The treatment method involves retaining bacteria naturally present in high concentration or population of wastewater treatment plant. It comprises several types of bacteria and protozoa found in treatment plant collectively referred as activated sludge [2]. The essence of the treatment is that bacteria break down organic carbon as a source of energy and food. As a result, bacteria can grow while the wastewater is being cleansed. Treated sewage at treatment plant is usually safe to be discharged into rivers or sea. Although the idea of applying bacteria into this treatment looks simple, the process is actually more complex considering many parameters that affect the treatment system. These include the changes in composition of bacteria, external factors such as weather, temperature and sunlight for an exposed treatment plant, as well as the changes in sewage passing through the treatment plant.

Industrial wastewater containing toxic chemicals at very high concentration may also affect the treatment process as the bacteria are only able to slowly degrade the pollutant. This toxic shock may inhibit the growth of bacteria, resulting in the untreated effluent discharged by the treatment plant to the environment. In this case, treatment plant will become malfunction until the dead bacteria are replaced with the new bacterial seeds.

Normally, the composition of effluents discharged to receiving waters is monitored by the national environment agencies. For example in Malaysia, the water quality standard must be in agreement with the Water Environment Partnership in Asia (WEPA). The legislation is concerned with the prevention of pollution and therefore sets concentration limits on dissolved organic carbon as biochemical oxygen demand (BOD), chemical oxygen demand (COD), nitrogen and phosphates (PO_4^{-3}) that can cause eutrophication if excessive [2].

Most wastewaters largely comprised organic carbon either in solution or particulate matter. Extremely small particle ranging from one nanometer (nm) to 100 micrometers (μm) will remain in colloidal suspension and become adsorbed to the activated sludge during treatment. It is quite straightforward to experimentally measure the amount of organic carbon in the sewage. There are two types of different measurement namely combustion and chemical oxidation, which can be described as total organic carbon (TOC) and chemical oxygen demand (COD), respectively. TOC is calculated by the accumulation of carbon dioxide (CO_2) produced by combustion at a very high temperature. Meanwhile for COD, the sample is heated in strong sulphuric acid (H_2SO_4) containing potassium dichromate ($K_2Cr_2O_7$). The oxidized carbon is determined by the amount of dichromate used in the reaction and the result is expressed in unit of oxygen [2].

However, both measurements have their own weaknesses, since they overestimate the organic carbon compounds that cannot be broken down biologically. Conversely, some aromatic compounds including benzene (C_6H_6), toluene (CH_3) and pyridine (C_5H_5N) are only partly oxidized in the procedure. Overall, TOC and COD will overestimate the carbon that can be removed by activated sludge. The more accurate method that can be used to determine the biodegradable carbon is the 5-day biological oxygen demand (BOD_5) (refer to Figure 1.1). This method is used to measure the oxygen uptake over a 5-day period by a small seed of bacteria that are confined in the dark, in a bottle containing the wastewater. During this time, the biodegradable organic carbon was taken up, and a corresponding decrease in the dissolved oxygen can be observed as some carbons were used for the respiration of the bacteria. The values obtained for BOD_5 are always lower than those for COD by two reasons. The first reason is that activated sludge bacteria cannot chemically degrade some of the compounds oxidized in the COD test, while the second one is that some of the carbon removed during the BOD test is not oxidized, but ends up in a new bacterial biomass. Thus, it can be stated that BOD is the measurement of biodegradable carbon that is actually oxidized by the bacteria [17].

The mathematical models of the activated sludge process from previous studies will be discussed in Chapter 2. This section also includes relevant mathematical background and concepts necessary for modelling the process and interpret the solutions. The modelling equations as well as constitutive relations represent the physical effects that are present in the process. Basically, solving these mathematical models, requires the discretization in both space and time.

There are many studies done using ordinary differential equation models for constructed wetland including that conducted by Rousseau [18]. However, it was found that these studies have not considered the stability analysis of the models including the steady-state and bifurcation analyses, which are crucial in determining the reliability and importance of the proposed models. Therefore, a model is proposed introducing a more simplified form for mass transport of oxygen through plant roots which is the difference between saturated oxygen concentration and the current concentration of dissolved oxygen (DO) compared to exponential term used by Rousseau [18].

Meanwhile, for oxidation pond model, the advection-diffusion-reaction equation model proposed by Pimpunchat *et al.* [19] is extended to three state variables in this study, which are phototrophic bacteria (PSB), chemical oxygen demand (COD) and DO. On top of that, it has been discovered that there is no available simulation published on the combined treatment systems (pond-constructed wetland) despite the experimental studies done [20–23]. Thus, this present study is conducted simulating the aforementioned system to act as a reference in improving the efficiency of wastewater treatment system.

1.3 Problem Statement

The sources of wastewater can be either from the industrial or non-industrial area. The major source of pollution comes from the non-industrial parts and the waste produced by human contributes the largest part of the non-industrial pollution. If it cannot be well handled, many problems will arise including epidemics. Hence, pollution level should be maintained at a very low level or at least controllable. Wastewater with human sources can be efficiently treated by oxidation pond. In addition, wetland system can effectively control industrial wastewater; for instance, the wastewater discharging from construction sites. Currently, there are several mathematical models available for simulating oxidation pond process where some important parameters are considered such as bacteria (cleansing agent), pollutants and dissolved oxygen (DO). However, previous results did not provide good approximation on the required parameters. Moreover, stability analysis was rarely considered for constructed wetland models. However, the steady-state and bifurcation analyses are usually crucial in determining the reliability of the models that is under study. Thus, dynamic mathematical models are developed in this study to allow the simulation and prediction of wastewater treatment process for both oxidation pond and CW case studies. Furthermore, the nonlinear system of ordinary differential equations (ODE) using multiple substrate limiting factors with interactive reactions and partial differential equations (PDE) using advection-diffusion-reaction equations are implemented for CW and oxidation pond, respectively.

1.4 Objectives of the Research

The objectives of this research are as follows:

1. To develop a mathematical model for wastewater treatment process of an oxidation pond with microbe-based product (mPHO) produced by Johor Biotechnology & Biodiversity Corporation (J-Biotech).

2. To develop a mathematical model for horizontal subsurface flow constructed wetlands system with vegetation type (*Typha Angustifolia*) based on the study given in Chew [15].
3. To construct numerical simulation and analysis of the models for validation.
4. To combine wastewater treatment processes by using pond-constructed wetland system.

1.5 Scope of the Research

This research is divided into two major parts, which are oxidation pond treatment process using microbe-based product in liquid form (mPHO) and horizontal subsurface flow constructed wetland system using plant type (*Typha Angustifolia*).

1.6 Significance of the Research

The significance of this research are as follows:

1. This study emphasises the ability of mathematical modelling to facilitate the process of wastewater treatment system using oxidation pond, which has become an important treatment procedure in Malaysia governed by Indah Water Consortium (IWK).
2. This study provides a mathematical model to understand the wastewater treatment process of constructed wetland and allow it to predict the output if the model is used for a long period.
3. The mathematical models are able to help the preservation and conservation of environment to run smoothly in the sense that it can save a lot of maintenance cost as well as being more efficient.

1.7 Thesis Organization

This thesis is organized as follows:

The first chapter explains in depth on the issue of water pollution, which has become our main concern. It includes motivation, the background of study, problem statement, objectives, significance as well as the scope of study to be carried out.

Chapter 2 reviews the biological processes related to water treatment process as other studies have obtained relevant methods to treat wastewater in the past until the present such as that conducted by Rousseau [18], Pimpunchat *et al*. [19] and Wang *et al*. [23].

In Chapter 3, the proposed solutions are discussed in detail. It covers the construction of model, parameter estimation and the method for nondimensionalization of the model.

Chapter 4 presents the oxidation pond problems involving ODE as well as the PDE models. This chapter presents four types of different models. The first model is three competing species model, which includes three types of bacterium known as E.coli, Coliform, and PSB. The second model is the coupled-reaction equations model, which includes COD, DO, PSB, and Coliform. The other two are the PDE models comprising advection-reaction equations and advection-diffusion-reaction equations models for competing species and transport of pollutant, respectively.

Chapter 5 presents the constructed wetland problems. In this chapter, three models are constructed based on the ODE model. The first model is the nonlinear ordinary differential equations model consisting six state variables. The second model is considered as the simplified model for the purpose of model analysis. Thus, the state variables for the model were reduced to only three variables including DO, mixed culture bacteria (cleansing agent) and COD. Lastly, the dimensionless model is analysed to show the behaviour of the proposed model.

Chapter 6 presents the simulation of pond-constructed wetland system. The simulation was carried out by combining an advection-diffusion-reaction equations model for oxidation pond with the simplified model for constructed wetland.

Finally, Chapter 7 summarizes the study with a conclusion, re-stating the contributions as well as some suggestions for future studies.

REFERENCES

1. Abdel-Shafy, H. I. and Salem, M. A. Efficiency of Oxidation Ponds for Wastewater Treatment in Egypt. In: Wastewater Reuse-Risk Assessment, Decision-Making and Environmental Security. *Springer*, 2007. 175-184.
2. Davies, P. S. The Biological Basis of Wastewater Treatment. *Strathkelvin Instruments Ltd*, 2005. 3-11.
3. Ockendon, J. R., Wake, G., Teo, K. L., Loxtan, R., Araujo, A., Widodo, B., Murid, A. H. M., Hoe, Y. S., Banitalebi, A., Johar, F. and Siam, F. M. Mathematical Modeling and Optimization for Biological-Based Treatment of Taman Timor Oxidation Pond, Johor. *Malaysian 2nd Mathematics in Industry Study Group (MISG 2014)*. March 17-21, 2014. Universiti Teknologi Malaysia, Johor Bahru, Malaysia. 2014. 1-37.
4. Maehlum, T. Treatment of Landfill Leachate in On-Site Lagoons and Constructed Wetlands. *Wat. Sci. Tech.*, 1995. 32(3): 129-135.
5. Campbell, C. S. and Ogden, M. H. *Constructed Wetlands in the Sustainable Landscape*. USA: John Wiley & Sons. 1999
6. Knight, R. L. Wildlife Habitat and Public Use Benefits of Treatment Wetlands. *Water Science and Technology*, 1997. 35(5): 35-43.
7. Kivaisi, A. K. The Potential for Constructed Wetlands for Wastewater Treatment and Reuse in Developing Countries: A Review. *Ecological Engineering*, 2001. 16(4): 545-560.
8. Kadlec, R. H. and Knight, R. L. *Treatment Wetlands*. Florida, USA: Lewis-CRC Press. 1996

9. Gearheart, R. A. The Use of Free Surface Constructed Wetlands as an Alternative Process Treatment Train to Meet Unrestricted Water Reclamation Standards. *Wat. Sci. Tech.*, 1999. 40(4-5): 375-382.
10. Pankratz, T. M. *Environmental Engineering Dictionary and Directory*. New York: Lewis Publishers. 2001
11. Karlovsky, P. Secondary Metabolites in Soil Ecology. *Springer*, 2008. 1-19.
12. Wang, L. K., Tay, J. H., Tay, S. T. L. and Hung, Y. T. *Environmental Bioengineering*. New York: Springer Science & Business Media. 2010
13. Dimirezen, D. and Aksoy, A. Accumulation of Heavy Metals in Typha Angustifolia (L.) and Potamogeton Pectinatus (L.) Living in Sultan Marsh (Kayseri, Turkey). *Chemosphere*, 2004. 56: 685-696.
14. Kaseva, M. E. Performance of a Sub-Surface Flow Constructed Wetlands in Polishing Pre-treated Wastewater-A Tropical Case Study. *Water Research*, 2004. 38(3): 681-687.
15. Chew Ai Ling. *Nutrient Removal from Leachate Using Horizontal Subsurface Constructed Wetlands*. Master Dissertation. Universiti Teknologi Malaysia, Malaysia; 2005
16. Fakhru′l-Razi, A., Zahangir A. M., Idris, A., Abd-Aziz, S. and Molla, A. H. Filamentous Fungi in Indah Water Konsortium (IWK) Sewage Treatment Plant for Biological Treatment of Domestic Wastewater Sludge. *Journal of Environmental Science and Health, Part A*, 2002. 37(3): 309-320.
17. Gray, N. F. *Biology of Wastewater Treatment, Series on Environmental Science and Management*. London: Imperial College Press. 2004
18. Diederik Rousseau. *Performance of Constructed Treatment Wetlands: Model-Based Evaluation and Impact of Operation and Maintenance*. Ph.D. Thesis. Ghent University, Ghent, Belgium; 2005
19. Pimpunchat, B., Sweatman, W. L., Wake, G. C., Triampo, W. and Parshotam, A. A Mathematical Model for Pollution in a River and Its Remediation by Aeration. *Applied Mathematics Letters*, 2009. 22: 304-308.

20. Wang, X., Bai, X., Qiu, J. and Wang, B. Municipal Wastewater Treatment with Pond-Constructed Wetland System: A Case Study. *Water Science and Technology*, 2005. 51(12): 325-329.
21. Peng, J. F., Wang, B. Z. and Wang, L. Multi-Stage Ponds-Wetlands Ecosystem for Effective Wastewater Treatment. *Journal of Zhejiang University. Science. B*, 2005. 6(5): 346-352.
22. Matamoros, V. and Salvadó, V. Evaluation of the Seasonal Performance of a Water Reclamation Pond-Constructed Wetland System for Removing Emerging Contaminants. *Chemosphere*, 2012. 86(2): 111-117.
23. Wang, X., Tian, Y., Zhao, X., Peng, S., Wu, Q. and Yan, L. Effects of Aeration Position on Organics, Nitrogen and Phosphorus Removal in Combined Oxidation Pond-Constructed Wetland Systems. *Bioresource Technology*, 2015. 198: 7-15.
24. Beck, M. B. and Young, P. C. A Dynamic Model for DO-BOD Relationships in a Non-Tidal Stream. *Water Research*, 1975. 9(9): 769-776.
25. Moreno-Grau, S., Garcia-Sanchez, A., Moreno-Clavel, J., Serrano-Aniorte, J. and Moreno-Grau, M. D. A Mathematical Model for Waste Water Stabilization Ponds with Macrophytes and Microphytes. *Ecological Modelling*, 1996. 91(1): 77-103.
26. Kayombo, S., Mbwette, T. S. A., Mayo, A. W., Katima, J. H. Y. and Jorgensen, S. E. Modelling Diurnal Variation of Dissolved Oxygen in Waste Stabilization Ponds. *Ecological Modelling*, 2000. 127(1): 21-31.
27. Senzia, M. A., Mayo, A. W., Mbwette, T. S. A., Katima, J. H. Y. and Jorgensen, S. E. Modelling Nitrogen Transformation and Removal in Primary Facultative Ponds. *Ecological Modelling*, 2002. 154(3): 207-215.
28. Abdulkareem, A. Modeling of Microbial Growth in a Wastewater Treatment Plant: A Case Study of Textile Industry in Kaduna, Nigeria. *AU Journal of Technology*, 2004. 8: 45-54.
29. Beran, B. and Kargi, F. A Dynamic Mathematical Model for Wastewater Stabilization Ponds. *Ecological Modelling*, 2005. 181: 39-57.
30. Abbas, H., Nasr, R. and Seif, H. Study of Waste Stabilization Pond Geometry for the Wastewater Treatment Efficiency. *Ecological Engineering*, 2006. 28(1): 25-34.

31. Peng, J. F., Wang, B. Z., Song, Y. H. and Yuan, P. Modeling N Transformation and Removal in a Duckweed Pond: Model Development and Calibration. *Ecological Modelling*, 2007. 206(1): 147-152.
32. Bhutiani, R. and Khanna, D. Ecological Study of River Suswa: Modeling DO and BOD. *Environmental Monitoring and Assessment*, 2007. 125: 183-195.
33. Dippner, J. W. Mathematical Modeling of the Transport of Pollution in Water. In: Kuchment, L. S. and Singh, V. P. *Hydrological Systems Modeling*. United Kingdom: EOLSS Publishers. 204; 2009.
34. Seetha, N., Bhargava, R. and Kumar, P. Effect of Organic Shock Loads on a Two-Stage Activated Sludge-Biofilm Reactor. *Bioresource Technology*, 2010. 101(9): 3060-3066.
35. Thalla, A. K., Bhargava, R. and Kumar, P. Nitrification Kinetics of Activated Sludge-Biofilm System: A Mathematical Model. *Bioresource Technology*, 2010. 101(15): 5827-5835.
36. Mwegoha, W. J. S., Kaseva, M. E. and Sabai, S. M. M. Mathematical Modeling of Dissolved Oxygen in Fish Ponds. *African Journal of Environmental Science and Technology*, 2010. 4(9): 625-638.
37. Abbasi T. and Abbasi S. A. Enhancement in the Efficiency of Existing Oxidation Ponds by Using Aquatic Weeds at Little or No Extra Cost–The Macrophyte-Upgraded Oxidation Pond (MUOP). *Bioremediation Journal*, 2010. 14(2): 67-80.
38. Sebenik Paul Gregory. *Relationships of Dissolved Oxygen and Biochemical Oxygen Demand in Sewage Effluent Releases*. Ph.D. Thesis. The University of Arizona, Tucson, Arizona; 1975
39. Edelstein-Keshet, L. *Mathematical Models in Biology*. Philadelphia: SIAM. 1988
40. Morley, D. A. *Mathematical Modelling in Water and Wastewater Treatment*. London: Applied Science Publishers. 1979
41. Seviour, R. J. and Nielsen, P. H. *Microbial Ecology of Activated Sludge*. New York: IWA publishing. 2010
42. Wu, B. and Chen, Z. An Integrated Physical and Biological Model for Anaerobic Lagoons. *Bioresource Technology*, 2011. 102(8): 5032-5038.

43. Ssegane, H., Tollner, E. W., Mohamoud, Y. M., Rasmussen, T. C. and Dowd, J. F. Advances in Variable Selection Methods II: Effect of Variable Selection Method on Classification of Hydrologically Similar Watersheds in Three Mid-Atlantic Ecoregions. *Journal of Hydrology*, 2012. 438-439(2012): 26-38.
44. Huesemann, M. H., Van Wagenen, J., Miller, T., Chavis, A., Hobbs, S. and Crowe, B. A Screening Model to Predict Microalgae Biomass Growth in Photobioreactors and Raceway Ponds. *Biotechnology and Bioengineering*, 2013. 110(6): 1583-1594.
45. Ukpaka, C. P. The Concept of Chemical and Biochemical Oxygen Demand in Inhibiting Crude Oil Degradation in Fresh Water Pond System. *Merit Research Journal of Environmental Science and Toxicology*, 2013. 1(7): 36-46.
46. Ahmed, M. G. and Mayo, A. W. Modeling of Mortality Rate of Heterotrophic Bacteria Due to Chromium in Waste Stabilization Pond. *Journal of Science and Technology*, 2013. 14(2).
47. Khusravi, R., Khodadadi, M., Gholizadeh, A., Mehrizi, E. A., Shahriary, T. and Shahnia, A. BOD_5 Removal Kinetics and Wastewater Flow Pattern of Stabilization Pond System in Birjand. *European Journal of Experimental Biology*, 2013. 3(2): 430-6.
48. Martinez, F. C., Cansino, A. T., Garcia, M. A., Kalashnikov, V. and Rojas, R. L. Mathematical Analysis for the Optimization of a Design in a Facultative Pond: Indicator Organism and Organic Matter. *Mathematical Problems in Engineering*, 2014. 20.
49. Batstone, D. J., Puyol, D., Flores-Alsina, X. and Rodriguez, J. Mathematical Modelling of Anaerobic Digestion Processes: Applications and Future Needs. *Reviews in Environmental Science and Bio-Technology*, 2015. 14(4): 595-613.
50. Verbyla, M. E. and Mihelcic, J. R. A Review of Virus Removal in Wastewater Treatment Pond Systems. *Water research*, 2015. 15(71): 107-24.
51. Song, R. Y., Qian, Y. F., Zheng, L. J., Zhao, Y. P., Wang, X. and Wei, J. Diffusion of Single Oxidation Pond. *Thermal Science*, 2016. 20(3): 849-53.

52. Laber, J., Perfler, R. and Haberl, R. Two Strategies for Advanced Nitrogen Elimination in Vertical Flow Constructed Wetlands. *Water Science and Technology*, 1997. 35(5): 71-77.

53. Cooper, P. A Review of the Design and Performance of Vertical-Flow and Hybrid Reed Bed Treatment Systems. *Water Science and Technology*, 1999. 40(3): 1-9.

54. Pastor, R., Benqlilou, C., Paz, D., Cardenas, G., Espuña, A. and Puigjaner, L. Design Optimisation of Constructed Wetlands for Wastewater Treatment. *Resources, Conservation and Recycling*, 2003. 37(3): 193-204.

55. Bader, F. G. Analysis of Double-Substrate Limited Growth. *Biotechnology and Bioengineering*, 1978. 20(2): 183-202.

56. Chen-Charpentier, B. Numerical Simulation of Biofilm Growth in Porous Media. *Journal of Computational and Applied Mathematics*, 1999. 103(1): 55-66.

57. Rousseau, D. P. L., Vanrolleghem, P. A. and Pauw, N. D. Model-Based Design of Horizontal Subsurface Flow Constructed Treatment Wetlands: A Review. *Water Research*, 2004. 38(6): 1484-1493.

58. Tomenko, V., Ahmed, S. and Popov, V. Modelling Constructed Wetland Treatment System Performance. *Ecological Modelling*, 2007. 205(3): 355-364.

59. Langergraber, G. Modeling of Processes in Subsurface Flow Constructed Wetlands: A Review. *Vadose Zone Journal*, 2008. 7(2): 830-842.

60. Samsó, R. and Garcia, J. The Cartridge Theory: A Description of the Functioning of Horizontal Subsurface Flow Constructed Wetlands for Wastewater Treatment, Based on Modelling Results. *Science of the Total Environment*, 2014. 473: 651-658.

61. Albalawneh, A., Chang, T. K., Chou, C. S. and Naoum, S. Efficiency of a Horizontal Sub-Surface Flow Constructed Wetland Treatment System in an Arid Area. *Water*, 2014. 8(2): 2-14.

62. Abed, S. N., Mahmoud, N. and Sharma, S. K. Potential of Horizontal Subsurface-Flow Constructed Wetlands for Polishing of Treated Sewages. *Journal of Environmental Engineering*, 2014. 142(6): 1-7.

63. Haberman, R. *Mathematical Models: Mechanical Vibrations, Population Dynamics, and Traffic Flow*. Philadelphia: Society for Industrial and Applied Mathematics (SIAM). 1977
64. Bird, R. B. *Transport Phenomena*. 2nd. ed. New York: John Wiley & Sons. 2007
65. Ulf Jeppsson. *Modelling Aspects of Wastewater Treatment Processes*. Ph.D. Thesis, Lund University, Sweden; 1996
66. Fowler, A. C. *Mathematical Models in the Applied Sciences*. USA: Cambridge University Press. 1997
67. Smith, G. D. *Numerical Solution of Partial Differential Equations: Finite Difference Methods*. 3rd. ed. Oxford: Clarendon Pr. 1985
68. Klipp, E. *Systems Biology in Practice: Concepts, Implementation and Application*. Weinheim : Wiley-VCH. 2005
69. Bellman, R. On the Poincaré-Lyapunov Theorem. *Nonlinear Analysis: Theory, Methods & Applications*, 1980. 4(2): 297-300.
70. Mark Kot. *Elements of Mathematical Ecology*. Cambridge: Cambridge University Press. 2001
71. Hom, R. A. and Johnson, C. R. *Topics in Matrix Analysis*. New York: Cambridge UP. 1991
72. Beck, M. B. Identification, Estimation and Control of Biological Waste-Water Treatment Processes. *In IEE Proceedings D-Control Theory and Applications*, 1986. 133(5): 254-264.
73. Baker, F. B. *Item Response Theory: Parameter Estimation Techniques*. 2nd. ed. New York: Marcel Dekker. 2004
74. Samarskii, A. A. and Mikhailov, A. P. *Principles of Mathematical Modelling: Ideas, Methods, Examples*, CRC Press. 2001
75. Steel and Robert, G. D. *Principles and Procedures of Statistics: A Biometrical Approach*, McGraw-Hill, New York. 1997
76. Butcher, J. C. *Numerical Methods for Ordinary Differential Equations*. Sussex: John Wiley & Sons, Ltd 2008

77. Crank, J. and Nicolson, P. A Practical Method for Numerical Evaluation of Solutions of Partial Differential Equations of the Heat-Conduction Type. *Advances in Computational Mathematics*, 1996. 6(1): 207-226.
78. Strikwerda, J. C. *Finite Difference Schemes and Partial Differential Equations*. Philadelphia: Society for Industrial and Applied Mathematics. 2004
79. Holmes, M. H. *Introduction to Numerical Methods in Differential Equations*. New York: Springer. 2007
80. Thomas, J. W. *Numerical Partial Differential Equations: Finite Difference Methods*. New York : Springer-Verlag. 1995
81. May, R. M. and Leonard, W. J. Nonlinear Aspects of Competition between Three Species. *SIAM Journal on Applied Mathematics*, 1975. 29: 243-253.
82. Abramowitz, M. and Stegun, I. A. *Handbook of Mathematical Functions: With Formulas, Graphs, and Mathematical Tables*. New York: Dover Publications. 1965
83. Burden, R. L., Faires, J. D. and Reynolds, A. C. *Numerical Analysis*, Boston: Prindle, Weber & Schmidt Inc. 1981
84. Lagally, E. (ed.) *Microfluidics and Nanotechnology: Biosensing to the Single Molecule Limit*. Boca Raton, FL: CRC Press. 2014
85. Shukla, J. B., Goyal, A., Tiwari, P. K. and Misra, A. K. Modeling the Role of Dissolved Oxygen-Dependent Bacteria on Biodegradation of Organic Pollutants. *World Scientific, International Journal of Biomathematics*, 2014. 7(01):1450008(1-16).
86. Howison, S. *Practical Applied Mathematics: Modelling, Analysis, Approximation*. United Kingdom: Cambridge University Press. 2005
87. Rhee, H. K., Aris, R. and Amundson, N. R. *First-Order Partial Differential Equations: Theory and Application of Single Equations*. Englewood Cliffs, N.J.: Prentice-Hall. 1986
88. Tchobanoglous, G. and Burton, F. *Wastewater Engineering: Treatment, Disposal, and Reuse*. 3rd. ed. New York: McGraw-Hill. 1991
89. Henze, M. (Ed.). *Activated Sludge Models ASM1, ASM2, ASM2d and ASM3*. London: IWA publishing. 2000

90. Romero, J. A., Brix, H. and Comin, F. A. Interactive Effects of N and P on Growth, Nutrient Allocation and NH_4 Uptake Kinetics by Phragmites Australis. *Aquatic Botany*, 1999. 64(3): 369-380.
91. Kadlec, R. H. and Wallace S. *Treatment Wetlands*. Boca Raton, FL: CRC Press. 2009
92. Hammer, D. A. (Ed.). *Constructed Wetlands for Wastewater Treatment: Municipal, Industrial and Agricultural*. Chelsea, Mich.: Lewis Publishers. 1989
93. Dunnivant, F. M. and Anders, E. *A Basic Introduction to Pollutant Fate and Transport*. New Jersey: John Wiley & Sons Inc. 2006
94. Cho, S. H., Colin, F., Sardin, M. and Prost, C. Settling Velocity Model of Activated Sludge. *Water Research*, 1993. 27(7): 1237-1242.